Available Online at http://www.journalajst.com

ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ASIAN JOURNAL
ISSN: 0976-3376

*Asian Journal of Science and Technology*
*Vol. 4, Issue 07, pp.004-009, July, 2013*

## RESEARCH ARTICLE

# SECOND LAW ANALYSIS OF HEAT AND HALL EFFECT OF AN OSCILLATING PLATE IN A POROUS MEDIUM

***[1]Okedoye M. Akindele and [2]Ayandokun O. Oluropo**

[1]Department of Mathematics, Covenant University Km. 10, Idiroko Road, P. M. B. 1023, Ota, Nigeria
[2]Department of Mathematics, Emmanuel Alayande College of Education, Oyo, Nigeria

## ARTICLE INFO

*Article History:*
Received 02[nd] April, 2013
Received in revised form
11[th] May, 2013
Accepted 20[th] June, 2013
Published online 19[th] July, 2013

*Key words:*
Stokes problem; Porous medium;
Hall effects; Heat transfer;
Entropy generation; Fluid friction;
MHD flow; Viscosity; Irreversibility.

## ABSTRACT

This paper reports calculation of the entropy generation due to heat transfer and fluid friction on a porous plate for the conjugate problem of an electrically conducting fluid in the presence of strong magnetic field by introducing the Hall currents. The momentum and energy balance equations are solved analytically, using perturbation technique. The fluid half-space is considered to be porous. The influences of the thermal Grashof numbers, heat generation/absorption and Hartmann number on total entropy generation were investigated, reported and discussed.



## INTRODUCTION

The study of magnetohydrodynamic flows with Hall currents has important engineering applications in problems of magnetohydro- dynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics [1-3]. Unfortunately, the results of these investigations cannot be applied to the flow of ionized gases. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiralling of electrons and ions about the magnetic lines of force before severing collisions; also, a current is induced in a direction normal to both the electric and magnetic fields. The phenomena, well known in the literature, are called the Hall Effect [4 - 7]. Recently, several researchers have worked on Hall Effect of an oscillating plate in a porous medium and Okedoye [8] has a good review of some of this work. In the traditional approach in numerical computation of double diffusive convection problems, the quantities to be computed are usually temperature, pressure, concentration, mass and heat flow rates, but infrequently involving entropy properties. The contemporary trend in the field of heat transfer and thermal designs is the second Law (of Thermodynamics) analysis and its design-related concept of entropy generation minimization [9]. Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Nag and Kumar [10] studied second Law optimization for convective heat transfer through a duct with constant heat flux.

***Corresponding author:** Okedoye M. Akindele
Department of Mathematics, Covenant University Km. 10, Idiroko Road, P. M. B. 1023, Ota, Nigeria

In their study, they plotted the variation of entropy generation versus the temperature difference of the bulk and the surface flow, using a dusty parameter. The dissipation of energy takes the form of a sum of products of conjugate forces and fluxes associated to the problem under consideration; this was presented by the text of De Groot [11]. Entropy generation in Magneto Hydro Dynamic (MHD) flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction was studied and reported by Okedoye *et al.* [12]. Although the various topics investigated about entropy generation and its minimization, the determination of total irreversibility in Heat and Hall Effect of an oscillating plate in a porous medium has not been encountered. In this context, the present investigation aims at obtaining an analytical determination of the entropy generation of Heat and Hall Effect of an oscillating plate in a porous medium.

**Nomenclature**

| | |
|---|---|
| $A_1$ | Rivlin-Ericksen tensor; |
| $B$ | magnetic field; |
| $B_0$ | applied magnetic field; |
| $E$ | electric field current; |
| $e$ | electron charge; |
| $grad$ | the gradient operator; |
| $J$ | the current density; |
| $K$ | constant permeability; |
| $M$ | MHD parameter; |
| $n_e$ | number density of electrons; |

| | |
|---|---|
| $p$ | scalar pressure; |
| $p_e$ | electron pressure; |
| $t$ | time; |
| $T_c$ | Cauchy stress tensor; |
| $u_w$ | main stream velocity/free stream; |
| $u,\ v$ | the velocity components; |
| $V$ | velocity vector; |
| $v_w$ | suction/blowing velocity; |
| $x, y$ | the coordinate axis; |
| $C_p$ | specific heat, |
| $Q_{0'}$ | Heat source/Sink parameter |
| $\nabla$ | nabla/del operator; |
| $i = \sqrt{-1}$ | complex identity |

**Greek letters**

| | |
|---|---|
| $\beta$ | acceleration/deceleration parameter; |
| $\mu$ | dynamic viscosity; |
| $\mu_m$ | magnetic permeability; |
| $\upsilon$ | kinematic viscosity; |
| $\rho$ | density; |
| $\sigma$ | electrical conductivity; |
| $\tau_e$ | electron collision time; |
| $\tau_i$ | ions collision time; |
| $\varphi$ | porosity; |
| $\phi$ | Hall parameter; |
| $\omega_i$ | cyclotron frequency of ions; |
| $\omega_e$ | cyclotron frequency of electrons; |
| $\omega$ | oscillating frequency. |
| $\rho$ | fluid density |
| $\beta_\tau$ | coefficient of thermal expansion |
| $\Gamma$ | entropy generation |

**Formulation of the basic equations**

In Cartesian co – ordinate system, x-axis is assumed to be along the plate in the direction of the flow and y – axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In the analysis, it is assumed that the magnetic Reynold number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. Therefore, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. When $t = 0$, the temperature of the plate is instantaneously raised (or lowered) to $T_w$ and that the plate is accelerating with a velocity $u_w e^{(\beta_1 - i\omega)t}$ in its own plane. Making reference to Cowling [13], when the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current the ion-slip and thermoelectric effects are not included. Further, it is assumed $\omega_e \tau_e \approx O(1)$ and $\omega_i \tau_i << 1$, where $\omega_i$ and $\tau_i$ are the cyclotron frequency and collision time for ions respectively. We have also assumed that the flow is confined $(y > 0)$ in a porous medium with constant permeability $K(> 0)$ and porosity $\varphi(0 < \varphi < 1)$.The MHD equations governing the unsteady flow of an incompressible fluid together with Brinkman's empirical modification of Darcy's law are [8]

$$\nabla \cdot V = 0 \tag{2.1}$$

$$\rho\left(\frac{\partial V}{\partial t} + (V \cdot \nabla)V\right) = -\frac{\mu\varphi}{K}V + \mu\nabla^2 V - \frac{\sigma B_0^2}{1 - i\phi}V + g\beta(T - T_\infty) \tag{2.2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + (V \cdot \nabla)T\right) = \alpha\nabla^2 T + (T - T_\infty)Q_0 \tag{2.3}$$

On disregarding the Joulean heat dissipation, the boundary conditions are given by

$$\left.\begin{aligned} &u = 0,\ \ T = T_\infty,\ \ for\ \ all\ \ y,\ t \le 0 \\ &u = u_w e^{(\beta_1 - i\omega)t},\ v = -v_w, t > 0 \\ &T = T_w + \varepsilon e^{(\beta_1 - i\omega)t},\ \ y = 0,\ t > 0 \\ &u = 0,\ T = T_\infty,\ \ as\ \ y \to \infty,\ t > 0 \end{aligned}\right\} \tag{2.4}$$

We consider the fluid lying in the upper half space. The $x-$ axis is taken along the flow direction and $y-$ axis perpendicular to it: such that there is simultaneous suction/blowing at the boundary $y = 0$. In fact, it follows from the continuity equation (2.1) that

$$\frac{\partial v}{\partial y} = 0$$

which implies $v = v_w = const$

so that the velocity field takes the form

$$V = (u(y,t), v_w) \tag{2.5}$$

Let us introduce the non-dimensional variables

$$u' = \frac{u}{u_w},\ t' = \frac{t v_w^2}{\upsilon},\ y' = \frac{y v_w}{\upsilon},\ \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{2.6}$$

where all the physical variables have their usual meanings.

With the help of (2.5), (2.6), on dropping primes ( $'$ ) the governing equations (2.2) and (2.3) with the boundary conditions (2.4) reduce to

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\left(\lambda+\frac{M^2}{1-i\phi}\right)u+Grt\theta \qquad (2.7)$$

$$\Pr\left(\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}\right)=\frac{\partial^2 \theta}{\partial y^2}-\Pr\beta\theta \qquad (2.8)$$

$$\left.\begin{array}{r} u=0,\ \ \theta=0,\quad for\,all\ y,\ t\le 0 \\ u=e^{(\beta_1-i\omega)t},\ \theta=1+\varepsilon e^{(\beta_1-i\omega)t},\ y=0,\ t>0 \\ u\to 0, \theta\to 0, as\ y\to\infty,\ t>0 \end{array}\right\} \qquad (2.9)$$

Where the flow control parameters are as defined below:

$$Gr\tau=\frac{g\beta_\tau\left(T_w-T_\infty\right)v}{\rho v_w^3},\ M=\frac{\sigma B_0^2\upsilon}{\rho v_w^2},$$

$$\Pr=\frac{\mu c_p}{\alpha_0},\quad \beta=\frac{Q\mu\upsilon}{\alpha_0 v_w^2\rho}, \lambda=\frac{\upsilon^2\varphi}{v_w^2 K}$$

where *Pr*, *Grt*, $\lambda$, $\beta$ and $M$ are Prandtl number, Grashof number for heat transfer, Porosity parameter, heat generation/absorption and Hartmann's number respectively.

## METHOD OF SOLUTION

To solve the problem posed in equations (2.7) – (2.9), we seek a perturbation series expansion in the limit of $\varepsilon$ for our dependent variables, ([15]).

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+o(\varepsilon^2)+.... \\ \theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y)+o(\varepsilon^2)+.... \end{array}\right\} \qquad (3.1)$$

Substituting equations (2.7) and (2.9) and the expression for the stream into equations (3.1), equating the harmonic and non – harmonic terms and neglecting the coefficient of $\varepsilon^2$, we obtain the equations governing the steady state motion and the equations governing the transient.

$$\frac{d^2\theta_0}{dy^2}+\Pr\frac{d\theta_0}{dy}-\Pr\beta\theta=0 \qquad (3.2)$$
$$\theta_0(0)=1,\ \theta_0(y)\to 0\ \ as\, y\to\infty$$

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-\left(\frac{M^2}{1-i\phi}+\lambda\right)u_0=-Gr\tau\theta_0 \qquad (3.3)$$
$$u_0(y)=e^{(\beta_1-i\omega)t}\ \ at\, y=0,\ \ u_0(y)\to 0\ \ as\, y\to\infty$$

and

$$\frac{d^2\theta_1}{dy^2}+\Pr\frac{d\theta_1}{dy}-\Pr(i\omega+\beta)\theta_1=0 \qquad (3.4)$$
$$\theta_1(y)=e^{(\beta_1-i\omega)t}\ at\, y=0,\ \theta_1(y)\to 0\ as\, y\to\infty$$

$$\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-\left(\frac{M^2}{1-i\phi}+\lambda+i\omega\right)u_1=-Gr\tau\theta_1 \qquad (3.5)$$
$$u_1(y)=0\ \ at\, y=0,\ \ u_1(y)\to 0\ \ as\, y\to\infty$$

These sets of equations are now solved analytically for the velocity and the temperature fields. The solutions of equations (3.2) – (3.5) are

$$\begin{array}{l}\theta_0(y)=e^{-ny},\ \ u_0(y)=a_2\,e^{-my}+a_3\,e^{-ny} \\ \theta_1(y)=e^{-n_1 y},\ \ u_1(y)=a_6\,e^{-m_1 y}+a_7\,e^{-n_1 y}\end{array} \qquad (3.6)$$

where

$$n=\frac{1}{2}\left(\Pr+\sqrt{\mathbf{Pr}^2-4\mathbf{P}r\beta}\right),$$

$$n_1=\frac{1}{2}\left(\Pr+\sqrt{\Pr^2-4\Pr\left(\beta-\beta_1+i\omega\right)}\right),$$

$$m=\frac{1}{2}\left(1+\sqrt{1+4\left(\lambda+\frac{M^2}{1-i\phi}\right)}\right),$$

$$m_1=\frac{1}{2}\left(1+\sqrt{1+4\left(\lambda+\frac{M^2}{1-i\phi}+\beta_1-i\omega\right)}\right),$$

with $\quad a_2=e^{(\beta_1-i\omega)t}-a_3,\ a_6=-a_7,$

$$a_3=\frac{-Grt}{n^2-n-\left(\lambda+\frac{M^2}{1-i\phi}\right)},$$

$$a_7=\frac{-Grt}{n_1^{\,2}-n_1-\left(\lambda+\frac{M^2}{1-i\phi}+\beta_1-i\omega\right)},$$

The functions $u_0(y)$ and $\theta_0(y)$ are the mean velocity and the mean temperature fields respectively; and $u_1(y)$ and $\theta_1(y)$ are, respectively, the velocity oscillatory part and the temperature oscillatory part fields. Now substituting equations (3.8) into equation (3.1), we obtain the required expressions for temperature and velocity fields;

$$\theta(y,t)=e^{-ny}+\varepsilon e^{(\beta_1-i\omega)t}e^{-n_1 y} \qquad (3.7)$$

$$\begin{array}{l}u(y,t)=a_2e^{-my}+a_3e^{-ny} \\ \qquad +\varepsilon e^{(\beta_1-i\omega)t}\left(a_6e^{-m_1 y}+a_7e^{-n_1 y}\right)\end{array} \qquad (3.8)$$

## ENTROPY GENERATION RATE

For an incompressible Newtonian fluid, the local entropy generation rate is given by De Groot [11]:

$$\begin{array}{l}\Gamma=\frac{\mu}{T}\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) \\ \quad -\frac{q}{T^2}\left(\frac{\partial T}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha S_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) \\ \quad -\frac{1}{T}\sum_\alpha K_\alpha\mu_\alpha\end{array}$$

On the right hand side of the above equation, the first term is due to fluid friction, the second is due to mass diffusion and the third term is due to heat conduction. The fourth term is due to heat transfer induced by mass diffusion and the fifth is due to chemical reactions. In convective heat and mass transfer and MHD flow, irreversibility arises due to the heat transfer, the viscous effects and the mass transfer. The entropy generation rate is expressed as the sum of contributions due to viscous, thermal and diffusive effects, and thus it depends functionally on the local values of temperature, velocity and concentration in the domain of interest. According to Okedoye et.al [12], the characteristic entropy transfer rate is given by:

$$\Gamma_0 = k\left(\frac{\Delta T}{LT_0}\right)$$

Where *k*, *L*, *To* and $\Delta T$ are respectively, the thermal conductivity, the characteristic length of the enclosure, a reference temperature and a reference temperature difference.

In the case of non – reactive mixture, the heat due to diffusion is negligible, Okedoye *et al* [12] defined the dimensionless entropy Generation rate as

$$\Gamma_n = \left(\frac{\partial \theta}{\partial y}\right)^2 + \lambda_1\left(\frac{\partial u}{\partial y}\right)^2 \tag{4.1}$$

Dimensionless terms denoted $\lambda_1$, and called irreversibilities distribution ratio, is given by:

$$\lambda_1 = \frac{\mu T_0}{k}\left(\frac{a}{L(\Delta T)}\right)^2$$

Where $T_o$ is respectively the reference temperature, which in our case, the bulk temperature.

The local entropy generation rate is a function of temperature and velocity gradients in the *y* directions in the entire calculation domain.

Using equation (4.1), on substituting equations (3.7) and (3.8) for irreversibilities, we have

$$\begin{aligned}\Gamma = &\left(-ne^{-ny} - \varepsilon e^{(\beta_1 - i\omega)t} n_1 e^{-n_1 y}\right)^2 \\ &+ \lambda_1\Big(-a_2 m e^{-my} - a_3 n e^{-ny} \\ &- \varepsilon e^{(\beta_1 - i\omega)t}\left(a_6 m_1 e^{-m_1 y} + a_7 n_1 e^{-n_1 y}\right)\Big)\end{aligned} \tag{4.2}$$

Having obtained expressions an expression for the entropy generation, we then use a computer software package (Mapple 11 release) to build up the real and imaginary parts and their graphical representation is presented for analysis.

## DISCUSSION OF RESULT

The entropy equation given in (4.2) is general (describing the combined effects of heat generation/absorption, porosity parameter, MHD parameter, Hall parameter, thermal Grashof number and acceleration/deceleration) and is independent of the form of the steady solution. In order to point out the effects of various parameters on the Entropy generation rate, the following considerations are made: To be realistic, the values of Prandtl number are chosen to be Pr = 0.71 which represents air and Pr=0.015 for mercury at temperature $25^{o}$C and one atmospheric pressure. For small thermal Grashof number, there is practically little or no convection and the entropy generation due to fluid friction is zero, consequently the total entropy generation is reduced to the entropy generation due to heat transfer. At higher Grashof number heat transfer due to convection begins to play a significant role increasing the flow velocity and in turn the entropy generation due to the viscous effects. Also the isotherms are deformed increasing the temperature gradient and consequently the entropy generation due to heat transfer. The positive values indicate that the impulsive velocity of the limiting surface is in a direction opposite to that of the flow.

In Figure 1, we show the distribution of entropy generation at various times. It could be seen that entropy generation rate decreases as time or increases. Figure 2 display effect of thermal Grashof number on the entropy generation rate. It is shown that for the case of heating of the plate, the entropy generation rate increases while it reduces for the case of cooling of the plate as thermal Grashof number increases.

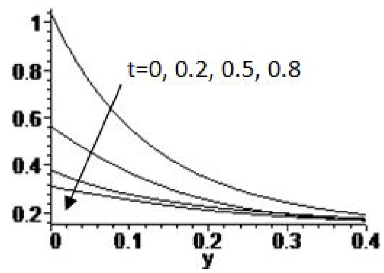


**Figure 1. Variation of Entropy Generation with time**

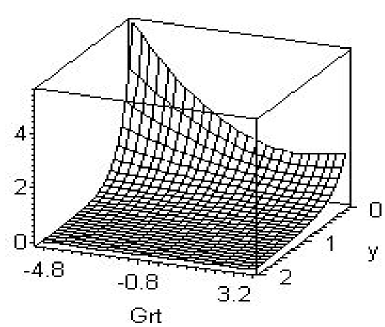


**Figure 2: Entropy Generation for various thermal Grashof number**

From Figure 3, we discovered that entropy generation rate decreases away from the surface and increases as Hartmann number increases. Hartmann number introduces a retarding force which makes the velocity to highest when Hartmann number is zero. As this opposing force increases the entropy generated increases, The same effect is observed in the case of plate acceleration as shown if Figure 4. We sow in Figure 5, the effect of porosity parameter on the entropy generation. It is discovered that entropy generation rate increases as porosity

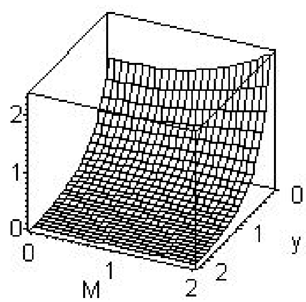


**Figure 3: Variation of Entropy Generation with Hartmann number**

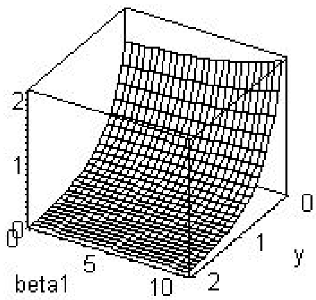


**Figure 4: Entropy Generation for various plate acceleration**

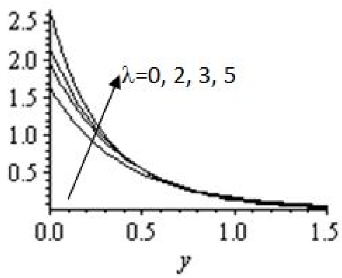


**Figure 5: Entropy Generation for various porosity parameter**

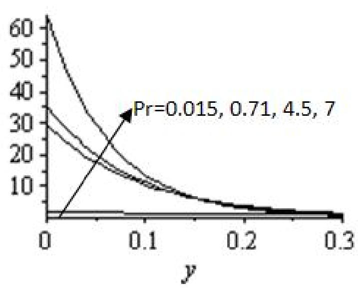


**Figure 6: Variation of Entropy Generation various Prandtl number**

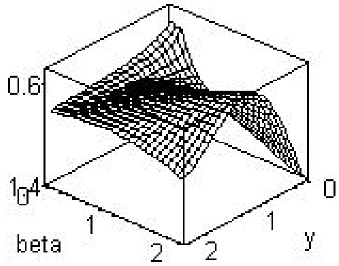


**Figure 7a**

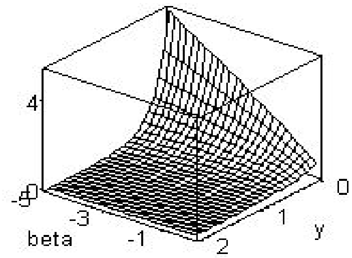


**Figure 7b**

**Figure 7(a, b): Variation of Entropy Generation with heat source/sink**

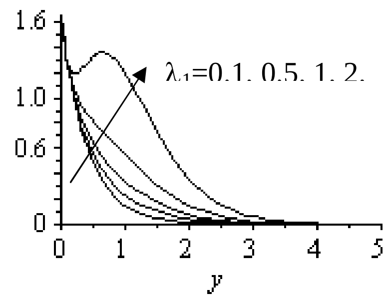


**Figure 8: Variation of Entropy Generation various Porosity parameter**

parameter increases. For a medium of high conductivity, irreversibility due to heat transfer is minimal and hence the entropy generation is minimal. Thus entropy generation rate increases as Prandtl number increase as shown in Figure 6. Figure 7 (a) and (b) sows entropy generation rate for the case of heat generation and eat absorption respectively. It is discovered that entropy generation decrease as heat generation increases for higher values of heat generation, the impulsive entropy of the limiting surface is in a direction opposite to that of the flow. While entropy generation rate increase as heat absorption increase. Variation of entropy generation rate wit irreversibility ratio is shown in Figure 8. It could be seen that entropy generation rate increase as irreversibility ratio increase.

In Table 1, the effect of Hall parameter $\phi$ on entropy generation rate is shown. It could be seen that entropy generation rate decreases as Hall parameter $\phi$ increase.

**Table 1: Variation of Entropy Generation various Hall parameter**

| $\phi$ =0 | $\phi$ =0.2 | $\phi$ =0.4 | $\phi$ =0.8 |
|---|---|---|---|
| 1.642473 | 1.642430 | 1.641686 | 1.641736 |
| 0.500729 | 0.500213 | 0.498768 | 0.494674 |
| 0.172236 | 0.172292 | 0.172415 | 0.172564 |
| 0.062959 | 0.063151 | 0.063653 | 0.065055 |
| 0.023391 | 0.023529 | 0.023903 | 0.025030 |
| 0.008630 | 0.008702 | 0.008903 | 0.009539 |
| 0.003132 | 0.003164 | 0.003256 | 0.003559 |

## REFERENCES


[1] Takhar H.S., Raptis A.A., Perdikis C.P. 1987: *MHD asymmetric flow past a semi-infinite moving plate.* Acta Mech. 65, 287 - 290.

[2] Lawrence P.S., Rao B.N. 1996. *Magnetohydro- dynamic flow past a semiinfinite moving plate.* Acta Mech. 117, 159 - 164.

[3] Pop I., Kumari M. and Nath G. 1994: *Conjugate MHD flow past a flat plate*, Acta Mech. 106, 215 - 220.

[4] Sato H. 1961: *The Hall effects in the viscous flow of ionized gas between parallel plates under transverse magnetic field*. J. Phys. Soc. Japan 16, 1427 – 1433.

[5] Hossain M.A. 1986: *Effect of Hall current on unsteady hydromagnetic free convection flow near an infinite vertical porous plate*, J. Phys. Soc. Japan 55 (7), 2183 – 2190.

[6] Asghar S., Mohyuddin M.R., and Hayat T., *Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disks*. Int. J. Heat Mass Transfer (to appear).

[7] Asghar, S. Parveen, S. Hanif, S. Siddiqui, A. M. and Hayat T. 2003: *Hall effects on the unsteady hydromagnetic flows of an Oldroyd-B fluid*, Int. J. Engng. Sci. 41, 609 – 619.

[8] Okedoye A. M. 2010. Heat and Hall Effect of an oscillating plate in a porous medium. J. Adv. in Theoretical and Mathematical Physics

[9] Bejan, A., 1996. Entropy generation minimization, CRC Press, Newyork,

[10] Nag, P. K. Kumar, N. 1989. Second Law optimization of convective heat transfer through a duct with constant heat flux. *Int. J. Energy Res*., 13, 537-543.

[11] De Groot, S. R. 1966, *Thermodynamics of irreversible processes*, North-Holland, Amesterdam, 28.

[12] Okedoye, A. M. Lamidi, O. T. and Ayeni, R. O. 2007. Entropy generation in MHD flow of stretched permeable surface in the presence of heat generation/absorption and chemical reaction. J. Math. Ass. Nigeria, vol. 34, Number 2A Mathematical series, pp. 52–57.

[13] Cowling, T.G. 1957. *Magnnetohydrodynamics*. Inter science, New York, pp. 101.

*******