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RESEARCH ARTICLE

HIGHLY HARSHAD NUMBERS

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ABSTRACT

In this paper we define a new type of number called the Highly Harshad number, that relates the sum of digits and prime divisors of a positive integer, then analyse some of its properties, its relation with prime numbers, and prove its infiniteness.

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INTRODUCTION

The Harshad number studied and defined by the Indian mathematician D.R. Kaprekar, is a positive integer which is divisible by the sum of its digits [1]. *For example:* 2023 is a Harshad number since 2+0+2+3=7, and 2023 is divisible by 7. The first few are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 20,...(OEIS A005349).

We now define Highly Harshad number as any positive integer that has at least *one of its prime divisors less than (or) equal to sum of its digits*.

1.1 Definition: Let X be a positive integer and let X be written with the digits $a_n, a_{n-1}, \dots, a_1, a_0$: that is,

$$X=\overline{a_n a_{n-1} \dots a_1 a_0},$$

Note that a_0 is the number of units, a_1 is number of tens, etc. Then X is expressed as

$$X=10^n a_n + 10^{n-1}a_{n-1} + \cdots + 10a_1 + a_0$$

$$\Rightarrow X = \sum_{k=0}^{n} a_k 10^k$$

Let $b_1, b_2 \dots b_m$ be all of the distinct prime divisors of X.

Then X is Highly Harshad, if $b_i \le a_0 + a_1 + \cdots + a_n$

$$(\text{i.e.})\mathrm{b_i} \le \sum_{j=0}^{n} a_j$$

For at least one i, where i =1, 2, 3...m.

Note: 1 and10^n, $n \in Z^+$ are never Highly Harshad. (Since sum of digits of 1 and 10^n are 1 and there is no prime divisor less than 1).

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1.2 Examples:

1) 9031
   - 9+0+3+1=13
   - Primes $\leq$ 13: 2,3,5,7,11,13

$\because 11|9031$, 9031 is Highly Harshad.

2) 121
   - 1+2+1=4
   - Primes $\leq$ 4: 2,3

Since, $2\nmid 121$, $3\nmid 121$, 121 is not Highly Harshad.

1.3 Lemma:

I. 1 is the only Non-Highly Harshad single digit number
II. The only primes that are Highly Harshad are 2, 3, 5 and 7.

Proof:

i. By definition, a number is Highly Harshad if,

$$\left.\begin{matrix} At\ least \\ one\ of\ the\ prime\ divisors \\ of\ the\ number \end{matrix}\right\} \leq \left\{\begin{matrix} Sum\ of\ digits \\ of\ that\ number \end{matrix}\right.$$

Since, 1 doesn't have a prime divisor, by definition it can't be Highly Harshad. Now, sum of digits of the prime numbers 2, 3, 5, 7 are 2, 3, 5, 7 respectively and they are divisible by themselves, here prime divisors of the numbers is equal to the sum of digits of the given numbers and thus 2, 3, 5 and 7 are Highly Harshad numbers. Now, for numbers 4, 6, 8 the sum of digits are 4, 6, 8 respectively and they are divisible by a prime less than themselves which is 2 and sum of digits of 9 is 9 and it is divisible by a prime less than itself which is 3, in both the cases, one of the prime divisors is less than the sum of digits of the given numbers and thus 4, 6, 8 and 9 are Highly Harshad numbers.

Hence, 1 is the only Non-Highly Harshad single digit number.

ii. From (i), 2,3,5,7 are Highly Harshad. Now let p be prime and p $\geq$ 11.

For p to be Highly Harshad, by definition,

$$\left.\begin{matrix} At\ least \\ one\ of\ the\ prime\ divisors \\ of\ the\ number \end{matrix}\right\} \leq \left\{\begin{matrix} Sum\ of\ digits \\ of\ that\ number \end{matrix}\right.$$

And by definition of a prime number, p has only two divisors 1 and itself, so for p to be a Highly Harshad number the sum of digits of p must be equal to itself. Thus, p can never be Highly Harshad, since sum of digits of p is always greater than 1 and less than itself.

$\therefore$ The only primes that are Highly Harshad are 2, 3, 5 and 7.

1.4 Lemma:

i. Every even positive integer with sum of digits greater than 1 is Highly Harshad.
ii. All positive integers divisible by 3 are Highly Harshad.
iii. Any positive integer with unit digit 5 is Highly Harshad.

Proof:

i. $$\left.\begin{matrix} \text{Even positive integers with} \\ \text{sum of digits } > 1 \end{matrix}\right\} \Longrightarrow \left\{\begin{matrix} \text{Set of all even positive integers} \\ \text{except positive integer powers of 10} \end{matrix}\right.$$

Then consider A= {2, 4, 6, 8, 12, 14, 16, 18, 20...}

By definition, a number is Highly Harshad if,

$$\left.\begin{matrix} At\ least \\ one\ of\ the\ prime\ divisors \\ of\ the\ number \end{matrix}\right\} \leq \left\{\begin{matrix} Sum\ of\ digits \\ of\ that\ number \end{matrix}\right.$$

Let p $\in$ A, then

- Every p $\in$ A has 2 as one of the prime divisors (since all are even).
- Every p $\in$ A has sum of digits > 1.

Sum of digits of p > 1 $\Longrightarrow$ Sum of digits of p $\geq$ 2
(Or) 2 $\leq$ Sum of digits of p.
$\therefore$ One of the prime divisors of p $\leq$ Sum of digits of p.

Thus, every p ∈ A is Highly Harshad.
Note: A is the set of all the even Highly Harshad numbers.

ii. Let B ={3,6,9,12,...}

Consider Divisibility theorem of 3 [2, pg 36, corollary 13.5 (3)],

An integer is divisible by 3 if and only if sum of digits of the integer is divisible of 3.....(1)

By definition, a number is Highly Harshad if,

$$\left.\begin{matrix} At\ least \\ one\ of\ the\ prime\ divisors \\ of\ the\ number \end{matrix}\right\} \leq \left\{\begin{matrix} Sum\ of\ digits \\ of\ that\ number \end{matrix}\right.$$

Let q ∈ B, then

- Every q ∈ B has 3 as one of the prime divisors, from (1)
- Every q ∈ B has sum of digits ≥ 3 (from (1), sum of digits is a multiple of 3 and sum of digits cannot be zero thus sum of digits is 3, 6, ... etc).

Sum of digits of q ≥ 3 ⟹ 3 ≤ sum of digits of q.

∴ One of the prime divisor of q ≤ sum of digits of q.

Thus, every q ∈ B is Highly Harshad.

iii. Let C ={5,15,25,...}

Consider divisibility rule of 5 [2, pg 36, corollary 13.5 (2)],

An integer is divisible by 5 iff its unit digit is 0 or 5......(2)

By definition, a number is Highly Harshad if,

$$\left.\begin{matrix} At\ least \\ one\ of\ the\ prime\ divisors \\ of\ the\ number \end{matrix}\right\} \leq \left\{\begin{matrix} Sum\ of\ digits \\ of\ that\ number \end{matrix}\right.$$

Let r ∈ C, then

- Every r ∈ C has 5 as one of the prime divisors (from 2).
- Every r ∈ C has sum of digits ≥ 5 (since unit digit is itself 5, sum of digits is greater than or equal to 5).

Sum of digits of r ≥ 5 ⟹ 5 ≤ sum of digits of r.

∴ One of the prime divisors of r ≤ sum of digits of r.

Thus, every r ∈ C is Highly Harshad.

1.5 Theorem: Infiniteness.

Highly Harshad numbers are infinite.

Proof: Highly Harshad are infinite is obvious from 1.4 Lemma (ii) *All positive integers divisible by 3 are Highly Harshad.* (Since B={3,6,9,12,...} is an infinite subset of the set of natural numbers).

1.6 Theorem:

For n ∈ Z^+ and n >1, n! (n factorial) is always Highly Harshad.

Proof:

Let n = 2, then n! = 2! = 2, which is Highly Harshad from 1.3 Lemma (ii)...(1)

For n ≥ 3, n! is always divisible by 3 (Since the factorial of every number greater than 2 will contain at least one multiple of 3).
By 1.4 Lemma (ii) *All positive integers divisible by 3 are Highly Harshad*, n! is Highly Harshad..............(2)
From (1) & (2), n! is always Highly Harshad, for n=2,3,4....

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