ISSN: 0976-3376

Available Online at http://www.journalajst.com

ASIAN JOURNAL OF
SCIENCE AND TECHNOLOGY

Asian Journal of Science and Technology
Vol. 14, Issue, 06, pp. 12552-12554, June, 2023

RESEARCH ARTICLE

NONLINEAR MODEL TO DESCRIBE THE MECHANICAL BEHAVIOR OF THE HUMAN KNEE MENISCI

Gilberto Piña Piña*[1] and Sabino Velázquez Trujillo[2]

[1]Technological Institute of Toluca; [2]Technological Institute of Tuxtla Gutiérrez, Chiapas

ARTICLE INFO

Article History:
Received 14th March, 2023
Received in revised form
15th April, 2023
Accepted 28th May, 2023
Published online 20th June, 2023

Keywords:
Biological tissue, Menisci,
Biphasic materials.

ABSTRACT

The nonlinear model is studied to describe the mechanical behavior of the human knee meniscus tissue, which is composed of a solid phase, a fluid phase and also positive and negative charges. The model presents the theoretical formulation and the constitutive equations. Meniscal tissue consists primarily of a solid phase and a fluid phase. The fluid plays an important role in the mechanical behavior of the meniscus [1]. The mechanical description of the meniscus is also considered a combined action of the solid phase and the fluid phase [2]. The theoretical formulation and description of the model equations are presented. The challenge of describing the human knee meniscus in its complex inhomogeneous microstructure, which consists primarily of ionized water and collagen fibers embedded in an extracellular network of charged protein compounds. To describe the physiological behavior of the meniscus at the macroscale, the electrochemical-mechanical couplings between the components must be considered, as well as the anisotropic properties of the extracellular and viscoelastic matrix.

Citation: ***Gilberto Piña Piña and Sabino Velázquez Trujillo.*** **2023.** "Nonlinear model to describe the mechanical behavior of the human knee menisci", *Asian Journal of Science and Technology*, 14, (06), 12552-12554.

INTRODUCTION

Model Formulation: The meniscal tissue consists mainly of four basic components: collagen fibers, proteoglycans, a fluid, and negatively charged particles. In the four-component mixture theory, the material is modeled as solid denoted by the letter s, it is saturated by fluid denoted by the letter f; in the fluid some ions move freely (cations and anions) [3,4]. In four-component mixture theory, the behavior of the meniscus is described by a series of equations that encompass balance equations and constitutive equations. The balance equations describe the conservation of mass, momentum and momentum can be described in the same way as in classical continuum mechanics [5]. In figure a bright thin fibril identified is seen at the edges of the collagen fibers with the number 4, which corresponds to the proteoglycan. Proteoglycans are hydrophilic molecules, negatively charged that could hold water above 50 times their body weight. The forces of the collagen fibers (3) in the interior of the meniscus portion froze these proteoglycans. During compression, they slowly dissipate the trapped water and hereby help to constrain the compressive forces, the fluid is loaded positively, the fluid is drawn into the interstitial spaces (2) when compressive loads are applied concerning the meniscus. The positive charges of the flow are attracted by the negative charges of the proteoglycans. The micrograph shows a deep area of the meniscus since it only consists of spherical chondrocytes, characteristic of the deep area. When age increases, changes in the vasculature, which include a decrease in vascular peripheral and central margin of the meniscus the number of cells of the meniscus decreases considerably, is by this reason we find few cells in the micrograph.

At the bottom of the ponds, we observe collagen oriented in a perpendicular direction (5), compared to collagen, which is identified by (3).

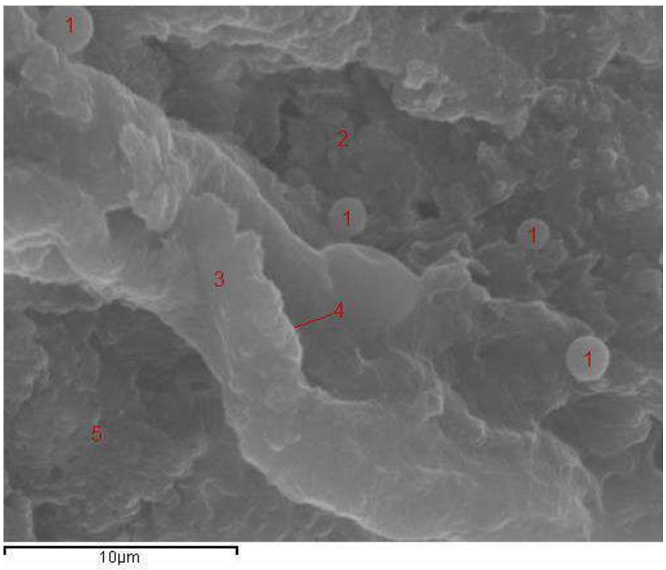

Figure 1. Principal components of the human knee meniscus

Balance of equations for the solid: For each α phase, the material satisfies the momentum equation

$$\nabla . T^{\propto} + \pi^{\propto} = f^{\propto} \quad \text{..........................1}$$

$\propto = s, f \quad s = solido, \; f = fluido$

where $T^{\propto}$ is the stress tensor of the phaseα,$\pi^{\propto}$ represents the interaction with other phases and $f^{\propto}$ represent the inertial terms and the forces of the body. The stress tensor is modeled as:

$$T^{\propto} = -n^{\propto} pI + \sigma^{\propto} \quad \ldots\ldots 2$$

where $n^{\propto}$ is the phaseα volume fraction, p is the fluid pressure, and $\sigma^{\propto}$ is the effective stress tensor of the phaseα. The volume fraction is defined by.

$$n^{\propto} = \frac{V^{\propto}}{V} \quad \ldots\ldots 3$$

where V is the representative volume of the mixture and $V^{\propto}$ is the volume occupied by the phaseα within the volume V. Conservation of momentum for the entire mixture leads to constraint$\pi^{s} + \pi^{f} = 0$. Therefore, the total momentum balance is simplified as follows.

$$\nabla.\left(T^{s} + T^{f}\right) = f^{s} + f^{f} \quad \ldots\ldots 4$$

$$\nabla.\left(\sigma^{s} + \sigma^{f}\right) - \nabla\left[\left(n^{s} + n^{f}\right)p\right] = f^{\alpha} \quad \ldots\ldots 5$$

From now on, we ignore the inertial terms and the body forces that are represented by $f^{\propto}$. Furthermore, $f^{\propto} = 0$we assume that the material is saturated.

$$n^{s} + n^{f} = 1 \quad \ldots\ldots 6$$

Constitutive equations for solids: The first equations define the effective stress tensor

$$\sigma^{\alpha} = 2\mu_{\alpha}\varepsilon(u) + \lambda_{\alpha}\nabla.\mathrm{uI} + 2m_{\alpha}d(v^{\alpha}) + \Lambda_{\alpha}\nabla.v^{\alpha}I \quad \ldots\ldots 7$$

dondeμ_{α} and λ_{α} are the Lamé parameters, m_{α} and Λ_{α} are the viscous stress parameters

The solid matrix is modeled as a linear elastic material. This means that the viscous stress parameters m_{α} and Λ_{α} are equal to zero. Therefore, mechanical behavior is described by Hooke's Law.

$$\sigma^{s} = 2\mu_{s}\varepsilon(u) + \lambda_{s}\nabla.\mathrm{uI} \quad \ldots\ldots 8$$

where μ_{s} and λ_{s} are the Lamé parameters for the solid, these Lamé parameters depend on Poisson's ratio and the stiffness of the elastic material.

The fluid is modeled as a viscous Newtonian fluid. So, $\mu_{f} = 0$ y $\lambda_{f} = 0$

$$\sigma^{f} = 2m_{f}d\left(v^{f}\right) + \Lambda_{\alpha}\nabla.v^{f}\mathrm{I} \quad \ldots\ldots 9$$

It is now assumed that the viscosity is negligible compared to the momentum transfer. So, $\sigma^{s} \gg \sigma^{f}$. This is generally the case, except in very thin layers of permeable materials [6]. Therefore, the momentum equilibrium is described as:

$$\nabla.\sigma - \nabla p = 0 \quad \ldots\ldots 10$$

donde$\sigma = 2\mu_{s}\varepsilon(u) + \lambda_{s}\nabla.\mathrm{uI}$

Balance of equations

In the absence of mass exchange due to chemical interactions, the balance equations that describe the conservation of the mass of the components of the mixture are written with the following expression (Oomens, 1985):

$$\frac{\delta(n^{\alpha}\rho^{\alpha})}{\delta t} + \nabla.(n^{\alpha}\rho^{\alpha}v^{\alpha}) = 0 \quad \ldots\ldots 11$$

This assumes that the phases are incompressible, ρ^{α} is constant. Assuming the mass balance for the solid and the fluid and using the saturation condition $\left(n^{s} + n^{f} = 1\right)$and the definition of speed of the solid $v^{s} = \delta_{u}/\delta_{t}$, where u are the solid displacements, t is the time and v^{s} is the speed of the solid. The equation 11 can be written as.

$$\frac{\delta n^{\alpha}}{\delta t} + \nabla.(n^{\alpha}v^{\alpha}) = 0 \quad \ldots\ldots 12$$

Molar Mass Balance Equations

$$\frac{\delta(n^{f}m^{\beta}c^{\beta})}{\delta t} + \nabla.\left(n^{f}m^{\beta}c^{\beta}v^{\beta}\right) = 0 \quad \ldots\ldots 13$$

The molar masses of the ions are constant m^{β}. So, the molar masses may be out of mass balance. Rewriting equation 13.

$$\frac{\delta(n^{f}c^{\beta})}{\delta t} + \nabla.\left(n^{f}c^{\beta}v^{\beta}\right) = 0 \quad \ldots\ldots 14$$

Constitutive equations for masses: The constitutive equations describe the flow of the fluid and the flow of ions. So, the flows are described by Darcy's extended law and Fick's extended law. These extended laws are derived by Huyghe and Janssen [7,8]

Darcy's Law

$$n^{f}(v^{l} - v^{s}) = \frac{k}{n^{f}}[n^{l}\nabla\mu^{l} + n^{+}\nabla\mu^{+} + n^{-}\nabla\mu^{-}] \quad \ldots\ldots 15$$

where n^{f} is the volume fraction of the fluid, μ^{β},with $(\beta = l, +, -)$ are the electrochemical potentials, v^{l} is the velocity of the liquid and k is the hydraulic permeability tensor.

Fick's Law

$$c^{\beta}\left(v^{\beta} - v^{l}\right) = -D^{\beta}\frac{\bar{V}^{\beta}c^{\beta}}{RT}\nabla\mu^{\beta}, \text{ with}\beta = +, -) \quad \ldots\ldots 16$$

where c^{β} is the concentration of ion β per unit volume of fluid, v^{β} is the velocity of component β, $D^{\beta}$is the diffusion tensor of ion β, $\bar{V}^{\beta}$is the volume occupied by one mole of component β, $T$is the absolute temperature and R is a constant.

Conclusions

The efficiency presented by the model can be applied not only to the description of the menisci of the human knee but also to other tissues of the body, to know their physiological and mechanical behavior, which will allow the development of prostheses that can be behave as real to human tissue.

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