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RESEARCH ARTICLE

INTEGER RIGHT TRIANGLE WITH AREA/PERIMETER AS A CANADA NUMBERS

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ABSTRACT

In each of these patterns of integer right triangles, the Canada number serves as a representation of the area-to-perimeter ratio. Also provided are a few intriguing relationships between the sides.

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INTRODUCTION

The characteristics of the positive integers (1, 2, 3,...) are the focus of number theory. It is one of the oldest and most natural branches of mathematics and is occasionally referred to as "higher arithmetic". Mathematicians have been enthralled by the Pythagorean Theorem for centuries despite its seeming simplicity. Mathematics is still being enriched by the solutions to numerous issues. In addition to polygonal numbers, we also have the Jarasandha numbers, Nasty numbers, and Dhuruva numbers, which are all fascinating patterns of numbers [2, 3, 6, 9, 10, 11& 13]. Numerous mathematicians have shown interest in the process of finding three non-zero integers s, t and r under specific circumstances satisfying the connection $s^2 + t^2 = r^2$ [1, 12 & 14]. Special Pythagorean problems are studied in [4,5,7 & 8]. Canada numbers are those n such that the sum of the squares of the digits of n is equal to the sum of the non- trivial divisors of n, i.e. $\sigma(n) - n - 1$. The Canada numbers are 125, 581, 8549 and 16999.The name of these numbers is due to the fact they were defined by some mathematicians from Manitoba University to celebrate the 125*th* anniversary of Canada. In this communication, we search for patterns of integer right triangles, where each triangle's area-to-perimeter ratio is symbolised by a Canada number. A few intriguing relationships between the sides are also provided.

Basic Definitions

Definition: 1

The ternary quadratic Diophantine equation given by $s^2 + t^2 = r^2$ is known as Integer right equation where s, t and r are natural numbers. The equation above is also known as Integer right triangle and denote it by $\Delta(s,t,r)$.

Also, in Integer righttriangle $\Delta(s,t,r) : s^2 + t^2 = r^2$, s and t are called its legs and r its hypotenuse.

Definition: 2

Most cited solution of the Integer right equation is $s = a^2 - b^2,\ t = 2ab,\ r = a^2 + b^2$,

where $a > b > 0$. If a and b have opposing parities and $\gcd(a,b) = 1$, this solution is referred to as primitive.

Definition: 3

A Harshad or Niven Number is a positive integer that can be divided by the sum of its digits.

Definition: 4

A positive integer having at least four different factors such that the difference between one pair of factors equals the sum of another pair of factors and the product of each pair is equal to that number is called Nasty number.

Analysis Technique

Area and perimeter of the triangle are denoted by Λ and P, respectively, with the hypothesis that $\frac{\Lambda}{P}$ = Canada number.

The relationship mentioned above leads to the expression $\frac{b(a-b)}{2}$ = Canada number. (1)

Case: 1

$$\frac{b(a-b)}{2} = 125\,(1^{st}\text{ Canada number}) \quad (2)$$

$$\Rightarrow b(a-b) = 250 \quad (3)$$

Following assessment, Table 1 shows the values of the generators *a* and *b* obeying (3):

Table 1.

S.No	*a*	*b*	*r*	*s*	*t*	Λ	P	$\frac{\Lambda}{P}$
1.	251	1	63000	502	63002	15813000	126504	125
2.	127	2	16125	508	16133	4095750	32766	125
3.	55	5	3000	550	3050	825000	6600	125
4.	35	10	1125	700	1325	393750	3150	125
5.	35	25	600	1750	1850	525000	4200	125
6.	55	50	525	5500	5525	1443750	11550	125
7.	127	125	504	31750	31754	8001000	64008	125
8.	251	250	501	125500	125501	31437750	251502	125

There are 8 integer right triangles, it is observed. Out of them, only two of the triangles are primitive, while the other six are not.

Case: 2

Take into account the 2^{nd} Canada number 581.

$$\therefore b(a-b) = 1162 \quad (4)$$

Following assessment, Table 2 is used to express the values of the generators *a* and *b* fulfilling (4)

Table 2.

S.No	*a*	*b*	*r*	*s*	*t*	Λ	P	$\frac{\Lambda}{P}$
1.	1163	1	1352568	2326	1352570	1573036584	2707464	581
2.	583	2	339885	2332	339893	396305910	682110	581
3.	173	7	29880	2422	29978	36184680	62280	581
4.	97	14	9213	2716	9605	12511254	21534	581
5.	97	83	2520	16102	16298	20288520	34920	581
6.	173	166	2373	57436	57485	68147814	117294	581
7.	583	581	2328	677446	677450	788547144	1357224	581
8.	1163	1162	2325	2702812	2702813	3142018950	5407950	581

It is noted that there are 8 integer right triangles. Four of the triangles are non-primitive, and the remaining four are primitive.

Case: 3

Choose the 3^{rd} Canada number 8549.

$$\therefore b(a-b) = 17098 \quad (5)$$

Following assessment, Table 3 lists the values of the generators *a* and *b* satisfies (5)

Table 3.

S.No	A	b	r	s	t	Λ	P	$\frac{\Lambda}{P}$
1.	17099	1	292375800	34198	292375802	4999333804200	584785800	8549
2.	8551	2	73119597	34204	73119605	1250491347894	146273406	8549
3.	289	83	76632	47974	90410	1838171784	215016	8549
4.	269	103	61752	55414	82970	1710962664	200136	8549
5.	269	166	44805	89308	99917	2000722470	234030	8549
6.	289	206	41085	119068	125957	2445954390	286110	8549
7.	8551	8549	34200	146204998	146205002	2500105465800	292444200	8549
8.	17099	17098	34197	584717404	584717405	9997790532294	1169469006	8549

There are 8 integer right triangles, it is observed. Out of them, 4 of the triangles are non-primitive, and the other 4 are primitive.

Case: 4

Take the 4th Canada number 16999.

$$\therefore b(a-b)=33998 \quad (6)$$

Following evaluation, Table 4 contains the values of the generators *a* and *b* satisfying (6)

Table 4.

S.No	a	b	r	s	t	Λ	P	$\frac{\Lambda}{P}$
1.	33999	1	1155932000	67998	1155932002	39300532068000	2311932000	16999
2.	17001	2	289033997	68004	289034005	9827733965994	578136006	16999
3.	471	89	213920	83838	229762	8967312480	527520	16999
4.	369	178	104477	131364	167845	6862258314	403686	16999
5.	369	191	99680	140958	172642	7025346720	413280	16999
6.	471	382	75917	359844	367765	13659138474	803526	16999
7.	17001	16999	68000	577999998	578000002	19651999932000	1156068000	16999
8.	33999	33998	67997	2311796004	2311796005	78597596441994	4623660006	16999

There are 8 integer right triangles, it is observed. Out of them, 4 of the triangles are non-primitive, and the other 4 are primitive.

FASCINATING FINDINGS

In all the four cases it is observed that,

1. All are having 8 Integer right triangles.
2. In all the 8 Integer right triangles leg *r* and hypotenuse *t* has same parity.
3. In all the 8 Integer right triangles leg *s*, area Λ and perimeter P are even.
4. If *b, a* are consecutive then *s, t* are also consecutive numbers.
5. Each cases $r+s-t$ having the same number.
6. $s+t, 2(r+t), 2(t-s)$ are a perfect square.
7. $6(s+t), 12(r+t), 12(t-s)$ are nasty numbers.
8. $\frac{r+s-t}{4}$ is a Canada number.
9. For the Canada number 125, $r+s-t-100$ is a perfect square.
10. For the Canada number 125, $r+s-t$ is a Harshad number.

CONCLUSION

In conclusion, one can look for relationships between integer right triangles and other unique numbers and number patterns.

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