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RESEARCH ARTICLE

APPLYING SHRINKAGE TECHNIQUE FOR ESTIMATE THE SCALE PARAMETER OF WEIGHTED RAYLEIGH DISTRIBUTION

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ABSTRACT

This paper is included the estimation for the scale parameter of weighted Rayleigh distribution using well known methods of estimation (classical and Bayesian).The proposed estimators were compared using Monte Carlo simulation based on mean squared error (MSE) criteria. Then all the results of simulation and comparison are performed in tables.

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INTRODUCTION

Rayleigh distribution is one of the failure distribution which has been used in reliability, life- testing and survival analysis. Being first presented by (Lord Rayleigh), (1) this statistical properties were originally derived in connection with a problem in acoustics more details on the Rayleigh distribution can be found in Johnson et al, (2). Weighted distributions are applied in research associated with reliability meta analysis, bio – medicine, econometrics , renewal processes ,physics, ecology and branching processes which are found in Zelen and Feinleib (3), Patil and Ord (4), Patil and Rao (5), Gupta and Keating (6), Gupta and Kirmani (7), and Oluyede, (8). The weighted Rayleigh distribution has been published by Reshi et al. (9). They presented a new class of size – biasedGeneralized Rayleigh distribution and also investigated the various structural and characterizing properties of that model, also Das and Roy, (10). Rashwan, (11) introduced the double Weighted Rayleigh distribution properties and estimation , AL-Kadim and Hussein, (12) published research for estimation the reliability of Weighted Rayleigh distribution through five methods using simulation and a comparison between the proposed estimators were made. Ahmed and Ahmed, (13), presented the characterization and estimation of double Weighted Rayleigh distribution.Salman and Ameen, (14) estimate the shape parameter of Ganeralize Rayleigh Distribution using Bayesian-Shrinkage Technique. Ajami and Jahanshahi, (15) introduced the parameter estimation in Weighted Rayleigh distribution. The aim of this paper is to estimate the scale parameter of Weighted Rayleigh distribution using shrinkage estimation methods depends on classical estimators; Moment (mom) and maximum likelihood (ML) methods.

Weighted Rayleigh Distribution (WRD): To present the concept of a weighted distribution, suppose that T is a non – negative random variable follows Rayleigh Distribution with one parameter $\beta\{T \sim RD(\beta)\}$, then the (pdf) of T is given by

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$$f(t,\beta) = 2t\beta e^{-t^2\beta} \quad ; t > 0 \ , \beta > 0 \tag{1}$$

And w (t) be a non- negative weight function satisfying the condition μ_w= E (w(t)) exist . then the r v T **w** whichis defined on the interval (L,U) having pdf as below :

$$f\omega\ (t) \frac{\omega(t)\, f(t)}{E[w(t)]} \ , \ l < t < U \tag{2}$$

$$\text{Where, } \omega(t) = e^{t^2} \ and \ E(\omega(t)) = \int_0^\infty \omega(t)\, f(t) dt$$

From equations (1) and (2), we get the probability density function of the random variable Tw will be

$$f\omega\ (t;0) - 2t(\beta - 1)e^{-t^2(\beta-1)} \ ; t > 0 \ , \beta > 1 \tag{3}$$

And the cumulative distribution function (cdf) of Tw will be

$$F\omega\ (t;\theta) = 1\text{-}e^{-t^2(\beta-1)} \tag{4}$$

By putting β -1 = θ>0 in equations (3) we get

$$f_w(t;\theta) = 2t\theta e^{-t^2\theta} \quad ; t > 0 \ , \theta > 0 \tag{5}$$

And the cumulative distribution function (cdf) of T_W will be

$$F_W\ (t;\theta) = 1\text{-}e^{-t^2\theta} \tag{6}$$

Accordingly, the reliability and hazard functions will be respectively as follows:

$$R\omega\ (t;\theta) = 1\text{-}F\omega\ (t;\theta) = e^{-t^2\theta} \tag{7}$$

$$h\omega\ (t;\theta) - \frac{f_\omega(t;0)}{R_\omega\ (t;\theta)} - 2t\theta \tag{8}$$

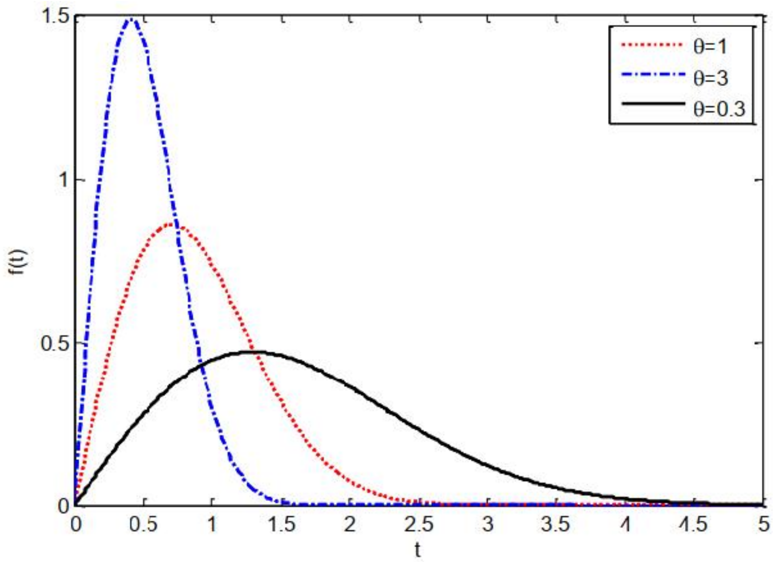

Figure 1. The Plot of Raleigh Distribution (p.d.f)

Maximum Likelihood Estimation (MLE): Firstly , we find the likelihood function L(t1,t2,t3,…,tn; θ) based on the following

Method of Moments

$$l = L(t_1, t_2, t_3, \ldots, t_n; \theta) = \prod_{i-1}^{n} f\omega(t, \theta)$$

$$l = \theta^n \prod_{i=1}^{n} 2t_i e^{-\theta \sum_{i=1}^{n} t_i^2}$$

$$\ln l \quad n\ln\theta + \sum_{i=1}^{n} \ln 2t_i - \theta \sum_{i=1}^{n} t_i^2$$

$$\frac{\partial \ln l}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n} t_i^2$$

$$\text{Put } \frac{\partial \ln l}{\partial \theta} = 0$$

$$\therefore \hat{\theta}_{mle} = \frac{n}{\sum_{i=1}^{n} t_i^2} \qquad (9)$$

Let t1, t2,…, tn refer to a random sample of size n from the WRD with pdf (5) then the moment estimator $\theta\_$ of is obtained by setting the mean of the distribution equal to the sample mean ,

$$\text{i.e., } E(T^K) = \sum_{i-1}^{n} t^k / n$$

The Moment estimator $\hat{\theta}_{mom}$ of θ obtained as below

$$\frac{\Gamma(1+\frac{k}{2})}{(\theta)^{k/2}} = \frac{\sum_i^n t_i^k}{n}$$

For k = 1 we have

$$\frac{\Gamma(1+\frac{1}{2})}{\theta^{1/2}} = \frac{\sum_{i=1}^{n} t_i}{n}$$

$$\therefore \hat{\theta}_{mom} = \frac{\pi}{4(\bar{t})^2} \qquad (10)$$

Shrinkage Method: Thompson in 1968,(16) has studied the problem of shrink a usual estimator $\hat{\theta}$ of the parameter θ which depend on observation of random sample and prior studies and previous experiences through merge the usual estimator $\hat{\theta}mle$ and initial estimate θo as a linear mixture via shrinkage weight factor Ø($\hat{\theta}$mle), 0 ≤ ($\hat{\theta}$mle) ≤ 1 , and the result estimator is so called shrinkage estimator which has the form below

$$\hat{\theta}\,sh = Ø(\hat{\theta}mle)\,\hat{\theta}mle + (1 - Ø(\hat{\theta}mle))\theta o \qquad (11)$$

Where Ø ($\hat{\theta}$mle) denotes to the trust of $\hat{\theta}$mle and(1 − Ø ($\hat{\theta}$mle)) signifies the trust of θ_O, which might be constant or a function of $\hat{\theta}$mle , function of sample size or could be find by reducing the mean square error for $\hat{\theta}$ sh . Thompson refers to the significant reasons to use initial value.

- supposing the initial value θo near to the true value and then it is essential to use it
- if the initial value θo near to the actual value of the parameter θ , we get bad situation , see (15) ,(17) , (18) , (19) , (5) , and (1) in this state there is no doubt to take the moment method as initial value , consequently theequation (11) becomes :

$$\theta\,sh = Ø1(\hat{\theta}mle)\hat{\theta}mle + (1 - Ø1(\hat{\theta}mle))\hat{\theta} \qquad (12)$$

The shrinkage weight function (sh1): In this section we claim the shrinkage weight factor as a function of sample size n, as below.

$$Ø1(\hat{\theta}mle) = e^{-n}$$

Consequently, shrinkage estimator of Ø, which is defined in (12) will be

$$\hat{\theta}\,sh1 = e^{-n}\,\hat{\theta}mle + (1 - e^{-n})\,\hat{\theta}mom \qquad (13)$$

In this subsection we suggest a constant shrinkage weight factor as below

$$Ø2(\hat{\theta}mle) = 0.01$$

Accordingly, shrinkage estimator of θ will be:

$$\hat{\theta}\, sh2 = (0.01)\ \hat{\theta}mle + (0.99)\ \hat{\theta}mom \tag{14}$$

Modified Thompson type shrinkage weight function (sh3)

In this subsection, we consider the modified shrinkage weight factor introduced by Thompson as follows.

$$\emptyset_3(\hat{\theta}_{mle}) = \frac{(\hat{\theta}_{mle}-\hat{\theta}_{mom})^2}{(\hat{\theta}_{mle}-\hat{\theta}_{mom})^2+var(\hat{\theta}_{mle})} * (0.001) \tag{15}$$

Where, $var(\hat{\theta}_{mle}) = \frac{n^2\theta^2}{(n-1)^2.(n-2)}$ (16)

Hence, shrinkage estimator of 5 became:

$$\hat{\theta}_{sh3} = \emptyset_3(\hat{\theta}_{mle})\hat{\theta}_{mle} + \left(1-\emptyset_3(\hat{\theta}_{mle})\right)\hat{\theta}_{mom} \tag{17}$$

Simulation Study: In this section, Monte Carlo simulation study were studied to compare the performance of the considered estimators for scale parameter θ which were obtained using deferent sample size (n=10,30,50,100), based on 1000 replication through MSE criteria as steps below (6) .

Step1: Generate random samples follows the continuous uniform distribution from interval (0,1) say as u**1**, u**2**, …, u**n**.

Step2: Transform the uniform random samples to random samples follows WRD using the cumulative distribution function (c.d.f) as follows:

$$u \sim Uinf(0,1)$$

$$F(t) = 1 - R(t)$$

$$u_i = 1 - e^{-t_i^2\theta}$$

$$t_i = \sqrt{-\frac{1}{\theta}\ln(1-u_i)}$$

Step3: Calculate maximum likelihood estimator of 5(t) from equation (9).Step4: Apply moment method of 5(t) via equation (11). Step5: Compute the three shrinkage estimators of 5(t) by equations (13), (14),(17).

Step6: Based on (L=1000) replication, the MSE for all proposed estimation methods of □(t) is utilized by:

$$MSE(\hat{\theta}_{mle}) = \frac{1}{L}\sum_{i=1}^{L}(\hat{\theta}_i - \theta)^2$$

Numerical results: We put all the results in the tables below

Table 1. Shown the estimation method of θ

N	θ	`	`	` 1	ˆ 2	ˆ 3
10	2	2.2839	2.1672	2.1672	2.1683	2.2839
	3	3.4259	3.2508	3.2508	3.2525	3.4259
	4	4.5678	4.3344	4.3344	4.3367	4.5678
	5	5.7098	5.4179	5.4180	5.4209	5.7098
30	2	2.0135	2.0977	2.0977	2.0968	2.0135
	3	3.0203	3.1465	3.1465	3.1453	3.0203
	4	4.0270	4.1954	4.1954	4.1937	4.0270
	5	5.0338	5.2442	5.2442	5.2421	5.0338
50	2	2.2096	2.2609	2.2609	2.2604	2.2096
	3	3.3145	3.3913	3.3913	3.3906	3.3145
	4	4.4193	4.5218	4.5218	4.5207	4.4193
	5	5.5241	5.6522	5.6522	5.6509	5.5241
100	2	2.3025	2.2893	2.2893	2.2895	2.3025
	3	3.4537	3.4340	3.4340	3.4342	3.4537
	4	4.6050	4.5787	4.5787	4.5789	4.6050
	5	5.7562	5.7233	5.7233	5.7237	5.7562

Table 2. Shown the MSE of estimation methods of θ

n	θ	mle	mom	sh1	sh2	sh3	best
10	2	0.0006864	0.00075275	0.00075275	0.0007515	0.0007180	Sh1
	3	0.0014082	0.001400023	0.001400019	0.0013991	0.0013878	Sh1
	4	0.0031097	0.003108731	0.003108720	0.0031063	0.0013878	Sh1
	5	0.0045856	0.004380536	0.004380531	0.0043795	0.0030178	Sh1
30	2	0.00015622	0.000170968	0.000170968	0.00017068	0.0043464	Sh3
	3	0.00032290	0.0003292038	0.0003292038	0.00032885	0.00016431	Sh3
	4	0.00058794	0.0006288918	0.0006288918	0.00062799	0.00032322	Sh3
	5	0.00115660	0.0012120170	0.0012120170	0.00121060	0.00061422	Sh3
50	2	0.000088041	0.0000967384	0.0000967384	0.000096577	0.00093266	Sh3
	3	0.00020739	0.0002223561	0.0002223561	0.00022203	0.00021604	Sh3
	4	0.00036650	0.0003935831	0.0003935831	0.00039299	0.00038074	Sh3
	5	0.00060234	0.0006305347	0.0006305347	0.00062978	0.00061549	Sh3
100	2	0.000041813	0.00004597264	0.00004597264	0.000045893	0.000044647	Sh1&Sh2
	3	0.000094374	0.000103385	0.000103385	0.00010321	0.00010006	Sh1&Sh2
	4	0.00017602	0.0001863072	0.0001863072	0.00018606	0.00018206	Sh1&Sh2
	5	0.00024768	0.0002786088	0.0002786088	0.00027805	0.00026833	Sh1&Sh2

RESULT ANALYSIS

- It is clear from table (1) that (MSE) has minimum value in case of mle for all the values of θ = 2 , 3 , 4 , 5 when the size of samples are 30 , 50 , 100, this implies that MLE the best. For n=10 (small sample size) the mean squared error (MSE) of the scale parameter $\hat{\theta}$ sh3 is minimum than the other estimators and follows by θ=2,3,4,5.
- $\hat{\theta}$ sh2 and, $\hat{\theta}$ sh1 hence the best estimator in this case is $\hat{\theta}$ sh3 for all
- For n=30, 50, 100 (medium sample size), the mean squared error (MSE) of the scale parameter $\hat{\theta}$ sh3 has minimum value than the other estimators follows by $\hat{\theta}$ sh2 and $\hat{\theta}$ sh1, consequently the best estimator in this case is $\hat{\theta}$ sh3 for all θ=2,3,4,5.
- For all n, the mean squared error (MSE) for all proposed estimators are approximately fixed with respect to θ.

Conclusion

From the results analysis the maximum likelihood method was the best because it has minimum mean squared error (MSE) for all values of θ when the size of sample are 30 , 50 , 100.

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