Available Online at http://www.journalajst.com

ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN: 0976-3376

Asian Journal of Science and Technology
Vol. 11, Issue, 10, pp.11319-11324, October, 2020

RESEARCH ARTICLE

A PRODUCTION INVENTORY MODEL WITH NEGATIVE EXPONENTIAL DEMAND RATE WITH NO SHORTAGE

[1]Janardan Behera and [2]Sudhir Kumar Sahu

[1]Research Scholar, Department of Statistics, Sambalpur University, Odisha, India
[2]Departments of management Sciences, Mahatma Gandhi Central University, Bihar, India

ARTICLE INFO

Article History:
Received 27th July, 2020
Received in revised form
19th August, 2020
Accepted 28th September, 2020
Published online 30th October, 2020

Key words:

Inventory, Probabilistic-Dependent Demand, Negative-Exponential Without Shortage, Deterioration.

ABSTRACT

This study contains a production inventory model, which developed for deteriorating items with negative exponential probabilistic demand rate without shortage. An algebraic way is applied to find the minimum total inventory cost (TIC). The idea of this study is to minimize the cost. A numerical example, graphical representation of the model, graphical representation of numerical example and sensitivity analysis are given. For calculation and sketching the graphs mathematica software has been used.

Citation: ***Janardan Behera and Sudhir Kumar Sahu*. 2020.** "A Production Inventory Model with Negative Exponential Demand rate with no Shortage.", *Asian Journal of Science and Technology*, 11, (10), 11319-11324.

INTRODUCTION

In present days, demand of many products stay in pick for a period of time and later its fall down with a high rate, such as mobile phones, hi-tech products, fashion apparels, auto mobiles. These products comes with a high demand rate at the beginning of release, later demand rate gradually decreases and after some period of time its collapse by new version of products. Shortage has not considered in this model, as production ends with another product. Therefore, it is important to discuss the market position of such type of items. In this direction, Ghare and Schrader (1) were considered the effect of deteriorating items in inventory model. They discussed the general economic order quantity model with direct spoilage and exponential deterioration. Covert and Philip (2) extended the work of Ghare and Schrader (1) with Weibull distribution and gamma distribution. Philip (3) deduced a three parameter Weibull distribution for the deteriorating time. Goyal (9), Datta and Pal (10), Sahu, Bishi and Behera (6), Goswami and Chaudhuri (11), Hariga (12), Chang and Dye (13), Goyal and Giri (14), Skouri and Papachristos (15), Sahu, Bishi and Behera (5), Skouri et al. (16), Sarkar (17), Sahu, Bishi and Behera (4) etc., extended the inventory models with different types of deterioration rates. Basically, inventory costs were optimized with the help of differential calculus. Many researchers have discussed different types of algorithm for inventory model. Supply chain management generally contains buyers and suppliers, producers and distributors, distributors and retailers, etc., in many different forms of customers. The idea, to consider a supply chain management, is to optimize the whole system at a time. In this direction, Goyal (18) developed an integrated inventory model for a single supplier-single buyer problem. Banerjee (19) found out a joint economic lot size model for the purchaser and the vendor. Khouja (20) presented optimizing inventory decisions in a multi-stage multi-customer supply chain. Shin, Kim and Lee (8) discussed on production and inventory control of auto parts on predicted probabilistic distribution of inventory. Cárdenas and Barrón (21) developed optimum manufacturing batch size with rework in a single-stage production system. Wee and Widyadana (22) found out economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time. Sarkar (7) presented a production-inventory model with probabilistic deterioration in two-echelon supply chain management. Chung and Cárdenas-Barrón (23) found out a complete solution procedure for the economic production quantity and economic production quantity inventory models with linear and fixed backorder costs. Our paper is also closely related to Sarkar (7) and Shin, Kim and Lee (8) , in which an inventory model for deteriorating products with probabilistic deterioration in two-echelon supply chain management discussed and also discussed on production and inventory control of auto parts on predicted probabilistic distribution of inventory. The objective function of our model is not a concave function in general. The problem considered in our paper is a production inventory system with probabilistic demand rate, which is also different from the inventory model Sarkar (7), and Shin, Kim and Lee (8). In this model, we try to optimize the cost with the help of differential calculus. We want to find the mathematical expression of the total inventory costs and to minimize it. This paper is designed as follows: introduction is given in section 1, notation is given in section 2, assumption is given in section 3, section 4 contains mathematical formulation of model and its solution, and in section 5 numerical examples and sensitivity analysis are presented to illustrate the model. At the last section, conclusion and the future extensions of the model have been made.

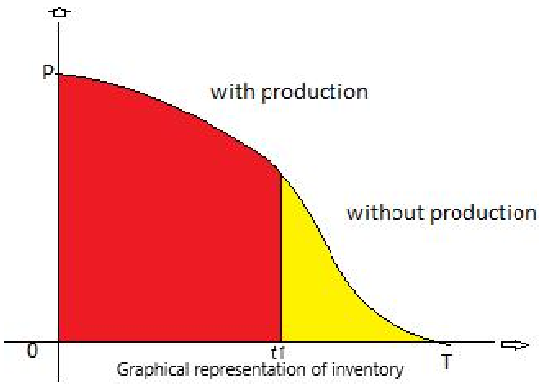
Notations: The notation in this paper is listed below:
P: production quantity.
p: production cost.
h: Inventory holding cost per unit per time.
$I(t)$: Inventory level of a time point.
d: deterioration cost.
O: Ordering cost per order. $D(t)$: Demand rate. *where* $D(t) = a e^{-at}$; $0 \le t \le T$.
TIC: Total inventory cost.

Assumptions:

The model in this paper is built on the base of the following assumptions.

- Market demand is negative exponential to time.
- Market demand only exists in a limited time horizon T.
- Demand cannot be backlogged.
- Ordering lead time is zero.
- Deteriorated products have no value, and there is no cost to dispose or store them.

Mathematical model: This study considers a production inventory policy in which the deterioration rate is affected by the preservation technology investment. The decision variables are the market demand, the production cost, and the preservation technology investment parameter. Ordering cost is not treated as separately it included with production cost in this model. According to the assumption, the time length is equal in all the ordering periods. The inventory level $I(t)$ can be depicted as Figure given below and formulated as follows:



$$\frac{dI(t)}{dt} = P - D(t); t \in [0, t_1] \text{........} \qquad \text{(A)}$$

$$\frac{dI(t)}{dt} = -D(t); t \in [t_1, T] \text{..............} \qquad \text{(B)}$$

Where $D(t) = a e^{-at}; t \in [0, T]$ with boundary Conditions $I(0) = P \,\&\, I(T) = 0$.

$$\frac{dI(t)}{dt} = P - a e^{at}; t \in [0, t_1] \text{............} (1)$$

$$\frac{dI(t)}{dt} = -a e^{at}; t \in [t_1, T] \text{...............} (2)$$

By solving equations (1) & (2) with boundary conditions $I(0) = P$ & $I(T) = 0$ respectively we get:

$$I(t)=P(t+1)+e^{-\alpha t};t\in[0,t_1]..........(3)$$

$$I(t)=e^{-\alpha t}-e^{-\alpha T};t\in[t_1,T]..............(4)$$

By using the continuity property of $I(T)$ at $t=t_1$ we have $I(t_1)_{W.P}=I(t_1)_{W.O.P}$

By solving we get $P=-\frac{e^{-\alpha T}}{(1+t_1)}$...........(5)

$$I(t)_{W.P}=e^{-\alpha t}-\frac{(1+t)e^{-\alpha T}}{(1+t_1)};t\in[0,t_1]......(6)$$

$$I(t)_{W.O.P}=e^{-\alpha t}-e^{-\alpha T};t\in[t_1,T]..............(7)$$

Holding Cost (HC):= $h\int_0^T I(t)dt=h[\int_0^{t_1} I(t)_{W.P}\,dt+\int_{t_1}^T I(t)_{W.O.P}\,dt]$

$$=h[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(t_1+\frac{t_1^{\ 2}}{2})e^{-\alpha T}}{(1+t_1)}$$

$$+\frac{(e^{-\alpha t_1}-e^{-\alpha T})}{\alpha}-e^{-\alpha T}(T-t_1)]$$

Production Cost (PC):= $p\int_0^T I(t)dt=p\int_0^{t_1} I(t)_{W.P}\,dt$

$$=p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(t_1+\frac{t_1^{\ 2}}{2})e^{-\alpha T}}{(1+t_1)}]$$

Deterioration Cost (DC):= $d\int_0^T I(t)dt=d[\int_0^{t_1} I(t)_{W.P}\,dt+\int_{t_1}^T I(t)_{W.O.P}\,dt]$

$$=d[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(t_1+\frac{t_1^{\ 2}}{2})e^{-\alpha T}}{(1+t_1)}$$

$$+\frac{(e^{-\alpha t_1}-e^{-\alpha T})}{\alpha}-e^{-\alpha T}(T-t_1)]$$

Total Invenotory Cost (TIC):= HC+DC+PC

$$(h+d)[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(t_1+\frac{t_1^{\ 2}}{2})e^{-\alpha T}}{(1+t_1)}$$

	% change	Changed value	Original TIC	Changed TIC	%change in TIC
a = 0.4	+50	0.6	23484	25496	+8.56
	+25	0.5	23484	24481	+4.24
	+0	0.4	23484	23484	0.00
	-25	0.3	23484	22464	-4.34
	-50	0.2	23484	21287	-9.36
h = 4	+50	6.0	23484	27494	+17.07
	+25	5.0	23484	24983	+6.38
	+0	4.0	23484	23484	0.00
	-25	3.0	23484	21164	-9.87
	-50	2.0	23484	20001	-14.83
d = 0.4	+50	0.6	23484	22898	-2.49
	+25	0.5	23484	23189	-1.25
	+0	0.4	23484	23484	0.00
	-25	0.3	23484	23697	+0.91
	-50	0.2	23484	23881	+1.69
p = 5000	+50	12.0	23484	27305	+16.2
	+25	10.0	23484	25177	+7.21
	+0	8.0	23484	23484	0.00
	-25	6.0	23484	21633	-7.81
	-50	4.0	23484	19266	-17.96
a = 0.4, p = 5000	+50	.6 12	23484	35017	+49.1
	+25	.5 10	23484	29411	+25.2
	+0	.4 8	23484	23484	0.00
	-25	.3 6	23484	19247	-18.04
	-50	.2 4	23484	15007	-36.09

$$+\frac{(e^{-a t_1}-e^{-a T})}{a}-e^{-a T}(T-t_1)]$$

$$+\,p[\frac{(1-e^{-a t_1})}{a}-\frac{(t_1+\frac{t_1^{\,2}}{2})e^{-a T}}{(1+t_1)}] \quad \text{............ (8)}$$

Algorithm of Optimality

1. Finding $\frac{\partial TIC}{\partial t_1}$ & $\frac{\partial TIC}{\partial T}$
2. Equating with zero and solving for t_1 & T .
3. Finding $\frac{\partial^2 TIC}{\partial t_1^{\,2}}$ & $\frac{\partial^2 TIC}{\partial T^2}$ at corresponding values of t_1 & T .
4. Checking $\begin{bmatrix} \frac{\partial^2 TIC}{\partial t_1^{\,2}} & \frac{\partial^2 TIC}{\partial T t_1} \\ \frac{\partial^2 TIC}{\partial t_1 T} & \frac{\partial^2 TIC}{\partial T^2} \end{bmatrix} > 0$ & $\frac{\partial^2 TIC}{\partial t_1^{\,2}} > 0$ at corresponding values of t_1 & T .
5. If TIC follows all above criteria then it will attain minimum.

$$\frac{\partial TIC}{\partial t_1}=\frac{e^{-a(t_1+T)}}{2(1+t_1)}[2p(1+t_1)^2 e^{aT}$$

$$+(d+h)t_1(1+t_1)e^{a t_1}-p(2+2t_1+t_1^{\,2})]=0 \quad \frac{\partial TIC}{\partial T}=\frac{e^{-aT}}{2(1+t_1)}[pt_1(2+t_1)$$

$$-(d+h)a(t_1^{\,2}-2T-2t_1T)]=0$$

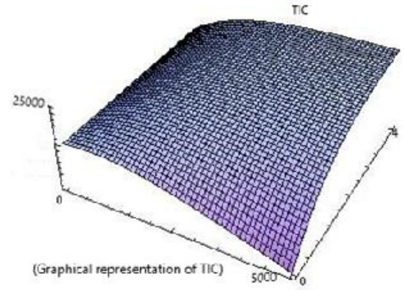
$$\frac{\partial^2 TIC}{\partial t_1^{\,2}}=\frac{e^{-a(t_1+T)}}{(1+t_1)^3}[de^{a t_1}(h+p)-ap(1+t_1)^3 e^{aT}] \quad \frac{\partial^2 TIC}{\partial T^2}=\frac{-a e^{-aT}}{2(1+t_1)}[pt_1(2+t_1)-(d+h)a(t_1^{\,2}-2T-2t_1T)+2(d+h)(1+t_1)]$$

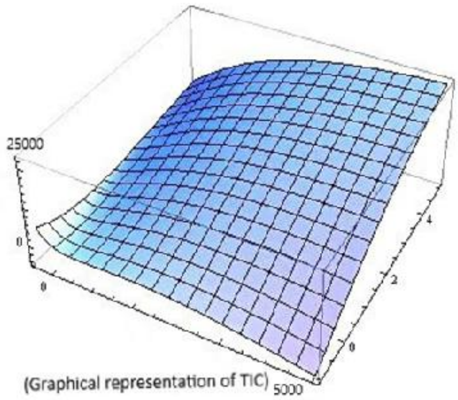
$$\frac{\partial^2 TIC}{\partial t_1 T} = \frac{a^2 e^{-aT}}{2(1+t_1)^2}[(d+h)t_1(2+t_1) - p(2+2t_1+t_1^2)] \quad \frac{\partial^2 TIC}{\partial T t_1} = \frac{a^2 e^{-aT}}{2(1+t_1)^2}[(d+h)t_1(2+t_1) - p(2+2t_1+t_1^2)]$$

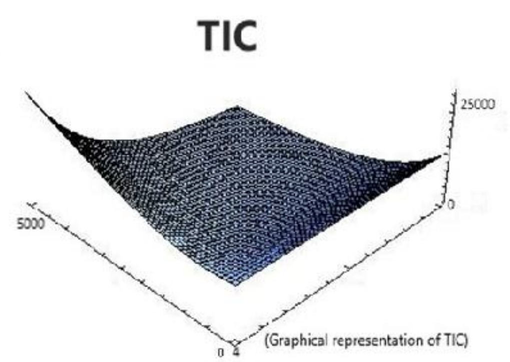
Numerical

Example: In this section, the optimality of total inventory cost and rate of production with respect to demand rate has been tested for a supply chain with the help of numerical data. Based on the following data, the numerical example is used here to illustrate the model. To solve the model help of MATHEMATICA software we have taken. Based on the step-by-step procedure developed above, the optimal values of decision variables and objective function are computed for given inventory model and the results are tabulated. The inventory parameter values: Let $a = 0.4$, $h = Rs4/unit$ $p = Rs5000$ $d = Rs0.4/unit$

Then $t_1 = 0.709601\, year$ & $T = 3.73345\, year$ $TIC = 23{,}484$







Sensitivity Analysis: We have studied the effect of change in parameter values on total inventory cost. The percentages of change in parameters are taken in healthy difference of 25%. From the above analysis we observed that the change in holding cost makes a significant contribution towards change in total inventory cost and change in deteriorating cost has negligible effect on total inventory cost, which are obvious (in electronics gadget, deterioration is negligible and holding price always maintained high). Also, we have been observed that, if we vary two or more parameters at a time, then it will provide sometimes mixed result and sometime comes with better result, which depends on the choice of parameters. In the last numerical table we found around 49% of growth in total inventory cost just by contributing 50% more at p & a .

Conclusion

In this study, we represent an algebraical method to obtain the minimum cost. The demand function follows negative exponential distribution on the time horizon $[0,T]$. In sensitivity analysis of numerical example, we have omitted some fractional values from the observed values to make our calculation little easier and those will not affect our result in a great form. The main contribution of the model is to find minimum total inventory cost. A numerical study of this model is shown graphically. The planned method is simple and does not require boring computational effort. To the author's best knowledge, such type of model has not yet been discussed in the existing literature. There are several extensions of this work could be done by taking different suitable demand rate for any physical situation.

Acknowledgements

I would like to thank my guide for their very helpful corrections suggested in revising the paper. I wish to thanks my parents for their valuable co-operation. I would like to thank Sambalpur University, Odisha, India for infrastructural assistance. Finally, I would also like to thanks to the Honourable Editor-in-Chef and his Editorial Board.

REFERENCES

Ghare, G.F. P.M. 1963. Schrader, A model for exponentially decaying inventory, J. Ind. Eng. 14 238–243.

Covert, R.P. G.C. Philip, An EOQ model for items with Weibull distribution deterioration, AIIE Trans. 5 (1973) 323–326.
Philip, G.C. 1974. A generalized EOQ model for items with Weibull distribution, AIIE Trans. 6 (1974) 159–162.
Bishi, B. J. Behera, S. Sahu, 2018. A deteriorating inventory model with linear, exponential demand with shortage, International Journal of Research in Humanities, Arts and Literature 6, 7, 171-182.
Bishi, B. J. Behera, S. Sahu, An Inventory Production Policy for a Single Parameter Weibull Deteriorating Item With Exponential Demand and Shortages, International Journal of Scientific Research in Computer Science Applications and Management Studies 7 (2018),3, 1-6.
Behera, J. B. Bishi, S. Sahu, Optimal replenishment polices for time dependent deteriorating items with time dependent demand, Journal of Emerging Technologies and Innovative Research, 6 (2019), 6, 282-288.
Sarkar, B. 2013. A production-inventory model with probabilistic deterioration in two-echelon supply chain management, applied mathematical modelling 37(2013) 3138-3151.
Shin, J. S. Kim, Lee, J. 2015. Production and inventory control of auto parts based on predicted probabilistic distribution of inventory, digital communications and networks (2015) 1, 292-301.
Goyal, S.K. 1988. A heuristic for replenishment of trended inventories considering shortages, J. Oper. Res. Soc. 39 885–887.
Datta, T.K. Pal, A.K. 1988. Order level inventory system with power demand pattern for items with variable rate of deterioration, Ind. J. Pure Appl. Math. 19 (1988) 1043–1053.
Goswami, A. Chaudhuri, K.S. 1991. An EOQ model for deteriorating items with shortages and a linear trend in demand, J. Oper. Res. Soc. 42 1105–1110.
Hariga, M. 1996. Optimal EOQ models for deteriorating items with time-varying demand, J. Oper. Res. Soc. 47 1228–1246.
Chang, H.J. C.Y. Dye, 1999. An EOQ model for deteriorating items with time varying demand and partial backlogging, J. Oper. Res. Soc. 50 1176–1182
Goyal, S.K. B.C. Giri, 2001. Recent trends in modeling of deteriorating inventory, Eur. J. Oper. Res. 134 1–16.
Skouri, K. S. Papachristos, 2003. Four inventory models for deteriorating items with time varying demand and partial backlogging: a cost comparison, Opt. Cont. Appl. Methods 24 315–330.
Skouri, K., I. Konstantaras, S. Papachristos, I. Ganas, Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate, Eur. J. Oper. Res. 192 (2009) 79–92.
Sarkar, B. 2012. An EOQ model with delay in payments and time varying deterioration rate, Math. Comput. Mod. 218 (2012) 4881–4891.
Goyal, S.K. 1976. An integrated inventory model for a single supplier-single customer problem, Int. J. Prod. Res. 15 (1976) 107–111.
Banerjee, A. 1986. A joint economic-lot-size model for purchaser and vendor, Decision Sci. 17 292–311.
Khouja, M. 2003. Optimizing inventory decisions in a multi-stage multi-customer supply chain, Trans. Res., Part E: Log. Transp. Rev. 39 193–208.
Cárdenas-Barrón, L.E. Optimal manufacturing batsize with rework in a single-stage production system – a simple derivation, Comput. Ind. Eng. 55
(2008) 758–765
Wee, G.A. H.M. Widyadana, Economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time, Int. J. Prod. Res., in press.
Chung, K.J. L.E. 2012. Cárdenas-Barrón, The complete solution procedure for the EOQ and EPQ inventory models with linear and fixed backorder costs, Math. Comp. Model. 55 2151–2156.

****