RESEARCH ARTICLE

TRANSIENT ANALYSIS OF LEVEL DEPENDENT PERISHABLE INVENTORY SYSTEM IN SUPPLY CHAIN ENVIRONMENT

Mohammad Ekramol Islam[1], Mohammad Ataullah[2, *], Rupen Barua[3] and Ganesh Chandra Ray[4]

[1]Department of Business Administration, Northern University Bangladesh, Dhaka
[2]Assistant Professor of Mathematics, DSHE, Ministry of Education, Bangladesh, Dhaka
[3]Assistant Professor of Mathematics, DSHE, Ministry of Education, Bangladesh, Dhaka
[4]Department of Mathematics, Chittagong University, Bangladesh

ARTICLE INFO

ABSTRACT

This paper analyzes an time dependent(s,S) Inventory system in supply chain environment. In this paper, we consider a level dependent perishable inventory system, where raw materials arrive from two warehouses which are situated nearby the central processing unit. Arrival of demands follows Poisson process with rate. Production takes place when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero unit. It is assumed that the initially inventory level is in $S$and system is in OFF mode. Inventory level decreases due to demands and perish ability. When the inventory level reaches to s then the system converted ON mode from OFF mode. The production follows exponentially distributed with parameterμ. Perish ability follows exponentially distributed with parameter θ. Perish ability will be level dependent that is rate of perish ability will depend on the amount of inventory available in the stock. Transient State analysis is made and some system characteristics are evaluated by numerical illustration.

INTRODUCTION

The analysis of inventory systems is primarily focused on the tactical question of which inventory control policies to use and the operational questions of when and how much inventory to order. By and large, these are the main questions for managing the inventory of perishable items as well. A lot of work has been done in inventory modeling with remarkable consideration of perishability of the items. Huge literature can be found in Nahmias S. (1982) and later by the same author in (Nahmias, 2011). Karaesmen et al. (2009) also mentioned the inventory problem in future directions. But recently inventory system with supply chain management addressed by few researchers. Data and Pal (1990) extended the model to the case. in which the demand rate of an item is dependent on the instantaneous inventory level until a given inventory level is achieved, after which the demand rate becomes constant. They assumed that at the end of each cycle, the inventory level is zero. Hwang and Hahn (2000) dealt with an optimal procurement policy of perishable item with stock dependent demand rate and FIFO issuing policy. Since the stock-dependent demand rate implicitly implies that all items in inventory are displayed for sale, the customers enforce the issuing policy and last-in-first-out (LIFO) issuing is a natural choice with prudent customers who are always looking for the freshest ones among the displayed items. On the other hand, most retailers arrange their displayed goods from the oldest up or front to the new goods down or back hoping that customers may pick the oldest ones first, which results in first-in-first-out (FIFO) issuing. Consequently, among displayed goods some are sold by LIFO principle while others by FIFO principle, which we call mixed issuing policy. Sudesh et al. (13) consider a two-heterogeneous servers queue with system disaster, server failure and repair, the customers become impatient when the system is down.

*Corresponding author: **Mohammad Ataullah,**
Assistant Professor of Mathematics, DSHE, Ministry of Education, Bangladesh, Dhaka.

The customers arrive according to a Poisson process and service time follows exponential distribution. Each customer requires exactly one server for its service and the customers select the servers on fastest server first basis. Blackburn and Scudder (Blackburn, 2009) discussed their paper, the challenge for companies in managing the supply chain of perishable foods is that the value of the product deteriorates significantly over time at rates that are highly dependent on the environment. Leat (2013) discussed that in the future food system will have to joint four major characteristics: resilience, sustainability, competitiveness, and ability to manage and meet customer expectations. Ahmad Al Hanbali and Onno Boxma (2009) studied the transient behavior of a state dependent M/M/1/K queue during the busy period. We derive in closed-form the joint transform of the length of the busy period, the number of customers served during the busy period, and the number of losses during the busy period. For two special cases called the threshold policy and the static policy we determine simple expressions for their joint transform. Mohammad Ekramol Islam (2015), discussed a perishable *(s,S)* inventory system with postponed demands. They assume that customers arrive to the system according to a Poisson process with rate $\lambda > 0$ When inventory level depletes to S due to demands or decay or service to a pooled customer, an order for replenishment is placed. The lead time is exponentially distributed with parameter γ. When inventory level reaches zero, the incoming customers are sent to a pool of capacity *M*. Any demand that takes place when the pool is full and inventory level is zero, is assumed to be lost. After replenishment, as long as the inventory level is greater than *s*, the pooled customers are selected according to an exponentially distributed time lag, with rate depending on the number in the pool. G. Arul FreedaVinodhini and Vidhya (2014) consider an M/M/c queuing system, which occasionally suffers disastrous failure and all the customers are lost. The repair mechanism starts immediately. When the system is down, the stream of arrivals continues. However, the new arrivals become impatient and activate their own timer.

In this model we consider, a level dependent perishable inventory system, where raw materials arrive from two warehouses which are situated nearby the central processing unit. Production takes place when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero unit. It is assumed that the initial level is in *S* and system is in *OFF* mode. Inventory level decreases due to demands and perish ability. When the inventory level reaches to *s* then the system converted *ON* mode from *OFF* mode. The production follows exponentially distributed with parameter μ. Perishability follows exponentially distributed with parameter θ. Perishability will be level dependent. When inventory reaches to order level *S* system converted to *ON* to *OFF* mode. This paper is extension of the work of Mohammad Ekramol Islam et al. (2018).

The paper is arranged in the following ways. In section: 1. Introduction, 2. Assumptions, 3. Notations, 4. Model Analysis, 5. System Characteristics, 6. Cost Function of the system, 7. Numerical Illustration, 8. Graphical Presentation of the System, 9. Numerical Results and Discussion, 10. Conclusion, 11. References

Assumption

i)   Initially the inventory level is **S**
ii)   Demands arrive according to Poisson process with rate **λ**
iii)   Raw materials arrive from two warehouses, situated nearby the central processing unit,
iv)   Production occurred when at least one component of each category is available in both the warehouses,
v)   Replenishment for the warehouses instantaneous.
vi)   When inventory level reaches to re-order level *s*then the system converted to **OFF** mode to **ON** mode and production starts,
vii)   When the inventory level reaches to zero, the arriving demands are lost forever.
viii) Production will be **ON** until the inventory level reaches to order level*S*. Production follow exponentially distributed with parameter*μ*.
ix)   Perishability follows exponentially distributed with parameter *θ.*
x)   Perishability will be level dependent i.e., rate of Perishability will depend the amount of inventory available in the stock

3.Notations:

a)   **S** $\rightarrow$ Maximum Inventory Level (Order level)
b)   *s* $\rightarrow$ Re-order level
c) $\lambda \rightarrow$ Demand rate
d)   **Q_1** $\rightarrow$ Amount of first warehouse component
e)   **Q_2** $\rightarrow$ Amount of second warehouse component
f)   *$\mu \rightarrow$*   Production rate
g)   *I(t)* $\rightarrow$ Inventory Level at time *t*
h)   *X(t)*   =   *1* if production is in**ON** mode
                                0 if production is in**OFF** mode
i)   **$W_1(t)$** $\rightarrow$ Warehouse - 1
j)   **$W_2(t)$** $\rightarrow$ Warehouse – 2
k)   **E**$\rightarrow$ $E_1 \times E_2 \times E_3 \times E_4 \rightarrow$ the state space of the process

$E_1 = \{0, 1, ----- S\}$

$E_2 = \{0, 1, ----- Q_1\}$

$E_3 = \{0, 1, ------ Q_2\}$

$E_4 = \{0, 1\}$

i) *e* is the component column vector of *I*'s.
j) $i\theta$ -->Level dependent inventory perishable rate.

Model and Diagram Analysis: In this model, the inventory system starts with order level *S* units of the item on stock. Demands arrive according to Poisson process with rateλ. We consider a level dependent perishable inventory system, where raw materials arrive from two warehouses, situated nearby the central processing unit. Production occurred when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero. When inventory level reaches to re-order level *s* then the system converted to **OFF** mode to **ON** mode and production starts. When the inventory level reaches to zero then arriving demand is lost forever. Production will be **ON** until the inventory level reaches to order level**S**. Production up to the inventory *s+1*system is act as a death process as the system is in **OFF** mode. But from *s* onwards, the system is act

as a birth & death process as the system is in **ON** Mode. Production follows exponentially distributed with parameterμ. Perishability follows exponentially distributed with parameter θ.

The Infinitesimal generator $\tilde{A}$ of the four dimensional Markov Process:

$[I(t), W_1(t), W_2(t), X(t); t \geq 0]$ can be defined

$$\tilde{A} = (a(i, j, k, l: u, v, w, y)) : (i, j, k, l), (u, v, w, y) \in E$$

5.Transient state Analysis:

Let us assumed $I(0) = S$and $w_1(0) = 0, w_2(0) = 0, X(0) = 0$. Let us consider the transition probabilities:
$P_{(S,Q_1,Q_2,0)(i,j,k,l)}(t)$

$= P\{I(t), w_1(t), w_2(t), X(t) = (i, j, k, l) | I(0), w_1(t), w_2(t), X(0) = (S, Q_1, Q_2, 0)\}$

From now onwards we can write

$P_{(i,j,k,l)}(t) \text{for} P_{(S,Q_1,Q_2,0)(i,j,k,l)}(t)$

Kolmogorov difference differential equations for the system $P_{(i,j,k,l)}(t)$are given bellow:

When system is OFF mode:

$$P'(t)_{SQ_1Q_20} =- (\lambda + 5\theta)p'(t)_{SQ_1Q_20} + Q_2 p'(t)_{SQ_100} + Q_1 p'(t)_{S0Q_20} \tag{1}$$

$$P'(t)_{SQ_1(Q_2-1)0} =- (\lambda + 5\theta)p'(t)_{SQ_1(Q_2-1)0} + Q_1 p'(t)_{S0(Q_2-1)0} \tag{2}$$

$$P'(t)_{SQ_100} =- (\lambda + 5\theta + Q_2)p'(t)_{SQ_100} + Q_1 p'(t)_{S000} \tag{3}$$

$$P'(t)_{S(Q_1-1)Q_20} =- (\lambda + 5\theta)p'(t)_{S(Q_1-1)Q_20} + Q_2 p'(t)_{S(Q_1-1)00} \tag{4}$$

$$P'(t)_{S0Q_20} = -(\lambda + 5\theta + Q_1)p'(t)_{S0Q_20} + Q_2 p'(t)_{S000} \tag{5}$$

$$P'(t)_{S(Q_1-1)(Q_2-1)0} =- (\lambda + 5\theta)p'(t)_{S(Q_1-1)(Q_2-1)0} + \mu p'(t)_{(S-1)Q_21} \tag{6}$$

$$P'(t)_{S(Q_1-1)00} =- (\lambda + 5\theta + Q_2)p'(t)_{S(Q_1-1)00} + \mu p'(t)_{(S-1)Q_1(Q_2-1)1} \tag{7}$$

$$P'(t)_{S0(Q_2-1)0} =- (\lambda + 5\theta + Q_1)p'(t)_{S0(Q_2-1)0} + \mu p'(t)_{(S-1)(Q_1-1)Q_21} \tag{8}$$

$$P'(t)_{S000} =- (\lambda + 5\theta + Q_1 + Q_2)p'(t)_{S000} + \mu p'(t)_{S(Q_1-1)(Q_2-1)1}$$

$$\tag{9}$$

$$P'(t)_{SQ_1Q_20} = (\lambda + 5\theta)p'(t)_{SQ_1Q_20-}$$

$(\lambda + 4\theta)p'(t)_{(S-1)Q_1Q_20} + +Q_2x_{(S-1)Q_100} + Q_1p'(t)_{(S-1)0Q_20}$

$$(10)$$

$P'(t)_{5210} = -(\lambda + 4\theta) + (\lambda + 5\theta)p'(t)_{5210} + p'(t) + Q_1p'(t)_{4010} \tag{11}$

$P'(t)_{5200} = (\lambda + 5\theta)p'(t)_{5200} - (\lambda + 4\theta + Q_2)p'(t)_{4200} + +Q_1p'(t)_{4000} \tag{12}$

$P'(t)_{5120} = (\lambda + 5\theta)p'(t)_{5120} - (\lambda + 4\theta)p'(t)_{4120} + Q_2p'(t)_{4100} \tag{13}$

$P'(t)_{5020} = (\lambda + 5\theta)p'(t)_{5020} - (\lambda + 4\theta + Q_1)p'(t)_{4020} + Q_2p'(t)_{4000}$

$$(14)$$

$P'(t)_{5110} = (\lambda + 5\theta)p'(t)_{5110} - (\lambda + 4\theta)p'(t)_{4110} \tag{15}$

$P'(t)_{5100} = (\lambda + 5\theta)p'(t)_{5100} - (\lambda + 4\theta + Q_2)p'(t)_{4100} \tag{16}$

$P'(t)_{5010} = (\lambda + 5\theta)p'(t)_{5010} - (\lambda + 4\theta + Q_1)p'(t)_{4010} \tag{17}$

$P'(t)_{5000} = (\lambda + 5\theta)p'(t)_{5000} - (\lambda + 4\theta + Q_1 + Q_2)p'(t)_{4000} \tag{18}$

$P'(t)_{4220} = (\lambda + 4\theta)p'(t)_{4220} - (\lambda + 3\theta)p'(t)_{3220} + Q_2p'(t)_{3200} + Q_1p'(t)_{3020} \tag{19}$

$P'(t)_{4210} = (\lambda + 4\theta)p'(t)_{4210} - (\lambda + 3\theta)p'(t)_{3210} + Q_1p'(t)_{3010} \tag{20}$

$P'(t)_{4200} = (\lambda + 4\theta)p'(t)_{4200} - (\lambda + 3\theta + Q_2)p'(t)_{3200} + Q_1p'(t)_{3000} \tag{21}$

$P'(t)_{4120} = (\lambda + 4\theta)p'(t)_{4120} - (\lambda + 3\theta)p'(t)_{3120} + Q_2p'(t)_{3100} \tag{22}$

$P'(t)_{4020} = (\lambda + 4\theta)p'(t)_{4020} - (\lambda + 3\theta + Q_1)p'(t)_{3020} + Q_2p'(t)_{3000} \tag{23}$

$P'(t)_{4110} = (\lambda + 4\theta)p'(t)_{4110} - (\lambda + 3\theta)p'(t)_{3110} \tag{24}$

$P'(t)_{4100} = (\lambda + 4\theta)p'(t)_{4100} - (\lambda + 3\theta + Q_2)p'(t)_{3100} \tag{25}$

$P'(t)_{4010} = (\lambda + 4\theta)p'(t)_{4010} - (\lambda + 3\theta + Q_1)p'(t)_{3010} \tag{26}$

$P'(t)_{4000} = (\lambda + 4\theta)p'(t)_{4000} - (\lambda + 3\theta + Q_1 + Q_2)p'(t)_{3000} \tag{27}$

When the system is ON mode:

$P'(t)_{4221} = -(\lambda + 4\theta + \mu)p'(t)_{4221} + p'(t) + Q_1p'(t)_{4021} \tag{28}$

$P'(t)_{4211} = -(\lambda + 4\theta + \mu)p'(t)_{4211} + Q_1p'(t)_{4011} \tag{29}$

$P'(t)_{4201} = -(\lambda + 4\theta + Q_2)p'(t)_{4201} + Q_1p'(t)_{4001} \tag{30}$

$P'(t)_{4121} = -(\lambda + 4\theta + \mu)p'(t)_{4121} + Q_2p'(t)_{4101} \tag{31}$

$P'(t)_{4021} = -(\lambda + 4\theta + Q_1)p'(t)_{4021} + Q_2p'(t)_{4001} \tag{32}$

$P'(t)_{4111} = -(\lambda + 4\theta + \mu)p'(t)_{4111} + \mu p'(t)_{3221} \tag{33}$

$P'(t)_{4101} = -(\lambda + 4\theta + Q_2)p'(t)_{4101} + \mu p'(t)_{3211} \tag{34}$

$P'(t)_{4011} = -(\lambda + 4\theta + Q_1)p'(t)_{4011} + \mu p'(t)_{3121} \tag{35}$

$P'(t)_{4001} = -(\lambda + 4\theta + Q_1 + Q_2)p'(t)_{4001} + \mu p'(t)_{3111} \tag{36}$

$P'(t)_{4221} = (\lambda + 4\theta)p'(t)_{4221} - (\lambda + 3\theta + \mu)p'(t)_{3221} + Q_2p'(t)_{3201} + Q_1p'(t)_{3021} \tag{37}$
$P'(t)_{4211} = (\lambda + 4\theta)p'(t)_{4211} - (\lambda + 3\theta + \mu)p'(t)_{3211} + Q_1p'(t)_{3011} \tag{38}$

$P'(t)_{4201} = (\lambda + 4\theta)p'(t)_{4201} - (\lambda + 3\theta + Q_2)p'(t)_{3201} + Q_1p'(t)_{3001} \tag{39}$

$P'(t)_{4121} = (\lambda + 4\theta)p'(t)_{4121} + (\lambda + 4\theta)p'(t)_{4020} - (\lambda + 3\theta + \mu)p'(t)_{3121} + Q_2p'(t)_{3101} \tag{40}$

$$P'(t)_{4021} = (\lambda + 4\theta)p'(t)_{4021} - (\lambda + 3\theta + Q_1)p'(t)_{3021} + Q_2 p'(t)_{3001} \tag{41}$$

$$P'(t)_{4111} = (\lambda + 4\theta)p'(t)_{4111} - (\lambda + 3\theta + \mu)p'(t)_{3111} + \mu p'(t)_{2221} \tag{42}$$

$$P'(t)_{4101} = (\lambda + 4\theta)p'(t)_{4101} - (\lambda + 3\theta + Q_2)p'(t)_{3101} + \mu p'(t)_{2211} \tag{43}$$

$$P'(t)_{4011} = (\lambda + 4\theta)p'(t)_{4011} - (\lambda + 3\theta + Q_1)p'(t)_{3011} + \mu p'(t)_{2121} \tag{44}$$

$$P'(t)_{4001} = (\lambda + 4\theta)p'(t)_{4001} - (\lambda + 3\theta + Q_1 + Q_2)p'(t)_{3001} + \mu p'(t)_{2111} \tag{45}$$

$$P'(t)_{3221} = (\lambda + 3\theta)p'(t)_{3221} - (\lambda + 2\theta + \mu)p'(t)_{2221} + Q_2 p'(t)_{2201} + p'(t)Q_1 x_{2021} \tag{46}$$

$$P'(t)_{3211} = (\lambda + 3\theta)p'(t)_{3211} + (\lambda + 3\theta)p'(t)_{3210} - (\lambda + 2\theta + \mu)p'(t)_{2211} + Q_1 p'(t)_{2011} \tag{47}$$

$$P'(t)_{3201} = (\lambda + 3\theta)p'(t)_{3201} + (\lambda + 3\theta)p'(t)_{3200} - (\lambda + 2\theta + Q_2)p'(t)_{2201} + Q_1 p'(t)_{2001} \tag{48}$$

$$P'(t)_{3121} = (\lambda + 3\theta)p'(t)_{3121} + (\lambda + 3\theta)p'(t)_{3120} - (\lambda + 2\theta + \mu)p'(t)_{2121} + Q_2 p'(t)_{2101} \tag{49}$$

$$P'(t)_{3021} = (\lambda + 3\theta)p'(t)_{3021} + (\lambda + 3\theta)p'(t)_{3020} - (\lambda + 2\theta + Q_1)p'(t)_{2021} + Q_2 p'(t)_{2001} \tag{50}$$

$$P'(t)_{3111} = (\lambda + 3\theta)p'(t)_{3111} + (\lambda + 3\theta)p'(t)_{3110} - (\lambda + 2\theta + \mu)p'(t)_{2111} + \mu p'(t)_{1221} \tag{51}$$

$$P'(t)_{3101} = (\lambda + 3\theta)p'(t)_{3101} + (\lambda + 3\theta)p'(t)_{3100} - (\lambda + 2\theta + Q_2)p'(t)_{2101} + \mu p'(t)_{1211} \tag{52}$$

$$P'(t)_{3011} = (\lambda + 3\theta)p'(t)_{3011} + (\lambda + 3\theta)p'(t)_{3010} - (\lambda + 2\theta + Q_1)p'(t)_{2011} + \mu p'(t)_{1121} \tag{53}$$

$$P'(t)_{3001} = (\lambda + 3\theta)p'(t)_{3001} + (\lambda + 3\theta)p'(t)_{3000} - (\lambda + 2\theta + Q_1 + Q_2)p'(t)_{2001} + \mu p'(t)_{1111} \tag{54}$$

$$P'(t)_{2221} = (\lambda + 2\theta)p'(t)_{2221} - (\lambda + \theta + \mu)p'(t)_{1221} + Q_2 p'(t)_{1201} + Q_1 p'(t)_{1021} \tag{55}$$

$$P'(t)_{2211} = (\lambda + 2\theta)p'(t)_{2211} - (\lambda + \theta + \mu)p'(t)_{1211} + Q_1 p'(t)_{1011} \tag{56}$$

$$P'(t)_{2201} = (\lambda + 2\theta)p'(t)_{2201} - (\lambda + \theta + Q_2)p'(t)_{1201} + Q_1 p'(t)_{1001} \tag{57}$$

$$P'(t)_{2121} = (\lambda + 2\theta)p'(t)_{2121} - (\lambda + \theta + \mu)p'(t)_{1121} + Q_2 p'(t)_{1101} \tag{58}$$

$$P'(t)_{2021} = (\lambda + 2\theta)p'(t)_{2021} - (\lambda + \theta + Q_1)p'(t)_{1021} + Q_2 p'(t)_{1001} \tag{59}$$

$$P'(t)_{2111} = (\lambda + 2\theta)p'(t)_{2111} - (\lambda + \theta + \mu)p'(t)_{1111} + \mu p'(t)_{0221} \tag{60}$$

$$P'(t)_{2101} = (\lambda + 2\theta)p'(t)_{2101} - (\lambda + \theta + Q_2)p'(t)_{1101} + \mu p'(t)_{0211} \tag{61}$$

$$P'(t)_{2011} = (\lambda + 2\theta)p'(t)_{2011} - (\lambda + \theta + Q_1)p'(t)_{1011} + \mu p'(t)_{0121} \tag{62}$$

$$P'(t)_{2001} = (\lambda + 2\theta)p'(t)_{2001} - (\lambda + \theta + Q_1 + Q_2)p'(t)_{1001} + \mu p'(t)_{0111} \tag{63}$$

$$P'(t)_{1221} = (\lambda + \theta)p'(t)_{1221} - \mu p'(t)_{0221} + Q_2 p'(t)_{0201} + Q_1 p'(t)_{0021} \tag{64}$$

$$P'(t)_{1211} = (\lambda + \theta)p'(t)_{1211} - \mu p'(t)_{0211} + Q_1 p'(t)_{0011} \tag{65}$$

$$P'(t)_{1201} = (\lambda + \theta)p'(t)_{1201} - Q_2 p'(t)_{0201} + Q_1 p'(t)_{0001} \tag{66}$$

$$P'(t)_{1121} = (\lambda + \theta)p'(t)_{1121} - \mu p'(t)_{0121} + Q_2 p'(t)_{0101} \tag{67}$$

$$P'(t)_{1021} = (\lambda + \theta)p'(t)_{1021} - Q_1 p'(t)_{0021} + Q_2 p'(t)_{0001} \tag{68}$$

$$P'(t)_{1111} = (\lambda + \theta)p'(t)_{1111} - \mu p'(t)_{0111} \tag{69}$$

$$P'(t)_{1101} = (\lambda + \theta)p'(t)_{1101} - Q_2 p'(t)_{0101} \tag{70}$$

$$P'(t)_{1011} = (\lambda + \theta)p'(t)_{1011} - Q_1 p'(t)_{0011} \tag{71}$$

$$P'(t)_{1001} = (\lambda + \theta)p'(t)_{1001} - (Q_1 + Q_2)p'(t)_{0001} \tag{72}$$

We solve this system of ordinary differential equations by using the Runge-Kutta method of fourth order based on specific parameters. We have plotted the graphs based on ODE's and performance measures.
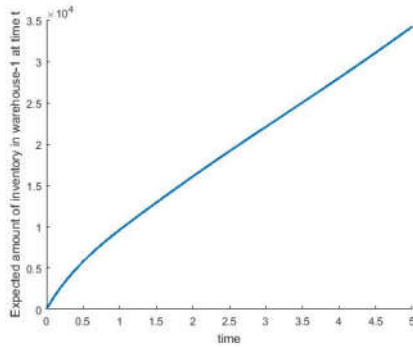
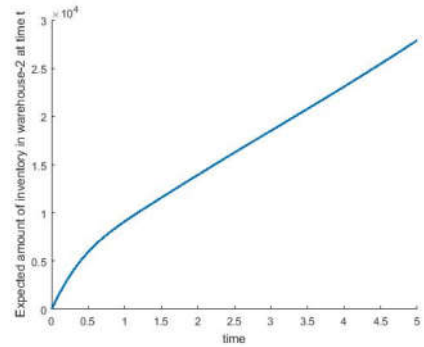**Fig.1: Expected inventory in warehouse-1 vs Time**

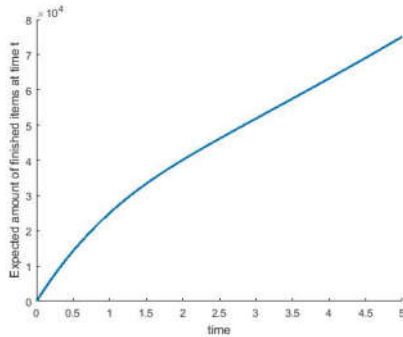**Fig.2: Expected inventory in warehouse-2 vs Time**

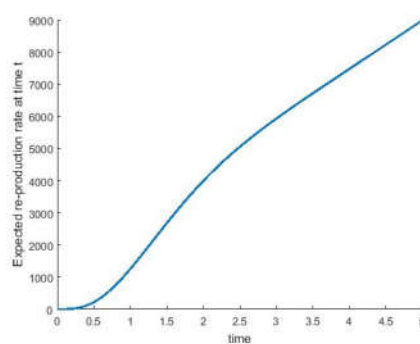**Fig.3: Expected inventory for finished items vs Time**

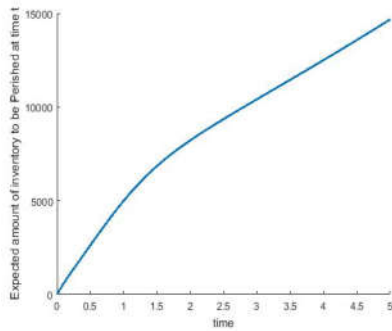**Fig.4: Expected reorder rate vs Time**

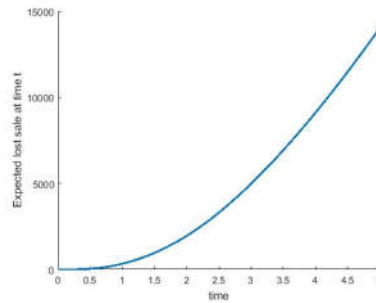**Fig.5: Expected amount of Perishable items vs Time**
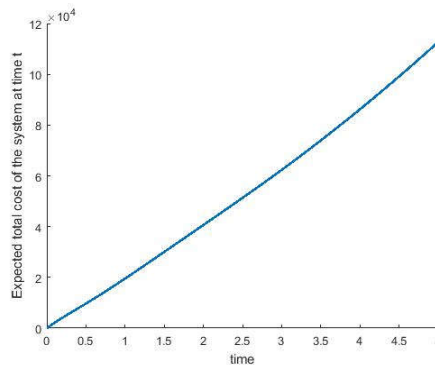
Fig.6: Expected lost sale vs Time

Fig.7: Expected total cost of the system vs Time

The effect of various parameters on the system performance measures such as expected number of customers in the system and mean waiting time in the system are studied. MATLAB software is used to develop the computational program.

6. System Characteristics:

a) Expected total inventory of the system:

$$L = \sum_{i=s+1}^{S} i \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{i,j,k,0} + \sum_{i=1}^{S-1} i \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{i,j,k,1}$$

b) Re-production rate of the system:

$$R = \lambda \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{s+1,j,k,0}$$

c) Number of customers lost in the system:

$$CL = \lambda \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{0,j,k,1}$$

d) Expected amount of inventory in warehouse-1:

$$W_1 = \sum_{j=1}^{Q_1} j \sum_{k=0}^{Q_2} \sum_{i=s+1}^{S} x^{i,j,k,0} + \sum_{j=1}^{Q_1} j \sum_{k=0}^{Q_2} \sum_{i=1}^{S-1} x^{i,j,k,1}$$

e) Expected amount of inventory in warehouse-2

$$W_2 = \sum_{k=1}^{Q_2} k \sum_{j=0}^{Q_1} \sum_{i=s+1}^{S} x^{i,j,k,0} + \sum_{k=1}^{Q_2} k \sum_{j=0}^{Q_1} \sum_{i=1}^{S-1} x^{i,j,k,1}$$

f) Expected amount to be perished.

$$P = \sum_{i=s+1}^{S} i\theta \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{i,j,k,0} + \sum_{i=1}^{S-1} i\theta \sum_{j=0}^{Q_1} \sum_{k=0}^{Q_2} x^{i,j,k,1}$$

7. Cost Function of the system:

c_1 =Holding cost of the system,
c_2 =Re-switching cost of the system
c_3 =Cost of customer lost in the system,
c_4 =Inventory holding cost in warehouse -1
c_5 = Inventory holding cost in warehouse -2,
c_6 = Expected amount to be perished

So expected total cost of the system: $E(TC) = C_1 L + C_2 R + C_3(CL) + C_4 W_1 + C_5 W_2 + C_6 P$

8. Numerical Illustration:

By giving values to the underlying parameters we provide some numerical illustrations. Take

S = 5, s = 2, $Q_1 = Q_2 = 2$, $\lambda = 2, \mu = 2.1, \theta = 0.2$,
c_1 =0.25, c_2 =2.5, c_3 =3, c_4 =0.35, c_5 =0.36, $c_6 = 0.5$

9. Graphical Presentation of the System:

10. Numerical Results and Discussion:

Numerical outcomes have been gotten by applying Runge-Kutta fourth request technique in the arrangement of standard differential conditions (1) through (72) with the assistance of computational programming MATLAB R2016a for the parametric qualities steady with time interim 0≤t≤5 as appeared in the diagrams. Diagrams of different parameters versus time have been demonstrated the fig.1 through fig.7. Fig.1 investigates expected inventory in warehouse-1 with time yet for long time interim it is

increase rate lower than from beginning stage. Fig.2 demonstrates expected inventory in warehouse-2, which raises quickly from starting and after some time passing, it continue slowly. Fib.3 Explain the quantity of finished product with time variety that reflects almost same characteristics in figure 2. From Fig.4 we see that the rate of reorder is low at the beginning but increases in a short time , which is reason by the way that underlying likelihood has been lessened essentially because of which some the probabilities have their qualities zero at first, after some time it increases with a parameter. Fib.5 exhibits Expected amount of Perishable items with time. Fig.6 shows the lost sale of the system, which increase almost parabolic shape, it should be take care for the business goodwill. It may be tackle by increasing production rate. Fig.7 demonstrate the normal aggregate expense of the framework which demonstrate that at first aggregate expense is higher than whatever remains of era considered in the model.

Conclusion

Investigation of single item inventory model under time subordinate entry and administration rates has been made .Under the examination the numerical outcomes for different execution measures have been acquired by utilizing Runge-kutta fourth request strategy with MATLAB R2016a. Our model can be considered under various servers' arrangement which may give progressively broad arrangement under time subordinate circumstances in order to make the model increasingly reasonable.

REFERENCES

Ahmad Al Hanbali and OnnoBoxma 2009. Transient Analysis of the State Dependent M/M/1/K Queue. EURANDOM, Eindhoven, The Netherlands {a.al.hanbali, o.j.boxma}@tue.nl

Arul FreedaVinodhini, G. and V. Vidhya(2014).Transient Solution of a Multi-Server Queue with Catastrophes and Impatient Customers when System is Down. Applied Mathematical Sciences, Vol. 8, 2014, no. 92, 4585 – 4592, HIKARI Ltd, www.m-hikari.com, http://dx.doi.org/ 10.12988/ ams.2014.45388

Blackburn, J. and Scudder, G. 2009. Supply Chain Strategies for Perishable Products: The Case of Fresh Produce. Production and Operations Management 18(2): 129-137.

Datta, T.K., A.K. Pal. 1990. A note an inventory model with inventory-level-dependent rate. Journal of the Operational Research Society 41 (10) 971-975.

Hwang. H., K.H. Hahn. 2000. An optimal procurement policy for items with an inventory level-dependent demand rate and fixed lifetime. European Journal of Operations Research 127 537-545.

Karaesmen, I., Scheller-Wolf A, Deniz B. Managing Perishable and Aging inventories: Review and Future Research Directions. Kempf K, Keshinocak A, Uzsoy P, editors. Handbook of Production Planning. Kluwer Academic Publishers; 2009. To appear.

Leat, P. and Revoredo-Giha, C. 2013. Risk and resilience in agri-food supply chains: the case of the ASDA Porklink supply chain in Scotland. Supply Chain Management: An International Journal 18(2): 219-231.

Mohammad Ekramol Islam, Perishable *(s,S)* Inventory System with Postponed Demands, NUB Journal of Applied sciences, Vol-1, No-1, 2015.

Mohammad Ekramol Islam, RupenBarua, Ganesh Chandra, Ray 2018: Level Dependent Production Inventory system in supply chain Environment: Global Scientific Journal, Vol.6, pp.40-58, December 2018.

Nahmias, S. 2011. Perishable Inventory Systems, International Series in Operations Research and Management, 160, springer (2011)

Nahmias, S. 1982. Perishable inventory theory: a review Oper Res 1982;30(4):680-708.

Sudhesh, R., P. Savitha and S. Dharmaraja, 2017. Transient analysis of a two-heterogeneous servers queue with system disaster, server repair and customers' impatience, TOP (2017) 25:179–205, DOI 10.1007/s11750-016-0428-x

****