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RESEARCH ARTICLE

FUZZY INVENTORY MODEL USING PENALTY COST AND SHORTAGE COST WITH FINITE AND INFINITE PRODUCTION RATE

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ABSTRACT

In this paper an inventory model using penalty cost and shortage cost are formulated. The aim of this research work is to minimize the time period, the order quantity and the total cost. The model is developed for both finite and infinite production rate. The demand is considered as constant demand. To achieve this the formulated inventory model is converted to fuzzy inventory model by considering the parameters holding cost, demand, shortage cost and setup cost as trapezoidal fuzzy numbers. To find the value of optimum time period, optimum order quantity and optimum total cost, signed distance method is used for defuzzification. Numerical examples have been given in order to explain the model clearly. Sensitivity analysis is also given.

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INTRODUCTION

An inventory consists of raw materials, work in progress or finished goods. Effective inventory control is essential for manufacturing organizations for many reasons. The objective of many inventory problems is to deal with minimization of totla cost . Thus it is essential to determine a suitable inventory model to meet the future demand. The first quantitative treatment of inventory was the simple EOQ model. This model was developed by Harris *et al*. (1915). Wilson (Wilson, 1934) showed interest in developing EOQ model in academics and industries. Hadely *et al*. (1963) analysed many inventory models. Uncertainity is most important phenomenon in real life inventory problems. This can be approached by probabilistic methods. But there are uncertainties that cannot be appropriately treated by usual probabilistic models. To obtain inventory optimization in such environment , fuzzy set theory is considered as more convenient than probability theory. Fuzziness in inventory problems was first introduced by Zadeh (1965). He proposed some strategies for decision making in fuzzy environment. Kacpryzk *et al*. (1982) discussed some long term inventory policy making through fuzzy decision making models. Dutta and Kumar (2012) developed fuzzy inventory model without shortages using fuzzy trapezoidal number and used Signed distance method for defuzzification. Products like fruits, vegetables and bakery items do not deteriorate at the beginning but deteriorate continuously after some time. As a result the selling price of the product decreases which is called as penalty cost. Jaggi *et al*. (2016) developed fuzzy inventory model with deterioration where demand was taken as time -varying. Kumar and Rajput (2015) developed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Saha and chakra borty (Saha, 2012) developed inventory model with time dependent demand and deterioration with shortages. Srivastava and Gupta (10) have proposed an EOQ model for time deteriorating items using penalty cost. Fujiwara and Perera (1993) developed an inventory model for time continuously deteriorating items using linear and penalty cost. Pevekar and Nagare (2015) discuused an inventory model for timely deteriorating items considering penalty cost and shortage cost. Nalini Prava Behera and Pradip Kumar Tripathy (2016) developed an inventory model for time deteriorating items with penalty cost using trapezoidal fuzzy numbers. G.C. Mahata and A. Goswami (2009) discussed an inventory model in which they have considered the demand as

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stock dependent demand and the holding cost as nonlinear holding cost. Hesham k. Alfares1 (2014) developed an inventory model with finite production rate and variable holding cost. M. Vijayashree and R. Uthayakumar proposed an inventory model with finite and infinite production rate , shortage cost and penalty cost also they included in the model. In this research paper, an inventory model for time deteriorating item with penalty cost and shortage cost is discussed. More over the holding cost, setup cost, shortage cost and demand are taken as trapezoidal fuzzy number . Also in this research paper signed distance methods is used for defuzzification. The aim of this paper is to optimize the time period, and to find the order quantity and total cost with the help of fuzzy numbers. This research paper reveals that trapezoidal fuzzy number yields optimum total cost than the total cost obtained in crisp model.

Preliminaries

Definition

Let X be a nonempty set. Then a fuzzy set A in X (ie., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0,1]$. Such a function μ_A is called the membership function and for each xϵX , $\mu_{\tilde{A}}(x)$is the degree of membership of x (membership grade of x) in the fuzzy set A.

In other words, A fuzzy set $\tilde{A} = \{(x, \mu_A(x))/ x\varepsilon X\}$where $\mu_A : X \to [0,1]$ Ғ(X) denotes the collection of all fuzzy sets in X, called the fuzzy power set of X.

Definition

A fuzzy set is a fuzzy number if it satisfies the following four conditions

- It is a convex set
- It is normalised
- It is defined on the real number R
- It is piecewise continuous

Definition (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A}$ =(a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}}(x)=\begin{cases} L(x) = \dfrac{x-a}{b-a}, & when\, a \le x \le b \\ 1 & when\ b \le x \le c \\ R(x) = \dfrac{d-x}{d-c} & when\, c \le x \le d \\ 0 & otherwise \end{cases}$$

Definition

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers then the arithmetical operations are defined as

1. $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\tilde{A} \Theta \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = \left(\dfrac{a_1}{b_4}, \dfrac{a_2}{b_3}, \dfrac{a_3}{b_2}, \dfrac{a_4}{b_1}\right)$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \ge 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) \alpha < 0 \end{cases}$

Definition (Signed distance method)

Let $\widetilde{A}$ be a fuzzy set defined on R. then the signed distance of $\widetilde{A}$ is defined as $d_F(\widetilde{A}) = \frac{A_1 + A_2 + A_3 + A_4}{4}$ for defuzzifying the trapezoidal fuzzy number.

Linear penalty cost function: A linear penalty cost function $P(t) = \pi(t-\theta)$, $t \geq \theta$, which gives the cost of keeping one unit of product in stock until age t , where θ be the time period at which deterioration of product starts and π is constant.

Assumptions and Notations

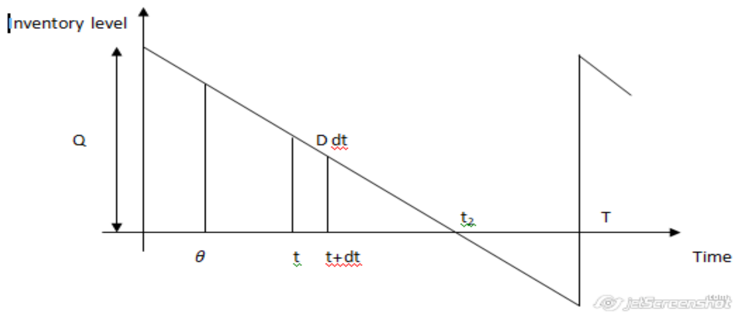
Assumptions

- The inventory system involves production of single item.
- Lead time is zero and shortages are allowed.
- Demand rate is constant.
- Replenishment is instantaneous.

Notations

Q	-	number of items at the beginning of the time period.
A	-	shortage cost per unit time.
$\widetilde{A}$	-	fuzzy shortage cost
θ	-	time period at which deterioration starts.
$\widetilde{\theta}$	-	fuzzy deterioration.
T	-	cycle length.
H	-	holding cost per unit time.
$\widetilde{H}$	-	fuzzy holding cost per unit time.
P(t)	-	penalty cost function.
D	-	Constant demand.
$\widetilde{D}$	-	fuzzy demand .
t_1	-	optimum time at which shortage reaches zero and inventory starts to accumulate.
t_2	-	optimum time at which shortage reaches its maximum and production process starts to meet the demand.
TC	-	total cost for the period (0,T)
$\widetilde{TC}$	-	fuzzy total cost for the period (0,T)
$d_F\widetilde{C}$	-	defuzzified value of $\widetilde{C}$

Crisp Model formulation



Case 1. Infinite production rate with shortages.

Let Q be the number of items in the inventory at the beginning of the time period. Then the inventory level starts decreasing due to the demand of the product. The demand rate is D units per unit time. Till the time period θ , there is no deterioration but after the time interval $(0, \theta)$, the product starts deteriorate. Hence penalty cost is considered in (θ, t_1). The above situation is depicted in figure 1. Since the demand rate is D units per unit time, the total demand in (0,T) is DT. The number of items at the beginning of the inventory is Q = DT.

The penalty cost due to the deterioration of the product in (θ, t_1) is given by

$$DC = \int_{\theta}^{t_1} \pi D(t-\theta)\, dt = \pi D(t_1 - \theta)^2 \tag{1}$$

The setup cost for the inventory in (0,T) is given by SC = $\frac{S}{T}$ (2)

The holding cost for the period (0,T) is given by HC= $\frac{HQT}{2} = \frac{HDT^2}{2}$ (3)

The shortage cost during the time interval (t_1 , T) is given by SHC= $A\int_{t_1}^{T} - D\,dt$ = -AD(T-t_1) (4)

The average total cost per unit time is given by TC=DC+SC+HC+SHC

$$TC= \frac{\pi D(t_1-\theta)^2}{2T} + \frac{S}{T} + \frac{HDT}{2} - \frac{AD(T-t_1)}{T} \quad (5)$$

Differentiating with respect to T we get $\frac{\partial TC}{\partial T} = -\frac{\pi D(t_1-\theta)^2}{2T^2} - \frac{S}{T^2} - \frac{ADt_1}{T^2} + \frac{HD}{2}$ (6)

Equate to zero we get the optimum solution $T^* = \sqrt{\frac{\pi D(t_1-\theta)^2 + 2S + 2ADt_1}{HD}}$ (7)

and the optimum order quantity Q = DT = $\sqrt{\frac{\pi D^2(t_1-\theta)^2 + 2SD + 2AD^2 t_1}{HD}}$ (8)

Case 2: Finite production rate with shortages

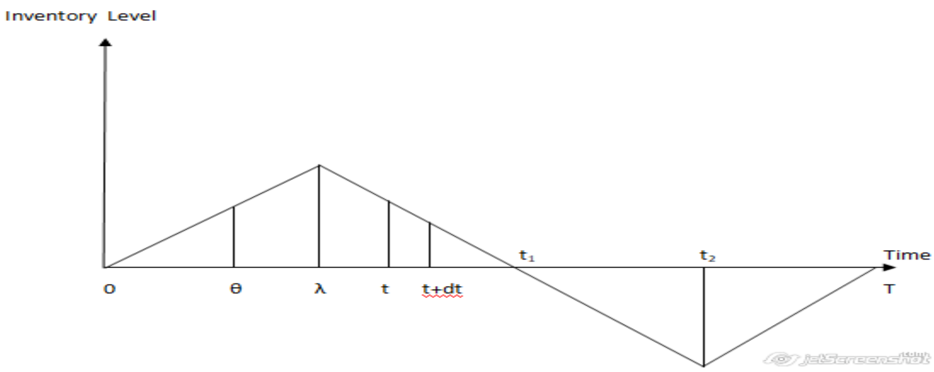


Figure 2: Finite Production rate with shortages

From figure 2 we observe that ,at first the inventory level is zero and the production starts with a finite rate P units per unit time(P>D). The inventory increases with the rate of (P-D) units per unit time. The inventory at time λ is I = (P-D) λ. Let Q be the number of items produced per production run then production will continue for λ = $\frac{Q}{P}$. The time of one complete cycle = $\frac{Q}{T}$.

After the completion of production , the inventory level is I = (P-D) $\frac{Q}{P}$ (9)

$$=Q\left(1-\frac{D}{P}\right) \quad (10)$$

Assume that the products do not deteriorate till the time period θ . After the time period θ, the product gets deteriorate and the penalty cost is incurred in the time interval (θ,t_1) . The total cost includes holding cost, setup cost, shortage cost and penalty cost.

Holding cost per cycle is given by HC= $\frac{HIT}{2} = \frac{HT^2D}{2}\left(1-\frac{D}{P}\right)$ (11)

The number of units delivered in time (t-θ) is D(t-θ) and the production time is $\frac{D(t-\theta)}{P}$, $\theta < t \le t_1$. (12)

The product delivered at time ‘t’ is given by (t-θ) $(t-\theta)\left(1-\frac{D}{P}\right)$ (13)

Cost due to deterioration of product is DC= $\int_{\theta}^{t_1} \pi D(t-\theta)\left(1-\frac{D}{P}\right)dt = \pi D\left(1-\frac{D}{P}\right)\frac{(t_1-\theta)^2}{2}$ (14)

Set up cost of inventory in the cycle length T is SC = $\frac{S}{T}$.

Shortage cost during the time interval (t_1,T) is SHC = $A\int_{t_1}^{T} -D\ dt$ = -AD(T-t_1).

The total cost TC= $\frac{HTD}{2}\left(1-\frac{D}{P}\right)+\pi D\left(1-\frac{D}{P}\right)\frac{(t_1-\theta)^2}{2T}+\frac{S}{T}-\frac{AD(T-t_1)}{T}$ (15)

Differentiate with respect to T we get $\frac{\partial TC}{\partial T}=\frac{HD}{2}\left(1-\frac{D}{P}\right)-\pi D\left(1-\frac{D}{P}\right)\frac{(t_1-\theta)^2}{2T^2}-\frac{S}{T^2}-\frac{ADt_1}{T^2}$

Equate to zero we get $T^*=\sqrt{\dfrac{\pi D\left(1-\frac{D}{P}\right)(t_1-\theta)^2+2S+2ADt_1}{HD\left(1-\frac{D}{P}\right)}}$ (16)

The optimum order quantity is given by $Q^*=\sqrt{\dfrac{\pi D^2\left(1-\frac{D}{P}\right)(t_1-\theta)^2+2SD+2AD^2t_1}{H\left(1-\frac{D}{P}\right)}}$ (17)

Fuzzy Model formulation

To face the uncertainty in the above developed inventory problem , let us consider the parameters demand, holding cost, set up cost and shortage cost as trapezoidal fuzzy numbers. Signed distance method is used for defuzzification.

Case 1 : Infinite production rate with shortages

$\tilde{A}=(A_1,A_2,A_3,A_4)$, $\tilde{H}=(H_1,H_2,H_3,H_4)$, $\tilde{D}=(D_1,D_2,D_3,D_4)$, $\tilde{\theta}=(\theta_1,\theta_2,\theta_3,\theta_4)$,
$\tilde{S}=(S_1,S_2,S_3,S_4)$ $\widetilde{TC}$ = (TC_1 ,TC_2 , TC_3 , TC_4)

Where TC_i= $\frac{\pi D_i(t_1-\theta_i)^2}{2T}+\frac{S_i}{T}+\frac{H_iD_iT}{2}-\frac{A_iD_i(T-t_1)}{T}$ i = 1, 2, 3,4 (18)

$\frac{d}{dT}(TC_i)=\frac{-\pi D_i(t_1-\theta_i)^2}{2T^2}-\frac{S_i}{T^2}+\frac{H_iD_i}{2}-\frac{A_iD_it_1}{T^2}$, i = 1,2,3,4 and

$\frac{d^2}{dT^2}(TC_i)=\frac{2\pi D_i(t_1-\theta_i)^2}{2T^3}+\frac{2S_i}{T^3}+\frac{2A_iD_it_1}{T^3}$, i = 1,2,3,4 .

To defuzzify the value of total cost, signed distance method is used. $d_F\widetilde{TC}=\frac{1}{4}(TC_1+TC_2+TC_3+TC_4)$ (19)

To find the optimum value differentiate (19) with respect to T

$$\frac{d(\widetilde{TC})}{dT}=\frac{1}{4}\left(\frac{dTC_1}{dT}+\frac{dTC_2}{dT}+\frac{dTC_3}{dT}+\frac{dTC_4}{dT}\right)$$

$$\frac{d^2}{dT}\left(d_F T\widetilde{C}\right)= \frac{1}{4}\left(\frac{dTC_1^{\,2}}{dT^2}+\frac{dTC_2^{\,2}}{dT^2}+\frac{dTC_3^{\,2}}{dT^2}+\frac{dTC_4^{\,2}}{dT^2}\right)=\frac{2\pi D_i(t_1-\theta_i)^2}{2T^3}+\frac{2S_i}{T^3}+\frac{2A_iD_it_1}{T^3}\text{ , i =1,2,3,4}$$

which is greater than zero .Therefore we get the minimum total cost.

Now let us find optimum solution of time period by putting $\frac{d}{dT}\left(d_F T\widetilde{C}\right)=0$

$$0=\frac{1}{4}\begin{bmatrix}\frac{-\pi D_1t_1^2}{2T^2}-\frac{-\pi D_1\theta_1^2}{2T^2}+\frac{\pi D_1t_1\theta_1}{T^2}-\frac{S_1}{T^2}+\frac{H_1D_1}{2}-\frac{A_1D_1t_1}{T^2}+\\ \frac{-\pi D_2t_1^2}{2T^2}-\frac{-\pi D_2\theta_2^2}{2T^2}+\frac{\pi D_2t_1\theta_2}{T^2}-\frac{S_2}{T^2}+\frac{H_2D_2}{2}-\frac{A_2D_2t_1}{T^2}+\\ \frac{-\pi D_3t_1^2}{2T^2}-\frac{-\pi D_3\theta_3^2}{2T^2}+\frac{\pi D_3t_1\theta_3}{T^2}-\frac{S_3}{T^2}+\frac{H_3D_3}{2}-\frac{A_3D_3t_1}{T^2}+\\ \frac{-\pi D_4t_1^2}{2T^2}-\frac{-\pi D_4\theta_4^2}{2T^2}+\frac{\pi D_4t_1\theta_4}{T^2}-\frac{S_4}{T^2}+\frac{H_4D_4}{2}-\frac{A_4D_4t_1}{T^2}\end{bmatrix}$$

On simplifying we get
$$T^*=\sqrt{\frac{\pi t_1^2(D_1+D_2+D_3+D_4)+\pi\left(D_1\theta_1^2+D_2\theta_2^2+D_3\theta_3^2+D_4\theta_4^2\right) -2\pi t_1(D_1\theta_1+D_2\theta_2+D_3\theta_3+D_4\theta_4)+2(S_1+S_2+S_3+S_4) +2t_1(A_1D_1+A_2D_2+A_3D_3+A_4D_4)}{H_1D_1+H_2D_2+H_3D_3+H_4D_4}} \tag{20}$$

and the optimum order quantity
$$Q^*=\sqrt{\frac{\pi t_1^2\left(D_1^{\,2}+D_2^{\,2}+D_3^{\,2}+D_4^{\,2}\right)+\pi\left(D_1^{\,2}\theta_1^2+D_2^{\,2}\theta_2^2+D_3^{\,2}\theta_3^2+D_4^{\,2}\theta_4^2\right) -2\pi t_1\left(D_1^{\,2}\theta_1+D_2^{\,2}\theta_2+D_3^{\,2}\theta_3+D_4^{\,2}\theta_4\right)+2(D_1S_1+D_2S_2+D_3S_3+D_4S_4) +2t_1(A_1D_1^{\,2}+A_2D_2^{\,2}+A_3D_3^{\,2}+A_4D_4^{\,2})}{H_1+H_2+H_3+H_4}} \tag{21}$$

Optimum total cost can be found out from (18)

Case 2 : Finite production rate with shortages

$\widetilde{A}=(A_1,A_2,A_3,A_4)$, $\widetilde{H}=(H_1,H_2,H_3,H_4)$, $\widetilde{D}=(D_1,D_2,D_3,D_4)$, $\widetilde{\theta}=(\theta_1,\theta_2,\theta_3,\theta_4)$,
$\widetilde{S}=(S_1,S_2,S_3,S_4)$ $\widetilde{TC}$ = (TC_1 ,TC_2 , TC_3 , TC_4) , $\widetilde{P}=(P_1,P_2,P_3,P_4)$

Where $TC_i=$
$$\frac{\pi D_it_1^{\,2}\left(1-\frac{D_i}{P_{5-i}}\right)}{2T}+\frac{\pi D_i\theta_i^2\left(1-\frac{D_i}{P_{5-i}}\right)}{2T}-\frac{\pi D_it_1\theta_i\left(1-\frac{D_i}{P_{5-i}}\right)}{T}+\frac{S_i}{T}+\frac{H_iD_iT\left(1-\frac{D_i}{P_{5-i}}\right)}{2}-\frac{A_iD_i(T-t_1)}{T}$$

i = 1, 2, 3,4---------(22)

$$\frac{d}{dT}(TC_i)=\frac{-\pi D_it_1^{\,2}\left(1-\frac{D_i}{P_{5-i}}\right)}{2T^2}-\frac{\pi D_i\theta_i^2\left(1-\frac{D_i}{P_{5-i}}\right)}{2T^2}+\frac{\pi D_it_1\theta_i\left(1-\frac{D_i}{P_{5-i}}\right)}{T^2}-\frac{S_i}{T^2}+\frac{H_iD_i\left(1-\frac{D_i}{P_{5-i}}\right)}{2}-\frac{A_iD_it_1}{T^2}$$

i = 1,2,3,4 ----(23)

$$\frac{d^2}{dT^2}(TC_i)=\frac{\pi D_it_1^{\,2}\left(1-\frac{D_i}{P_{5-i}}\right)}{T^3}+\frac{\pi D_i\theta_i^2\left(1-\frac{D_i}{P_{5-i}}\right)}{T^3}-\frac{\pi D_it_1\theta_i\left(1-\frac{D_i}{P_{5-i}}\right)}{T^3}+\frac{2S_i}{T^3}+\frac{2A_iD_it_1}{T^3} \tag{24}$$

i = 1,2,3,4

To defuzzify the value of total cost, signed distance method is used. $d_F\widetilde{TC} = \frac{1}{4}\left(TC_1 + TC_2 + TC_3 + TC_4\right)$

To find the optimum value differentiate with respect to T

$$\frac{d\left(\widetilde{TC}\right)}{dT} = \frac{1}{4}\left(\frac{dTC_1}{dT} + \frac{dTC_2}{dT} + \frac{dTC_3}{dT} + \frac{dTC_4}{dT}\right)$$

$$\frac{d^2}{dT}\left(d_F\widetilde{TC}\right) = \frac{1}{4}\left(\frac{dTC_1^{\ 2}}{dT^2} + \frac{dTC_2^{\ 2}}{dT^2} + \frac{dTC_3^{\ 2}}{dT^2} + \frac{dTC_4^{\ 2}}{dT^2}\right)$$

$$= \frac{\pi D_i t_1^{\ 2}\left(1 - \frac{D_i}{P_{5-i}}\right)}{T^3} + \frac{\pi D_i \theta_i^2\left(1 - \frac{D_i}{P_{5-i}}\right)}{T^3} - \frac{\pi D_i t_1 \theta_i\left(1 - \frac{D_i}{P_{5-i}}\right)}{T^3} + \frac{2S_i}{T^3} + \frac{2A_i D_i t_1}{T^3} \quad , \text{ i =1,2,3,4} \tag{25}$$

which is greater than zero .Therefore we get the minimum total cost.

Now let us find optimum solution of time period by putting $\frac{d}{dT}\left(d_F\widetilde{TC}\right)$=0

$$0 = \frac{1}{4}\left[\begin{array}{l} -\pi t_1^2\left(D_1\left(1-\frac{D_1}{P_4}\right) + D_2\left(1-\frac{D_2}{P_3}\right) + D_3\left(1-\frac{D_3}{P_2}\right) + D_4\left(1-\frac{D_4}{P_1}\right)\right) - \\ \pi\left(D_1\theta_1^2\left(1-\frac{D_1}{P_4}\right) + D_2\theta_2^2\left(1-\frac{D_2}{P_3}\right) + D_3\theta_3^2\left(1-\frac{D_3}{P_2}\right) + D_4\theta_4^2\left(1-\frac{D_4}{P_1}\right)\right) + \\ 2\pi t_1\left(D_1\theta_1\left(1-\frac{D_1}{P_4}\right) + D_2\theta_2\left(1-\frac{D_2}{P_3}\right) + D_3\theta_3\left(1-\frac{D_3}{P_2}\right) + D_4\theta_4\left(1-\frac{D_4}{P_1}\right)\right) - \\ \hline 2\left(S_1 + S_2 + S_3 + S_4\right) - 2t_1\left(A_1D_1 + A_2D_2 + A_3D_3 + A_4D_4\right) \\ \hline \qquad 2T^2 \\ + \dfrac{H_1D_1\left(1-\frac{D_1}{P_4}\right) + H_2D_2\left(1-\frac{D_2}{P_3}\right) + H_3D_3\left(1-\frac{D_3}{P_2}\right) + H_4D_4\left(1-\frac{D_4}{P_1}\right)}{2} \end{array}\right] - \tag{26}$$

On simplifying we get

$$T^* = \sqrt{\frac{\begin{array}{l} \pi t_1^2\left(D_1\left(1-\frac{D_1}{P_4}\right) + D_2\left(1-\frac{D_2}{P_3}\right) + D_3\left(1-\frac{D_3}{P_2}\right) + D_4\left(1-\frac{D_4}{P_1}\right)\right) + \\ \pi\left(D_1\theta_1^2\left(1-\frac{D_1}{P_4}\right) + D_2\theta_2^2\left(1-\frac{D_2}{P_3}\right) + D_3\theta_3^2\left(1-\frac{D_3}{P_2}\right) + D_4\theta_4^2\left(1-\frac{D_4}{P_1}\right)\right) - \\ 2\pi t_1\left(D_1\theta_1\left(1-\frac{D_1}{P_4}\right) + D_2\theta_2\left(1-\frac{D_2}{P_3}\right) + D_3\theta_3\left(1-\frac{D_3}{P_2}\right) + D_4\theta_4\left(1-\frac{D_4}{P_1}\right)\right) + \\ 2\left(S_1 + S_2 + S_3 + S_4\right) + 2t_1\left(A_1D_1 + A_2D_2 + A_3D_3 + A_4D_4\right) \end{array}}{H_1D_1\left(1-\frac{D_1}{P_4}\right) + H_2D_2\left(1-\frac{D_2}{P_3}\right) + H_3D_3\left(1-\frac{D_3}{P_2}\right) + H_4D_4\left(1-\frac{D_4}{P_1}\right)}} \tag{27}$$

and the optimum order quantity Q= DT. Optimum total cost is given by (22)

Numerical example

CRISP MODEL

Infinite production rate with shortages

θ=0.03, D =30, S=10,H=2,A=5

Optimum time period =**5.4** , Optimum total cost=7.53 Optimum order quantity =162

Finite production rate with shortages

θ=0.03, D =30, S=10,H=2,A=5 , P=50

Optimum time period =**7.2**, Optimum total cost=69.2 Optimum order quantity =216

Fuzzy model

Infinite production rate with shortages

$\widetilde{D}$ =(10,20,30,40), $\widetilde{S}$ =(10,12,14,16), $\widetilde{H}$ =(1,2,3,4), $\widetilde{A}$ =(3,5,7,9)

Optimum time period = **3.1 ,** Optimum total cost=56 Optimum order quantity =(31,62,93,124)

Finite production rate with shortages

$\widetilde{D}$ =(10,20,30,40), $\widetilde{S}$ =(10,12,14,16), $\widetilde{H}$ =(1,2,3,4), $\widetilde{A}$ =(3,5,7,9), $\widetilde{P}$ =(50,60,70,80)

Optimum time period = **6.97** ,Optimum total cost=96.81 , Optimum order quantity= (69.7,139.4,209.1,278.8)

Sensitivity Analysis

Table 1. Parameter $\widetilde{\theta}$ as Fuzzy number

$\widetilde{\theta}$	Infinite production rate with shortages			Finite production rate with shortages		
	T^*	TC	Q^*	T^*	TC	Q^*
(0.3,0.4,0.5,0.6)	2.90	42.5	(29,58,87,116)	6.78	74.45	(67.8,135.6,203.4,271.2)
(0.03,0.05,0.07,0.09)	3.10	56.0	(31,62,93,124)	6.97	96.81	(69.7,139.4,209.1,278.8)
(0.01,0.05,0.1,0.15)	3.06	37.8	(30.6,61.2,91.8,122.4)	6.96	95.82	(69.6,139.2,208.8,278.4)
(0.8,1,1.2,1.4)	2.73	30.2	(27.3,54.6,81.9,109.2)	6.5	43.31	(65,130,195,260)
(1,1.5,2,2.5)	2.80	35.5	(28,56,84,112)	6.32	22.64	(63.2,126.4,189.6,252.8)

Table 2. Parameter $\widetilde{D}$ as Fuzzy number

$\widetilde{D}$	Infinite production rate with shortages			Finite production rate with shortages		
	T^*	TC	Q^*	T^*	TC	Q^*
(5,12,17,22)	4.6	112.3	(46,92,138,184)	4.91	61.4	(49.1,98.2,147.3,196)
(20,23,26,29)	4.72	152.3	(47.2,94.4,122.7,188.8)	6.28	61.5	(62.8,125.6,188.4,251.2)
(10,20,30,40)	4.57	167.9	(45.7,91.4,137.1,182.8)	6.78	74.45	(67.8,135.6,203.4,271.2)
(32,34,36,38)	4.74	275.9	(151.7,161.2,170.6,18)	8.1	223.94	(81,162,243,324)
(15,21,27,33)	4.64	154.8	(46.4,92.8,139.2,185.6)	6.18	173.86	(61.8,123.6,185.4,247.2)

Table 3. Parameter $\widetilde{S}$ as Fuzzy number

$\widetilde{S}$	Infinite production rate with shortages			Finite production rate with shortages		
	T^*	TC	Q^*	T^*	TC	Q^*
(6,11,16,21)	4.57	168.0	(45.7,91.4,137.1,182.8)	6.8	74.75	(68,136,204,272)
(20,25,30,35)	4.6	171.1	(46,92,138,184)	6.85	86.7	(68.5,137,205.5,274)
(7,14,21,28)	4.58	168.9	(45.8,91.6,137.4,183.2)	6.79	77.11	(67.9,135.8,203.7,271.6)
(10,20,30,40)	4.61	170.5	(46.1,92.2,138.3,184.4)	6.84	81.5	(68.4,136.8,205.2,273.6)
(10,12,14,16)	4.56	167.9	(45.6,91.2,136.8,182.4)	6.78	74.45	(67.8,135.6,203.4,271.2)

Table 4. Parameter $\widetilde{H}$ as Fuzzy number

$\widetilde{H}$	Infinite production rate with shortages			Finite production rate with shortages		
	T^*	TC	Q^*	T^*	TC	Q^*
(2,5,8,11)	2.79	129.3	(27.9,55.8,83.7,111.6)	3.88	162.8	(38.8,77.6,116.4,155.2)
(6,12,18,24)	1.87	290.8	(18.7,374,56.1,74.8)	2.77	299.3	(27.7,55.4,83.1,110.8)
(5,6,7,8)	2.99	137.8	(29.9,59.8,89.7,119.6)	4.19	86.90	(41.9,83.8,125.7,167.6)
(3,5,7,9)	3.82	167.9	(38.2,76.4,114.6,152.8)	6.76	74.45	(67.6,135.2,202.8,270.4)

Table 5. Parameter $\widetilde{A}$ as Fuzzy number

$\widetilde{A}$	Infinite production rate with shortages			Finite production rate with shortages		
	T^*	TC	Q^*	T^*	TC	Q^*
(2,6,10,14)	4.54	90.60	(45.4,90.8,136.2,181.6)	7.85	165.4	(78.5,157,235.5,314)
(3,5,7,9)	3.82	24.04	(38.2,76.4,114.6,152.8)	6.76	72.92	(67.6,135.2,202.8,270.4)
(1,4,8,12)	4.10	105.1	(41,82,123,164)	7.1	15.86	(71,142,213,284)
(2,3,4,5)	2.93	120.2	(29.3,58.6,87.9,117.2)	5.49	56.92	(54.9,109.8,164.7,219.6)
(6,7,8,9)	4.07	105.7	(40.7,81.4,122.1,162.8)	7.15	17.53	(71,160.53,214.5,286)

Conclusion

In this research paper an inventory model with finite and infinite production rate using penalty cost and shortage cost is formulated. The model is converted into fuzzy model by considering the parameters D,H,A,θ,P,S as trapezoidal fuzzy numbers. Table 1 gives the value of optimum time period and optimum order quantity for the various values of θ. Table 2 gives optimum time and optimum order quantity for the values of $\widetilde{D}$. Table 3 gives optimum time and optimum order quantity for the values of $\widetilde{S}$. Table 4 gives optimum time and optimum order quantity for the values of $\widetilde{H}$. Table 5 gives optimum time and optimum order quantity for the values of $\widetilde{A}$. Also all the table compares the infinite production rate with shortages and finite production rate with shortages. The effect of change in the optimum values by changing the values of the parameter is shown in the table. This paper concludes that the fuzzy model will give optimum time period and optimum total cost when compared to crisp model.

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