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RESEARCH ARTICLE

ON THE PROPERTIES AND APPLICATIONS OF THE ODD LINDLEY-GOMPERTZ DISTRIBUTION

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ABSTRACT

The Gompertz distribution is both skewed to the left and to the right. It is an extension of the exponential distribution and is commonly used in many applied problems, particularly in life time data analysis. This paper proposes another extension of the Gompertz distribution called "the odd lindley-Gompertz distribution". Some properties of the new distribution have been derived and proposed distribution. The performance of the proposed model has been evaluated by using a real life dataset.

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INTRODUCTION

The Lindley distribution introduced by Lindley (1958) in the context of Bayesian analysis as a counter example of fiducial statistics, is defined by its cumulative distribution function (C.D.F) and probability density function (P.D.F) as

$$G(x)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x} \tag{1.3}$$

And

$$g(x)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x} \tag{1.4}$$

Respectively.

For $x > 0, \theta > 0$,

Where θ is the scale parameter of the Lindley distribution

The Gompertz distribution (*GD*) is both skewed to the right and to the left. It is a generalization of the Exponential Distribution (*ED*) and is commonly used in many applied problems, particularly in lifetime data analysis (Johnson *et al*., 1995).

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The Gompertz distribution with parameters α *and* γ has the cumulative distribution function (*cdf*) and probability density function (*pdf*) given by:

$$G(x) = 1 - e^{-\frac{\alpha}{\gamma}\left(e^{\gamma x}-1\right)} \tag{1.5}$$

and

$$g(x) = \alpha e^{\gamma x} e^{-\frac{\alpha}{\gamma}\left(e^{\gamma x}-1\right)} \tag{1.6}$$

Respectively.

For $x \geq 0, \alpha > 0, \gamma > 0$ where α *and* γ are the model parameters respectively.

1.2 Statement of the problem

One of the major problems in distribution theory and applications is that some data sets do not follow any of the existing and well known probability distributions appropriately and hence create a difficulty in the process of statistical analysis. Despite the applicability of the Gompertz distribution, limited work has been done in extending the distribution to increase its flexibility. Also, some existing extensions of the Gompertz distribution fail to manifest a decreasing failure rate useful for modeling lifetime data. Hence, the motivation for the development of this distribution is for modeling of lifetime data with a diverse model that takes into consideration not only shape, and scale but also skewness, kurtosis and tail variation.

1.3 Aim and objectives of the study

Aim: The aim of this work is to generalize a Gompertz distribution using the Lindley link function (Odd Lindley-G) by Gomes-Silva *et al*. (2017), derive its properties and evaluate its performance using simulated and real life data.

Specific Objectives

i. To derive some statistical properties of the proposed distribution such as the moments, the moment generating function and the characteristics function.
ii. To estimate the parameters of the new distribution using the method of maximum Likelihood estimation (MLE).
iii. To test the performance of the new distribution compared to other generalizations of the Gompertz distribution.

1.4 Significance of the Study

The main significance of this study is that it is a new extension of the Gompertz distribution with a derivation and study of many properties of the proposed distribution with different applications both in engineering and other fields of study. The proposed distribution shall be compared to some existing generalizations of the Gompertz distribution sand some other baseline distributions to rate its' strength using simulated and real life data sets.

2.1 The Definition of the proposed Distribution

For any continuous distribution with cdf $G(x;\xi)$ and *pdfs*, $g(x;\xi)$ Gomes-Silva *et al*. (2017) proposed the Odd Lindley-Generalized (denoted "Lindley-G) family of distributions that provides greater flexibility in modeling of real data sets. The cumulative distribution function (*cdf*) of the Odd Lindley-G family of distributions according to Gomes-Silva *et al*. (2017) is defined as

$$F_{OL-G}(x;\theta,\xi) = \int_{-\infty}^{\frac{G(x;\xi)}{G'(x;\xi)}} \frac{\theta^2}{\theta+1}(1+t)e^{-\theta t}dt$$

Where $G(x;\xi)$ is the cdf of any continuous distribution which depends on the parameter vector □, $G'(x;\xi) = 1 - G(x;\xi)$ and $\theta > 0$ is the scale parameter.
Using integration by substitution in the equation above and evaluating the integrand in equation yields

$$F_{OL-G}(x;\theta,\xi) = \frac{1-\theta+G'(x;\xi)}{(1+\theta)G'(x;\xi)}\exp\left\{-\theta\left[\frac{G(x;\xi)}{G'(x;\xi)}\right]\right\}, -\infty < x < \infty, \theta > 0 \tag{1.7}$$

Therefore, equation (3.1) is the cumulative distribution function (*cdf*) of the Odd Lindley-G family of distributions proposed by Gomes-Silva*et al*. (2017) and the corresponding *pdf* of the Odd Lindley-G family can be obtained from equation (3.1) by taking the derivative of the *cdf* with respect to *x* and is obtained as:

$$f_{OL-G}(x;\theta,\xi)=\frac{\theta^2 g(x;\xi)}{(1+\theta)(G'(x;\xi))^3}\exp\left\{-\theta\left[\frac{G(x;\xi)}{G'(x;\xi)}\right]\right\} \tag{1.8}$$

where $g(x;\xi)$ and $G(x;\xi)$ are the *pdf* and the *cdf* of any continuous distribution respectively which depends on the parameter vector ξ and $\theta>0$ is the scale parameter. The major benefit of (3.2) is to offer more flexibility to extremes of the *pdfs* and therefore it becomes suitable for analyzing data with high degree of asymmetry.

2.2 Some Properties of the Proposed Distribution

2.2.1 Moments: Let X denote a continuous random variable, the n^{th} moment of X is given by;

$$\mu_n' = E\left[X^n\right]=\int_0^\infty x^n f(x)dx \tag{1.9}$$

Where *f(x) is* the *pdf* of the proposed distribution

The mean, variance, skewness and kurtosis measures can also be calculated from moments using some well-known relationships.

2.3.2 Moment Generating Function

The moment generating function of a random variable X can be obtained as $M_x(t) = E\left[e^{tx}\right]=\int_0^\infty e^{tx} f(x)dx$ (10)

Where *f(x)* is the pdf of the newly proposed distribution.

2.4.3 Characteristics Function

The characteristics function of a random variable *X* is given by;

$$\varphi_x(x) = E\left[e^{itx}\right]=\int_0^\infty e^{itx} f(x)dx \tag{3.7}$$

Where $f(x)$ is the Pdf of the propose distribution.

2.3 Estimation of Parameters

The parameters of the proposed distribution will be estimated using the method of Maximum Likelihood Estimation (*MLE*) following the necessary steps.

2.4 Applications

This section presents three data sets and the test statistics to be used in the analysis with some selected distribution.

2.4.1 Data sets: We will simulate data from the quintile function of this distribution and use it to test the performance of the propose distribution and as well, use a real life data set and compare the two cases.

2.4.2 Test Statistics: To compare these distributions, we will consider some criteria: the value of the log-likelihood function evaluated at the MLEs (*ll*), *AIC* (Akaike Information Criterion), *CAIC* (Consistent Akaike Information Criterion), *BIC* (Bayesian Information Criterion), and *HQIC* (Hannan Quin Information Criterion).

Note: In decision making, any model with the lowest values of the above listed statistics would be chosen as the best model to fit the data set in question.

2.4.3 Related Distributions or models: The proposed distribution, Odd Lindley-Gompertz distribution (*OLGD*), will be compared to some other generalizations of the Gompertz distribution such as Generalized Gompertz distribution (*GGD*), odd generalized Exponential-Gompertz distribution (*OGEGD*), Transmuted Gompertz distribution (*TGD*) and the Gompertz distribution (*GD*).

2.5 Expected Results: This study will introduce a new extension of the Gompertz distribution called "Odd Lindley-Gompertz distribution (*OLGD*)". The same research shall derive and study extensively some important properties of the new model such as moments (mean, variance, skewness and kurtosis), moment generating function, characteristics function e.t.c. The method of maximum likelihood estimation will be used to estimate the parameters of the newly proposed distribution with some data applications to check the fitness or performance of the new distribution compared to related distributions previously studied.

3.0 RESULTS AND DISCUSSION

3.1 Analysis of Data: In this section, we present two data sets, their descriptive statistics, graphics and applications to some selected generalizations of the Gompertz distribution. We have compared the performance of the proposed distribution, Odd Lindley-Gompertz distribution to other generalizations of the Gompertz distribution such as Generalized Gompertz distribution (*GGD*), odd generalized Exponential-Gompertz distribution (*OGEGD*), Transmuted Gompertz distribution (*TGD*) and the Gompertz distribution (*GD*).

The density functions of these distributions are given as follows;

The Generalized Gompertz Distribution (*GGD*)

The *pdf* of the *GGD* distribution is given as;

$$f(x;\alpha,\beta,c)=c\alpha e^{\beta x}e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\left[1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\right]^{c-1} \tag{10.1}$$

The Odd Generalized Exponential Gompertz Distribution **(*OGEGD*)**

The *pdf* of the *OGEGD* is given as;

$$f(x;a,b,\alpha,\beta)=\frac{ab\alpha e^{\beta x}e^{-\frac{\alpha}{\beta}\left(e^{\gamma x}-1\right)}}{\left[1-\left(1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\right)\right]^{2}}\exp\left\{-a\left[\frac{1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}}{1-\left(1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\right)}\right]\right\}\left(1-\exp\left\{-a\left[\frac{1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}}{1-\left(1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\right)}\right]\right\}\right)^{b-1} \tag{10.2}$$

The Transmuted Gompertz Distribution (*TGD*)

The *pdf* of the *TGD* is given by;

$$f(x;\alpha,\beta,\lambda)=\alpha e^{\beta x}e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\left[1+\lambda-2\lambda\left(1-e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)}\right)\right] \tag{10.3}$$

The Gompertz Distribution (*GD*)

The *pdf* of the Gompertz distribution is given by

$$f(x;\alpha,\beta)=\alpha e^{\beta x}e^{-\frac{\alpha}{\beta}\left(e^{\beta x}-1\right)} \tag{10.4}$$

Data set

The following is the data set used for analysis and applications in this dissertation. It is given as follows:

Data set: This data represents the remission times (in months) of a random sample of 128 bladder cancer patients. It has previously been used by Lee and Wang (2003) and Rady *et al*. (2016). It is given and summarized as follows:

0.080, 0.200, 0.400, 0.500, 0.510, 0.810, 0.900, 1.050, 1.190, 1.260, 1.350, 1.400, 1.460, 1.760, 2.020, 2.020, 2.070, 2.090, 2.230, 2.260, 2.460, 2.540, 2.620, 2.640, 2.690, 2.690, 2.750, 2.830, 2.870, 3.020, 3.250, 3.310, 3.360, 3.360, 3.480, 3.520, 3.570, 3.640, 3.700, 3.820, 3.880, 4.180, 4.230, 4.260, 4.330, 4.340, 4.400, 4.500, 4.510, 4.870, 4.980, 5.060, 5.090, 5.170, 5.320, 5.320, 5.340, 5.410, 5.410, 5.490, 5.620, 5.710, 5.850, 6.250, 6.540, 6.760, 6.930, 6.940, 6.970, 7.090, 7.260, 7.280, 7.320, 7.390, 7.590, 7.620, 7.630, 7.660, 7.870, 7.930, 8.260, 8.370, 8.530, 8.650, 8.660, 9.020, 9.220, 9.470, 9.740, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05.

Table 4.1. Summary Statistics for the data set

parameters	N	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

We also provide a histogram and a density for the data as shown in **Figure 4.1** below.

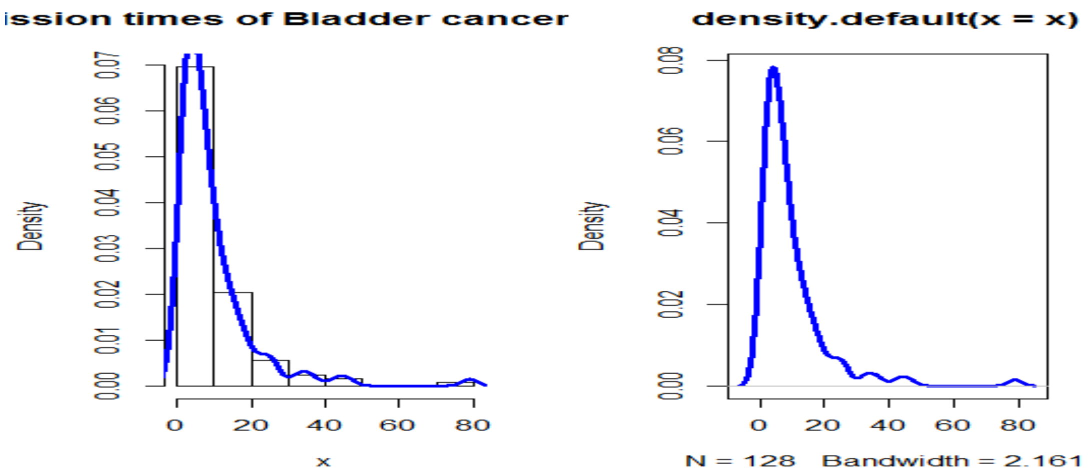

Figure 4.1. A Histogram and density plot on the remission times of bladder cancer patients

From the descriptive statistics in tables 4.1 as well as the histogram shown above in figures 4.1 we observed that the data set is positively skewed, therefore suitable for distributions that are skewed to the right. To assess these models above, we made use of some criteria: the *AIC* (Akaike Information Criterion), *CAIC* (Consistent Akaike Information Criterion), *BIC* (Bayesian Information Criterion) and *HQIC* (Hannan Quin information criterion). The formulas for these statistics are given as follows:

$$AIC = -2ll + 2k,$$

$$BIC = -2ll + k\log(n),$$

$$CAIC = -2ll + \frac{2kn}{(n-k-1)} \text{ and}$$

$$HQIC = -2ll + 2k\log\left[\log(n)\right]$$

Where ll denotes the log-likelihood function evaluated at the *MLEs*, k is the number of model parameters and n is the sample size. Decision statement: The model with the lowest values of these statistics would be chosen as the best model to fit the data.

Table 4.2: Performance evaluation of the Odd Lindley-Gompertz distribution with some generalizations of the Gompertz distribution using the *AIC*, *CAIC*, *BIC and HQIC* values of the models evaluated at the *MLEs* based on our data set.

Distributions	Parameter estimates	-ll=(-log-likelihood value)	*AIC*	*CAIC*	*BIC*	*HQIC*	Ranks of models performance
OLn GD	$\hat{\alpha}$=0.2593 $\hat{\beta}$=0.4411 $\hat{\theta}$=2.3755	347.8476	701.6952	701.8887	702.0168	697.6374	1
GGD	$\hat{\alpha}$=0.2215 $\hat{\beta}$=0.0932 $\hat{c}$=0.3262	739.5045	1485.0090	1485.2590	1492.8240	1488.1720	2
TGD	$\hat{\alpha}$=0.1950 $\hat{\beta}$=0.0217 $\hat{\lambda}$=-0.1190	365.8488	737.6975	737.9475	745.5130	740.8606	3
OGEGD	$\hat{\alpha}$=0.0347 $\hat{\beta}$=0.0063 $\hat{a}$=7.5647 $\hat{b}$=1.5793	659.9827	1327.9650	1328.3870	1338.3860	1332.1830	4
GD	$\hat{\alpha}$=2.0907 $\hat{\beta}$=0.0433	2894.2880	5792.575	5792.6990	5797.7850	5794.6840	5

Using the values of the parameter *MLEs* and the corresponding values of *-ll*, *AIC*, *BIC*, *CAIC* and *HQIC* for each model as shown in table **4.2**, we can understand that the *OLnGD*performs better with smaller values of the information criteria compared to the performance of the other models which are, the *GGD*, *TGD, OGEGD* and *GD*. The above performance can be traced to the fact that the proposed distribution is heavily skewed to the right with a high peak and our data set is also positively skewed with a large coefficient of kurtosis.

We recommend based on the findings of this research that the proposed distribution should be used to model both positively skewed data sets with large sample sizes. We also recommend that the new model should be used for analyzing time or age dependent variables based on the behavior of the survival and the hazard functions.

Conclusion

This paper introduced a new distribution called an Odd Lindley-Gompertz distribution. It studied some mathematical and statistical properties of the proposed distribution with some graphical demonstration appropriately. The derivations of some expressions for its moments, moment generating function, characteristics function and ordered statistics has been done effectively. Some plots of the distribution revealed that it is positively skewed and its degree of kurtosis depends on the values of the parameters. The model parameters have been estimated using the method of maximum likelihood estimation. The results of the applications showed that the proposed distribution performs better than some extensions of the Gompertz distribution however, depending on the nature of the data sets. It was revealed that this new distribution has better performance for positively skewed data sets with larger sample sizes. New model should be used for analyzing time or age dependent variables based on the behavior of the survival and the hazard functions.

Competing Interests: **A**uthors have declared that no competing interests exist.

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