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ASIAN JOURNAL OF
SCIENCE AND TECHNOLOGY

ISSN: 0976-3376

Asian Journal of Science and Technology
Vol. 09, Issue, 09, pp.8680-8685, September, 2018

RESEARCH ARTICLE

CHARACTERIZATIONS OF G*-OPEN FUNCTIONS IN TOPOLOGY

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ARTICLE INFO

Article History:
Received 08th June, 2018
Received in revised form
16th July, 2018
Accepted 17th August, 2018
Published online 30th September, 2018

Key words:

Preopen sets,
Semipreopen sets,
gsp-closed sets,
g*p-closed sets,
G*-closed sets
Just below the Keywords line,
type as below before 54A05 ,
54B05, 54D10

ABSTRACT

The purpose of this paper is to define and study the new classes of functions, called G*-open functions, pre-G*-open functions, G*-closed functions, always G*-open functions and always G*-closed functions via newly introduced G*-closed sets due to Navalagi (August – 2018). Also, some normality axioms are introduced and their preservations are studied.

Citation: ***Govindappa Navalagi.*** **2018.** "Characterizations of g*-open functions in topology", *Asian Journal of Science and Technology*, 09, (09), 8680-8685.

INTRODUCTION

Levine [13] generalized the closed set to generalized closed set (g-closed set) in topology for the first time. Since then it is noticed that many of the weaker forms of closed sets have been generalized. In 1982 and 1986, respectively, A.S.Mashhour *et al* [14] and D.Andrijevic [1] have defined and studied the concepts of preopen sets and semipreopen sets in topology. In 1993,1995, 1997, 1998, 2000, 2002, 2010, 2011 and 2014, respectively, Palaniappan *et al*. [22], Dontchev [8], Gnanambal [10], Noiri *et al* [21], M. K. R. S. Veera Kumar [26-28], M.Shyla Isac *et al*. [23], S. Bhattacharya [4] and K.Indrani *et al* [11], have defined and studied the concepts of rg-closed sets, gsp-closed sets, gpr-closed sets ,gp-closed sets, g*-closed sets, g*p-closed sets, Pre-semiclosed sets, rps-closed sets, gr-closed sets and gr*-closed sets in topological spaces .Quiet recently, Navalagi [20] has defined and studied the concepts of G*-closed sets, G*-open sets, G*-continuity and G*-irresoluteness in topology. In this paper, we define and study the new classes of functions, called G*-open functions, pre-G*-open functions, G*-closed functions, always G*-open functions and always G*-closed functions via G*-closed sets due to Navalagi [20]. Also, some normality axioms are introduced and their preservations are studied.

Preliminaries

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X. We denote the closure of A and the interior of A by Cl(A) and Int(A) respectively.

Definition 2.1 [25] : A subset A of a space X is called :

(i) regular open (in brief, r-open) if A = Int Cl(A) .
(ii) regular closed (in brief, r-closed) if A = Cl Int(A).

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The following definitions and results are useful in the sequel:

Definition 2.1 : A subset A of a space X is said to be:

(i) Preopen [14] if $A \subset$ Int Cl(A).

(ii) Semiopen [12] if $A \subset$ ClInt(A).
(iii) Semipreopen [1] if $A \subset$ Cl Int Cl(A).

The complement of a preopen (resp. semiopen, semipreopen) set of a space X is called preclosed [9] (resp. semiclosed [5], semipreclosed [1]).

Definition 2.2 [1]**:** The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by spInt(A) . pInt(A)[15] and sInt(A)[7], rInt(A) [2 &22] can be similarly defined.

Definition 2.3[2]: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by spCl(A).

pCl(A)[7] and sCl(A)[4] , rCl(A) [2 & 22] can be similarly defined.

DEFINITION 2.4 : A subset A of a space X is called :

(i) Generalized closed set (in brief, g-closed) set [13] if Cl(A)$\subset$u whenever $A \subset$ u and u is open in x .
(ii) Generalized semiclosed (in brief, gs-closed) set [3] if sCl(A)$\subset$u whenever $A \subset$ u and u is open in x .
(iii) Generalized semipreclosed (in brief, gsp-closed) set [8] if spCl(a)$\subset$u whenever $A \subset$ u and u is open in x .
(iv) Generalized preclosed (in brief, gp-closed) set [21] if pCl(A)$\subset$u whenever $a \subset$ u and u is open in x .
(v) Gpr--closed set [10] if pCl(A) $\subset$ u whenever $A \subset$ u and u is r-open in x .
(vi) Rg-closed set [23] if Cl(A) $\subset$ u whenever $A \subset$ u and u is r-open in x .
(vii) Gr-closed set [4] if rCl(A) $\subset$ u whenever $A \subset$ u and u is open in x .
(viii) Pre-semiclosed set [28] if spCl(A) $\subset$ u whenever $A \subset$ u and u is g-open in x .
(ix) G*-closed set [26] if Cl(A) $\subset$ u whenever $A \subset$ u and u is g-open in x .
(x) G*-preclosed (in brief, g*p-closed) set [27] if pCl(A)$\subset$u whenever $a \subset$ u and u is g-open in x.
(xi) G*sp-closed set [18] if spCl(A) $\subset$ u whenever $A \subset$ u and u is g-open in x .
(xii) Gr*-closed set [11] if rCl(A) $\subset$ u whenever $A \subset$ u and u is g-open in x .

Definition 2.5 [19]**:** The union of all gsp-open sets contained in A is called the gsp-interior of A and is denoted by gspInt(A).

Definition 2.6 [19]: The intersection of all gsp-closed sets containing A is called the gsp-closure of A and is denoted by gspCl(A).

Lemma 2.7 [19] : For every subset $A \subset X$, the following hold.

(i) gspCl(X-A) = X-gspInt(A) .
(ii) gspInt(X-A) = X-gspCl(A).

Definition 2.8 : A function $f : X \rightarrow Y$ is called:

(i) Precontinuous [14] if the inverse image of each open set of Y is preopen in X.
(ii) Semiprecontinuous [17] if the inverse image of each open set of Y is semipreopen in X.
(iii) Semipre-irresolute [17] if the inverse image of each semipreopen set of Y is semipre- open in X.
(iv) Semipreopen [17] if the image of each open set of X is semipreopen in Y.
(v) Semipreclosed [17] if the image of each closed set of X is semipreclosed in Y.

Definition 2.9 [16] : A space X is said to be sp-normal if for any pair of disjoint closed sets A and B of X, there exist disjoint semipreopen sets U and V in X such that $A \subset U$ and $B \subset V$.

3.Properties of G*-open functions

We, recall the following.

Definition 3.1[20]: A subset A of a space X is said to be G*-closed set if gspCl(A) $\subset$ U whenever $A \subset U$ and U is g-open in X.

The complement of a G*-closed set is called G*-open set.

Lemma 3.2[20]: Let X be a space. Then,

(i) Every closed (and hence preclosed, semipreclosed, r-closed) set is G*-closed set.
(ii) Every g*-closed (and hence g*p-closed, g*sp-closed, gr*-closed, rps-closed) set is G*-closed set.
(iii) Every G*-closed set is g-closed (and hence gs-closed, gsp-closed, gp-closed, gpr-closed, rg-closed, gr-closed, pre-semiclosed) set.

Lemma 3.3[20] : A subset A of a space X is called G*-open iff $F \subset$ gspInt(A) whenever F is g-closed and $F \subset A$.

We, recall the following.

Definition 3.4 [20]: A function $f:X\rightarrow Y$ is called G* -continuous if f^{1} (V) is G*-closed in X for every closed subset V of Y.

Definition 3.5 [20]: A function $f:X\rightarrow Y$ is called G*-irresolute if f^{1} (V) is G*-closed in X for every G*- closed subset V of Y. We, define the following.

Definition 3.6 : A function $f:X\rightarrow Y$ is called G* - open f(V) is G*-open in Y for every open subset V of X.

Definition 3.7 : A function $f:X\rightarrow Y$ is called pre-G*- open f(V) is G*-open in Y for every semipreopen subset V of X.

Theorem 3.8 : A surjective function $f:X\rightarrow Y$ is G*-open if and only if for each subset B of Y and each closed set F of X containing $f^{1}(B)$, there exists G*-closed set H of Y such that $B \subset H$ and $f^{1}(H) \subset F$.

Proof. Suppose f is G*-open function. Let B be subset of Y and F is closed set of X containing $f^{1}(B)$ .Put H = Y-(X-F). Then, H is G*-closed set in Y, $B \subset H$ and $f^{1}(H) \subset F$. Conversely, let U be any open set in X. Put B = Y –f(U), then we have $f^{1}(B) \subset$ X-U and X-U is closed set of X.There exists G*-closed set H in Y such that B = Y- f(U) $\subset$ H and $f^{1}(H) \subset$ X-U. Therefore, we obtain f(U) = X-H and hence f(U) is G*-open set in Y.This shos that f is G*-open function.

Theorem 3.9: A surjective function $f:X\rightarrow Y$ is pre-G*-open if and only if for each subset B of Y and each semipreclosed set F of X containing $f^{1}(B)$, there exists G*-closed set H of Y such that $B \subset H$ and $f^{1}(H) \subset F$.

Proof. Similar to Th.3.8.

We, define the following.

Definition 3.10: A function $f : X\rightarrow Y$ is called strongly A-semipreopen if the image of each semipreopen set of X is open in Y.

Definition 3.11: A function $f : X\rightarrow Y$ is called strongly G*-open if the image of each G*-open set of X is open in Y.

Clearly, every strongly G*-open function is strongly A-semipreopen function.

Theorem 3.12 : Let $f:X\rightarrow Y$ and $h:Y\rightarrow Z$ be two functions.

(i) If f is pre-G*-open function and h is strongly G*-open function, then hof is strongly A-semipreopen function.
(ii) If f is G*-open function and h is strongly G*-open function, then hof is open function.
(iii) If f is semipreopen function and h is pre-G*-open function, then hof is G*-open function.
(iv) If f is strongly G*-open function and h is G*-open function, then hof is pre-G*-open function.

Proof. (i) Let U be any semipreopen subset of X, then f(U) is G*-open set in Y since f is pre-G*-open function. Again, h is strongly G*-open function and f(U) is G*-open set in Y, then h(f(U)) = hof (U) is open set in Z. Therefore, h of is strongly A-semipreopen function.

(ii)-(iv) : Obvious.

Theorem 3.13 : Let $f:X\rightarrow Y$ and $h:Y\rightarrow Z$ be two functions such that hof is G*-open function:

(i) If f is continuous and surjective, then h is G*-open.
(ii) If h is G*-irresolute and injective, then f is G*-open.

Proof. (i): Let V be an arbitrary open subset of Y. Then, $f^{1}(V)$ is open set in X since f is continuous .Since h of is G*-open and f is surjective, h of ($f^{-1}(V)$) = h(V) is G*-open set in Z.

This shows that h is G*-open function.

(ii) As, we have f(A) = h^{-1}(hof (A)) this is true for every subset A of X, since h is given injective function. Let U be an arbitrary open subset of X, then hof(U) is G*-open set in Z .But h is given as G*-irresolute function and hence we obtain that,
f(U) = h^{-1}(hof (U)) is G*-open set in Y. Thus, f is G*-open function.
The routine proof of the following is omitted.

Theorem 3.14 : Let f :X→Y and h :Y→Z be two functions such that hof is pre-G*-open function

(i) If f is semipreirresolute and surjective, then h is pre-G*-open.
(ii) If h is G*-irresolute and injective, then f is pre-G*-open.

We, define the following.

Definition 3.15: A function f:X→Y is called G* - closed f(F) is G*-closed set in Y for every closed subset F of X.

Theorem 3.16 : A surjective function f :X→Y is G*-closed if and only if for each subset B of Y and each open set V of X containing f^{-1}(B), there exists G*-open set H of Y such that B ⊂ H and f^{-1}(H) ⊂ V.

Proof. Similar to Th.3.8.

Definition 3.17 : A function f:X→Y is called pre-G*- closed f(F) is G*-closed set in Y for every semi reclosed subset F of X.

Theorem 3.18: A subjective function f :X→Y is pre-G*-closed if and only if for each subset B of Y and each semi reopen set V of X containing f^{-1}(B), there exists G*-open set H of Y such that B ⊂ H and f^{-1}(H) ⊂ V.

Proof. Similar to Th.3.8

Definition 3.19 : A function f:X→Y is called always G* - open f(U) is G*-open set in Y for every G*-open subset U of X.
Definition 3.20 : A function f:X→Y is called always G* - closed f(F) is G*-closed set in Y for every G*-closed subset F of X.
We, prove the following.

Theorem 3.21 : A surjective function f :X→Y is always G*-open if and only if for each subset B of Y and each G*-closed set F of X containing f^{-1}(B), there exists G*-closed set H of Y such that B ⊂ H and f^{-1}(H) ⊂ F.

Proof is similar to Th.3.8 above.

Theorem 3.22: A surjective function f :X→Y is always G*-closed if and only if for each subset B of Y and each G*-open set V of X containing f^{-1}(B), there exists G*-open set H of Y such that B ⊂ H and f^{-1}(H) ⊂ V.

Proof is similar to Th.3.8 above.

Theorem 3.23: Let f :X→Y and h :Y→Z be two functions such that hof is always G*-open function:

(i) If f is G*-irresolute and surjective, then h is always G*-open.
(ii) If h is G*-irresolute and injective, then f is always G*-open.

We, recall the following.

Definition 3.24[24] : A space X is said to be ultra-normal if for any pair of disjoint closed sets A and B of X there exist disjoint G*-open sets U and V in X such that A ⊂ U and B ⊂ V.

We, define the following.

Definition 3.25 : A space X is said to be G*-normal if for any pair of disjoint closed sets A and B of X there exist disjoint G*-open sets U and V in X such that A ⊂ U and B ⊂ V.

Clearly, every sp-normal space is G*-normal space. We, prove the following.

Theorem 3.26 : The following statements are equivalent for a space X:

(i) X is sp-normal,
(ii) For any pair of disjoint closed sets A ,B of X, there exist disjoint G*-open sets U and V such that A ⊂ U and B ⊂ V,
(iii) For any closed set A and any open set V containing A, there exists a G*-open set U such that A ⊂ U ⊂ gspCl(U) ⊂ V.
The routine proof of the Theorem is omitted.

Theorem 3.27: If f :X→Y is a continuous G*-closed surjection and X is normal ,then Y is G*-normal.
Proof : Let A and B be any disjoint closed sets of Y.Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets of X since f is continuous. Since X is normal, there exist disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By Th.3.16, there exist G*-open sets M and N of Y such that $A \subset M$, $B \subset N$, $f^{-1}(M) \subset U$ and $f^{-1}(N) \subset V$. Then ,$f^{-1}(M) \cap f^{-1}(N) =$ □ and hence $M \cap N =$ □ .
Then by Th.3.26, Y is G*-normal.

Theorem 3.28 : If f :X→Y is a continuous pre-G*-closed surjection and X is G*-normal ,then Y is G*-normal.

Proof follows from Th.3.18 and Th.3.26.

Definition 3.29 : A space X is said to be strongly G*-normal if for any pair of disjoint semipreclosed sets A and B of X there exist disjoint G*-open sets U and V in X such that $A \subset U$ and $B \subset V$. Clearly, every strongly G*-normal space is G*-normal space.

Definition 3.30: A space X is said to be ultra semipre-normal if for any pair of disjoint closed sets A and B of X there exist disjoint semipre-closed sets U and V in X such that $A \subset U$ and $B \subset V$.

Definition 3.31: A space X is said to be ultra G*-normal if for any pair of disjoint closed sets A and B of X there exist disjoint G*-closed sets U and V in X such that $A \subset U$ and $B \subset V$.

Definition 3.32 : A space X is said to be ultra strongly G*-normal if for any pair of disjoint closed sets A and B of X there exist disjoint G*-closed sets U and V in X such that $A \subset U$ and $B \subset V$.

Clearly, every ultra strongly G*-normal space is ultra G*-normal space.

We, state the following.

Theorem 3.33 : The following properties are true for a space X :

(i) X is ultra-normal space
(ii) For any pair of disjoint closed sets A,B of X ,there exist disjoint G*-closed sets U and V in X such that $A \subset U$ and $B \subset V$.

Proof is clear since every closed set is G*-closed set.

Theorem 3.34 : The following properties are true for a space X :

(iii) X is ultra semipre-normal space
(iv) For any pair of disjoint closed sets A,B of X ,there exist disjoint G*-closed sets U and V in X such that $A \subset U$ and $B \subset V$.

Proof is clear since every semipre-closed set is G*-closed set.

Theorem 3.35: If f :X→Y is a continuous G*-open surjection and X is ultra – normal space, then Y is ultra G*-normal space.

Proof : Let A and B be any disjoint closed sets of Y. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets of X since f is continuous function. Since X is ultra –normal space, there exist disjoint closed sets F and H in X such that $f^{-1}(A) \subset F$ and $f^{-1}(B) \subset H$. By Th.3.8, there exist disjoint G*-closed sets M and N of Y such that $A \subset M$ and $B \subset N$, $f^{-1}(M) \subset F$ and $f^{-1}(N) \subset H$. Then, we have $f^{-1}(M) \cap f^{-1}(N) = \emptyset$ and hence $M \cap N = \emptyset$. Then from Th.3.28, Y is ultra G*-normal spa⊂ H.

Theorem 3.36: If f :X→Y is a semipreirresolute alays G*-open surjection and X is ultra – semipre-normal space, then Y is ultra strongly G*-normal space.

Proof follows from Th.3.34 and Th.3.35.

The routine proofs of the following are omitted.

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