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RESEARCH ARTICLE

SEMIGROUPS WITH CYCLIC PROPERTIES

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ABSTRACT

The theory of semigroups is one of the relatively young branch of algebra. This paper mainly deals with the structure theorems on semigroups. By using the cyclic properties in semigroups we determine some properties of semigroups like singular, normal, permutable etc.

INTRODUCTION

The algebraic objectives encountered in this paper are sets with associative binary operation defined on them, most often semigroups. Semigroups are important in many areas of mathematics; for example: Coding and Language theory, Automata theory, Combinatorics and Mathematical analysis. One of the fundamental concepts in Algebra is that of commutative property. Indeed special theories like the "Theory of semigroups" in which the elements satisfy the additional properties: commutative, Left(Right) identity, L(R) cyclic, L(R) cancellative along with identities are also studied.

The basic definitions of the properties of semigroups are as follows

- Definition: A semigroup (S,.) is said to be L- cyclic if it satisfies the identity

  $a.(b.c) = b.(c.a) = c.(a.b)$ for all a, b, c in S

- Definition: A semigroup (S,.) is said to be R- cyclic if it satisfies the identity

  $(a.b).c = (b.c).a = (c.a).b$ for all a, b, c in S

- Definition: A semigroup (S, .) is said to be commutative if it satisfies the identity $ab = ba$ for all a, b in S
- Definition: A semigroup (S, .) is said to be left(right) singular if it satisfies the identity $ab = a$ $(ab = b)$ for all a, b in S
- Definition: A semigroup (S, .) is rectangular if it satisfies the identity $aba = a$ for all a, b in S.
- Definition: A semigroup (S, .) is called regular if for each $a \in S$, there exist an element x∈S such that $axa = a$.
- Definition: A semigroup (S, .) is said to be normal if satisfies the identity $abca = acba$ for all a, b, c in S.
- Definition: A semigroup (S, .) is said to be left(right) cancellative if it satisfies the identity $ab = ac \Rightarrow b = c$ $(ab = cb \Rightarrow a = c)$ for all a, b, c in S.
- Definition: A semigroup (S, .) is said to be cross cancellative if it satisfies the identity $ab = bc \Rightarrow a = c$ and $ab = ca \Rightarrow b = c$ for all a, b, c in S.
- Definition: A semigroup (S, .) is said to be left(right) permutable if, $abc = acb$ $(abc = bac)$ for all a,b,c in S.

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- Definition: A semigroup (S, .) is said to admit conjugates if for all a,b ∈ S there exists an element c∈S such that $ab = bc$ then c is called conjugate of $a\ by\ b$ and is denoted by a^b .
- Definition: A semigroup (S, .) is said to be weakly separative if it satisfies the identity $a^2 = ab = b^2 \Rightarrow a = b$ for all a,b in S.
- Definition: A semigroup (S, .) is said to left (right) identity if for all a ∈ S there exists an element e∈S such that $ea = ae = a$.

Note: In any semigroup of S, S is L- cyclic ⇔ S is R- cyclic.

RESULTS

1 Let (S, .) be a left(right) singular semigroup. Then (S, .) is **cross cancellative** if (S, .) is **L(R) cyclic**.

Proof : Given (S, .) be a left(right) singular semigroup

i.e., $xy = x\ (xy = y)$ (1)

Let (S, .) is L- cyclic

i.e., $x.(y.z) = y.(z.x) = z.(x.y)$ for all x, y, z ∈ S (2)

Now $x.(y.z) = x.y = xy$

$y.(z.x) = y.z = yz$ (3)

$z.(x.y) = z.x = zx$

Using (2) in (3) i.e., $xy = yz = zx$

Case : 1 If $xy = yz$ { Using $xy = x\ and\ yz = z\ from$ (1) }

Then $x = z$ (left) (right)

⇒ (S,.) is cross cancellative

Case : 2 If $yz = zx$ { Using $yz = y\ and\ zx = x\ from$ (1) }

Then $y = x$ (left) (right)

⇒ (S,.) is cross cancellative

From cases (1) and (2) (S,.) is cross cancellative.

RESULT 2 Let (S, .) be a L-cyclic semigroup. Then (S, .) is R-cyclic semigroup if (S, .) is regular.

Proof : Let (S, .) is L- cyclic

For all x, y ∈ S $x.(x.y) = y.(x.x) = x.(y.x)$ (1)

Since (S,.) is regular, we have $axa = a\ for\ all\ a$ ∈ S (2)

⇒ $a.(x.a) = a$ [Using (1) in (2)] (3)

Using (3) in (1) we have $x.(x.y) = y.(x.x) = x.(y.x) = x$

To prove (S, .) is R-cyclic semigroup

i.e., $(x.x).y = (y.x).x = (x.y).x$

Now $x.(x.y) = xxy = (x.x).y = (x.y).x = xyx = x$ [Using (2) and (3)] (4)

Thus $x.(x.y) = x = (x.x).y$ [From (4)]

Similarly $y.(x.x) = x = (y.x).x$

And $x.(y.x) = x = (x.y).x$

Thus (S, .) is R-cyclic semigroup.

RESULT: 3 Let (S, .) be a L-cyclic semigroup then (S,.) is normal and (S,.) is commutative provided (S, .) is cross cancellative.
Proof: Let (S, .)be a semigroup with L- cyclic property.

i.e., $a.(b.c) = b.(c.a) = c.(a.b)$ for all $a, b, c \in$ S

$\Rightarrow abc = bca = cab$ (1)

To prove (S,.) is normal i.e., $xabx = xbax$

L.H.S: $xabx = x(abx)$

$= x(bax)$
[Using (1)]
$= xbax$
= R.H.S

Now if S is normal i.e., $xabx = xbax$ implies

$(xab)x = x(bax)$
$\Rightarrow xab = bax$ [Using cross cancellative]
$\Rightarrow ab = ba$
$\Rightarrow$ (S,.) is commutative.

RESULT: 4 Let (S, .) be a semigroup, where (S, .) is left(right) singular then (S, .) is L(R) cyclic
Proof: Let (S, .) be a semigroup.

If S is left(right) singular then ab = a (ab = b) for all $a, b \in$S

Case: (1) If $ab = a \Rightarrow abc = ac$ (1)

Also $ba = a \Rightarrow bac = ac$ (2)

From (1) and (2) $abc = bac$ (3)

Case: (2): If $ac = c \Rightarrow bac = bc$ (4)

Also $ca = a \Rightarrow cab = bc$ (5)

From (4) and (5) $bac = cab$ (6)
From (3) and (6) we have,
$abc = bac = cab$
$\Rightarrow a.(b.c) = b.(a.c) = c.(a.b)$
$\Rightarrow$ (S, .) is L(R) cyclic.

RESULT: 5 Let (S, .) be a semigroup, where (S, .) is right regular then (S, .) is regular if (S, .) is L(R) cyclic. Also it is commutative.

Proof: Given (S, .) be a semigroup, where S is right regular

i.e., $a^2x = a$
$\Rightarrow aax = a$
$\Rightarrow (a.a).x = a$
$\Rightarrow (a.x).a = a$ [Using L cyclic]
$\Rightarrow axa = a$
$\Rightarrow$ S is regular.
Again if $a^2x = a$

$\Rightarrow a^2xb = ab$
$\Rightarrow bxa^2 = ab$
$\Rightarrow bxaa = ab$
$\Rightarrow b(xaa) = ab$
$\Rightarrow b(axa) = ab$ [Using L cyclic]
$\Rightarrow ba = ab$
$\Rightarrow$ (S,.) is commutative.

RESULT: 6 Let (S, .) be a semigroup, where (S, .) is L(R) cyclic. Then (S, .) is commutative if it is cross cancellative.
Proof: Let (S, .)be a semigroup with L(R) cyclic property.

Applying repeated cyclic property,
$xyxy = x(yxy) = x(xyy)$
$= x(yyx)$
$\Rightarrow (xy)(xy) = (xy)\,(yx)$ [Using cross cancellative]
$\Rightarrow xy = yx$
$\Rightarrow$ (S, .) is commutative

RESULT: 7 Let (S, ., ≤) be a totally ordered semigroup, where ≤ is defined by a ≤ b if and only if $a = xb = by\,, a = xa$ for all a, b in S and for some x, y in S. If (S, .) is L- cyclic then (S, .) is normal. Also if (S, .) is normal then

$xabx = ab \text{ and } xbax = ba.$
Proof: Given (S, . , ≤) is a totally ordered semigroup.
Since S is L- cyclic we have

$x.(y.z) = y.(z.x) = z.(x.y)$ for all $x, y, z \in$ S.

Also S is totally ordered semigroup, we have $a \le b \text{ or } b \le a$.

Case:1 Let $a \le b$ then $a = xb = by\,, a = xa$ for all a, b in S
and for some x, y in S.

To prove (S, .) is normal.
L.H.S: $xabx = (xab)x$
$= (axb)x$
$= (xba)x$
$= xbax$ (1)
= R.H.S

Case:2 Let $b \le a$ then $b = xa = ay\,, b = xb$ for all a, b in S
and for some x, y in S.

To prove (S, .) is normal.
L.H.S: $xabx = x(abx)$
$= x(bxa)$
$= x(bax)$
$= xbax$ (2)
= R.H.S
From (1) and (2), (S, .) is normal.
Now if S is normal,
$xabx = (xa)(bx)$
$= a\,(bx)$
$= xab$
$\therefore xabx = ab$
Again, $xbax = (xb)(ax)$
$= bax$
$= xba$
$\therefore xbax = ba$

$$\therefore xabx = ab \text{ and } xbax = ba$$

SEMIGROUPS SATISFYING THE IDENTITY: $abc = ca$

RESULT: 8 Let (S, .) be a L-cyclic semigroup where "e" is the identity. If (S, .) satisfies the identity $abc = ca$ then S admits conjugates.

Proof: Let (S, .)be a semigroup.

S admits conjugates if for all $x, y \in S$ there exists an element $z \in S$ such that $xy = yz$

then z is called conjugate of x by y and is denoted by x^y.

Now S is L- cyclic.

i.e., $x.(y.z) = y.(z.x) = z.(x.y)$ for all $x, y, z \in$ S (1)
Now $x.(y.z) = xyz$
$\Rightarrow x.(y.e) = xye = xy$
Also $y.(z.x) = yzx$
$\Rightarrow y.(z.e) = yze = yz$
Also $z.(x.y) = zxy$
$\Rightarrow z.(x.e) = zxe = zx$
Thus from (1) $x.(y.z) = y.(z.x) = z.(x.y)$
implies $xy = yz = zx$
$\Rightarrow$ S admits conjugates.

RESULT: 9 Let (S, .) be a L-cyclic semigroup satisfying the identity $abc = ca$ then (S, .) is weakly separative.
Proof: Let (S, .)be a semigroup.

Also S is L- cyclic.

i.e., $x.(y.z) = y.(z.x) = z.(x.y)$ for all $x, y, z \in$ S (1)

Now $x.(y.z) = zx$ [From the identity $abc = ca$]
Put $z = x$ then $x.(y.z) = x^2$
Also $y.(z.x) = xy$ [From the identity $abc = ca$]
Now $z.(x.y) = yz$ [From the identity $abc = ca$]
Put $z = x$ then $z.(x.y) = xy$
Now $x.(y.z) = y.(z.x) = z.(x.y)$ implies
$x^2 = xy = xy$
$\Rightarrow$ S is weakly separative.

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