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ISSN: 0976-3376

ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

Asian Journal of Science and Technology
Vol. 09, Issue, 05, pp.8048-8057, May, 2018

RESEARCH ARTICLE

ANALYSIS OF A POISEUILLE FLOW OF AN INCOMPRESSIBLE FLUID BETWEEN CONCENTRIC CIRCULAR CYLINDERS

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ARTICLE INFO

Article History:
Received 17th February, 2018
Received in revised form
20th March, 2018
Accepted 06th April, 2018
Published online 30th May, 2018

Key words:

Analytical solution,
Incompressible Fluid, Poiseuille Flow,
Slip and no-slip boundary conditions,
Volume flow rate, Velocity profile.

ABSTRACT

This paper presents analytical solutions of a Poiseulle flow of incompressible fluid between two fixed upright concentric circular cylinders subject to slip boundary conditions by considering three different cases. In each case steady state velocity and volume flow rate distribution in the field are determined by applying slip boundary conditions. By utilizing MATLAB the obtained results are visualized in figure. The effect of slip length and a radius of a gap between the cylinders on steady state velocity and volume flow rate distribution of the fluid are discussed. It is found that, velocity profile and volume flow rate distribution of the fluid are affected by a radius of a gap between the cylinders and slip length. The result also indicates that, fluid velocity of distribution decreases when the distance between the two cylinder increases in the vertical fluid motion and maximum velocity distribution occurs at the center line of parallel flow.

INTRODUCTION

Fluid mechanics is the branch of science which deals with the behavior of the fluids (liquids or gases) at rest as well as in motion. Thus, this branch of science deals with the static, kinematics and dynamics aspects of fluids (Bansal, 2005). The inadequacy of the classical Navier-Stokes theory for describing rheological complex fluids has led to the development of several theories of non-Newtonian fluids. Non-Newtonian fluid flows play important role in several industrial manufacturing processes. Due to the prominent applications in the modern technology and industries, many researchers made attempt to study different non-Newtonian fluid flow problems (Devakar, 2014). The study of flows of non-Newtonian fluids with slip boundary condition has gained considerable attention in the recent past. Though many flow problems concerning Newtonian and several non-Newtonian fluids have been solved under the no-slip condition, the fluid slippage might occur at the solid boundary (Ashmawy, 2012; Thompson and Troian, 1997). Several investigations indicate the existence of slip at the solid boundary (Neill *et al.,* 1986). As a matter of fact(Navier, 1823) proposed a general boundary condition that permits the possibility of fluid slip at a solid boundary. This boundary condition assumes that the tangential velocity of the fluid relative to the solid at a point on its surface is proportional to the tangential stress acting at that point.

Due to its vast applications in engineering and industry, pressure driven flow or the Poiseuille flow has attracted attention of various researchers. Although pressure driven flows are unidirectional and have been studied earlier for Newtonian and some non-Newtonian fluids, they still attract special attention in a number of emerging problems (Tang, 2012). (Chen and Zhu, 2008) obtained the analytical solution of Couette-Poiseuille flow of Bingham fluids between two porous parallel plates with slip conditions. (Song and Chen, 2008) investigated the Poiseuille flow of simple fluids in cylindrical nanochannels. (Yang and Zhu, 2012) have analyzed the squeeze flow of Bingham fluid in the small gap between parallel disks with the Navier slip condition. (Ferrás et al., 2012) presented analytical solutions of both Newtonian and inelastic non-Newtonian fluids with slip boundary conditions for Couette and Poiseuille flows. (Hron et al.,2008) established closed form analytical solution for the flows of incompressible non-Newtonian fluids with Navier's slip conditions at the boundary. (Devakar et al., 2014) established the closed form analytical solutions of steady fully developed flows of couple stress fluid between two concentric cylinders, generated due to the constant pressure gradient, using the slip boundaryconditions. As far as we know, an analytical solution of Poiseuille flow of an Incompressible Fluid between two upright (or vertical) concentric cylinders subject to slip boundary conditions has been not established.

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The main objective of this study is to establish analytical solutions for Poiseuille flows of an incompressible fluid between two vertical concentric circular cylinders governed by a Navier-Stokes equation by utilizing slip boundary condition and toexaminethe effect of slip length and the radius of the gap between two cylinders on steady state velocity and volume flow rate distribution of fluid. The slip boundary conditions are applied at the boundaries of the inner and outer cylinders. In this paper, first analytical solution of the Poiseuille flow of an incompressible fluid between vertical concentric circular cylinders governed by a Navier-Stokes equation is established by applying appropriate assumptions subject to slip boundary conditions. Then the effect of slip length and the radius between the cylinders on the volume flow rate and velocity profile has been analyzed. Three cases have been discussed: In the first case it is assumed that the fluid slip at the surface of the two cylinders, in the second case the fluid does not slip (no-slip) at the surface of the inner cylinder and the fluid slip at the surface of the outer cylinder, and in the last case it is assumed that the fluid slip at the surface of the inner cylinder surfaces and the fluid does not slip (no-slip) at the surface of the outer cylinder. Ineachcase, it is assumed that both the inner and outer cylinders are at rest and the flow is due to the constant pressure gradient.

Governing equations and simplifying assumptions

Simplifying assumptions: Analytical solutions are a valuable tool to understand the complexity of fluid dynamics. But due to the great complexity of the full compressible Navier Stoke equations, no known analytical solution exists. Hence, it is necessary to simplify the equations either by making assumptions about the fluid, about the flow or about the geometry of the problem in order to obtain analytical solutions. In view of this, we consider steady, incompressible, fully developed laminar and viscous flow called Poiseuille flow of a Newtonian fluid of constant density ρ and viscosity μ contained between two straight up concentric circular cylinders driven by constant pressure gradient as shown in Fig.1. The radii of the inner and outer cylinders are respectively, R_1 *and* R_2 and the cylinders are assumed to be at rest. It is also assumed that the flow is parallel to the wall with slip on boundary condition $o = k_0 < k_1 < k_2; R_1 \le r \le R_2; \frac{R_1 - R_2}{L}$ □ $1, K \le 2$.

Because of the geometry, Poiseuille flow is analyzed using cylindrical polar coordinates (r, θ, z) with origin on the center-line of the cylinder entrance and z-direction aligned with the centerline.For this work we made the following realistic (or reasonable) simplifying assumptions:

1. Since the flow is parallel to the walls there is only one non-zero velocity component- that in the direction of flow, v_z .Thus, $v_r = v_\theta = 0$, where v_r and v_θ are, respectively, theradial (or tangential) and angular velocity of the fluid.
2. As the flow is steady the velocity of the fluid at a particular fixed pointdoes not change with time, subsequently $\frac{\partial(v_r)}{\partial t} = \frac{\partial(V_\theta)}{\partial t} = \frac{\partial(v_Z)}{\partial t} = \frac{\partial(\rho)}{\partial t} = 0$
3. Since the flows studied in this work are fully developedPoissulleflow, the velocity profile does not change in the fluid flow direction. Hence, $\frac{\partial(v_r)}{\partial z} = \frac{\partial(V_\theta)}{\partial z} = \frac{\partial(v_Z)}{\partial z} = 0$
4. As the flow is axisymmetric flow the cylindrical velocity components v_r, v_θ and v_z are independent of the angular variable θ. Thus $\frac{\partial(V_r)}{\partial \theta} = \frac{\partial(V_\theta)}{\partial \theta} = \frac{\partial(v_Z)}{\partial \theta} = 0$
5. Gravity actsvertically down wards. Therefore $g_Z = g,\ g_r = g_\theta = 0$,

Governing equations: This work concerns steady, viscous, fully developed incompressible fluids with constant density which are governed by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0 , \tag{1}$$

and the momentum equation

$$\rho \frac{\partial u}{\partial t} = \rho g - \nabla p + \mu \nabla^2 v \tag{2}$$

In eqn.(1) and eqn.(2) v is the velocity vector of the fluid , p is the pressure, ρ is the fluid density , μ is viscosity and g is the gravity.

In cylindrical polar coordinates eqn. (1) can be written as

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0 \quad (3)$$

In view of the constant-density assumption, eqn. (1) simplifies to Eqn. (4) and eqn. (3) simplifies to eqn. (5)

$$\nabla . v = 0 \,, \quad (4)$$

$$\rho\left(\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}\right)=0 \quad (5)$$

With the stated assumptions of a Newtonian fluid with constant density and viscosityeqn. (6) gives r, θ *and* z momentum balances:

$$\rho\left(\frac{\partial(v_r)}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta{}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\rho g_r-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right)-\frac{v_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\delta^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$\rho\left(\frac{\partial(v_\theta)}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)=\rho g_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial v_\theta{}^2}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

$$\rho\left(\frac{\partial(v_z)}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial v_z{}^2}{\partial z^2}\right] \quad (6)$$

RESULTS AND DISCUSSION

To analyze the steady state velocity and volume flow rate distribution, we start from the continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Which, for constant density and $v_r = v_\theta = 0$ reduced to:

$$\frac{\partial v_z}{\partial z}=0 \quad (7)$$

verifying that v_z is independent of distance from the inlet, and that the velocity profile $v_z = v_z(r)$ and $p = p(z)$ appears the same for all values of z.

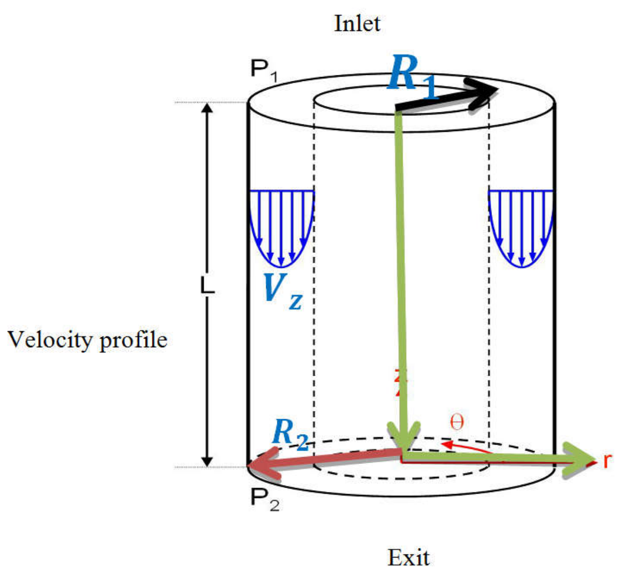

Fig. 1. Geometry for Poiseuille flow between two concentric cylinders

Now we are going to use the momentum equation to solve for v_z. From the Z momentum balance

$$\rho\left(\frac{\partial(\mathrm{v_z})}{\partial t}+\mathrm{v_r}\frac{\partial \mathrm{v_z}}{\partial r}+\frac{\mathrm{v_\theta}}{r}\frac{\partial \mathrm{v_z}}{\partial\theta}+\mathrm{v_z}\frac{\partial \mathrm{v_z}}{\partial z}\right)=\rho\boldsymbol{g}_z-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \mathrm{v_z}}{\partial\theta^2}+\frac{\partial \mathrm{v_z}^2}{\partial z^2}\right]$$

With $v_r = v_\theta = 0$ (from assumption 1), $\frac{\partial v_z}{\partial z} = 0$ (from equation 7), $\frac{\partial(v_Z)}{\partial\theta} = 0$ (assumption 4) and $g_Z = g$ (assumption 5), this momentum balance

$$\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \mathrm{v_z}}{\partial\theta^2}+\frac{\partial \mathrm{v_z}^2}{\partial z^2}\right]=-(\rho\boldsymbol{g}_z+p) \tag{8}$$

In which total derivatives are used because $\mathrm{v_z}$ depends only on r. Two successive integration of equation (8), yields

$$V_Z=\frac{-(\mathrm{P}+\rho \mathrm{g}_z)}{4\mu}r^2+A\ln r+B \tag{9}$$

In order to evaluate the two integration constants A and B we consider three different cases to apply boundary conditions at the walls of the cylinders.

Steady state velocity analysis in the first case

Suppose that a fluid slip at the surface of the two cylinders and both cylinders arefixed, then thetwo constants can be evaluated by applying boundary condition of non-zero velocity at the surface of the two cylinders,

$$\begin{cases} r=R_1 : \boldsymbol{v}_Z = U \\ r=R_2 : \boldsymbol{v}_Z = U \end{cases} \tag{10}$$

Eqn. (9) and eqn. (10) leading to:

$$\mathrm{A}=\frac{(\mathrm{P}+\rho \mathrm{g}_z)(R_2^2-R_1^2)}{4\mu\ln\frac{R_2}{R_1}},\ \mathrm{B}=U+\frac{(\mathrm{P}+\rho \mathrm{g}_z)R_1^2}{4\mu}-\frac{(\mathrm{P}+\rho \mathrm{g}_z)(R_2^2-R_1^2)}{4\mu\ln\frac{R_2}{R_1}}\ln R_1 \tag{11}$$

Eqn. (9) and eqn. (11) gives the final solution for velocity distribution;

$$V_Z=\frac{(P+\rho g)}{4\mu}\left[\frac{(R_2^2-R_1^2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}-(r^2-R_1^2)\right]+k\left(1-\left(\frac{r}{R_1}\right)^2\right) \tag{12}$$

Where $U=k(1-(\frac{r}{R_1})^2)$, is velocity specified, k is slip length or friction coefficient.

This solution is visualized by figure as follows by utilizing MATLAB.

Figure (2) shows that the graph of velocity profile is parabolic. This indicates thatas $r \to R_1$, and $r \to R_2$, $0 \le k \le 2$, the fluid velocity decreases and it is greatest at the centerline than near to slip boundaries. The graph also indicates as the gap between concentric cylinders decreases the fluid velocity decreases and the more close the fluid slips at the boundarythe less its velocity is affected by the slip length of boundary condition.

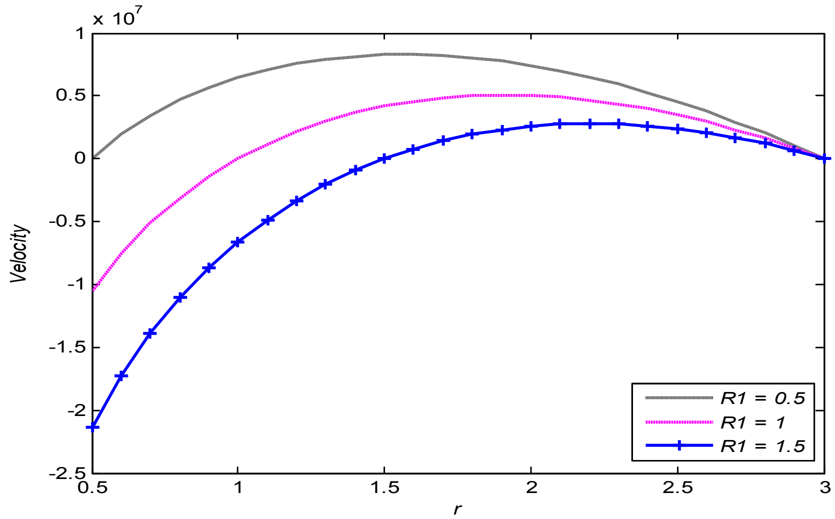

Fig. 2. Velocity profiles

Volume flow rate analysis in the first case

The flow rate through concentric circular cylinders or annulus of internal radius r and external radius $r + dr$ is

$$dQ = 2V_Z \pi r dr \tag{13}$$

From eqn. (12) and eqn. (13), we get the volume flow rate,

$$Q = \int_{R_1}^{R_2} V_Z 2\pi r dr$$

$$= \frac{2\pi\left(p + \rho g_z (R_2^{\,2} - R_1^{\,2}\right)}{4\mu \ln \frac{R_2}{R_1}}\left[\int_{R_1}^{R_2} \left(\ln \frac{r}{R_2}\right) r dr - \int_{R_1}^{R_2} r^3 dr + R_1^{\,2} \int_{R_1}^{R_2} r dr\right] + \int_{R_1}^{R_2} k\left(1 - \left(\frac{r}{R_1}\right)^2\right) 2\pi r dr$$

$$= \left(\frac{\pi\left((R_2^{\,2} - R_1^{\,2}\right)}{8}\left[\frac{(p + \rho g_z)}{\mu}\left[(R_2^{\,2} + R_1^{\,2} - \frac{(R_2^{\,2} - R_1^{\,2})}{\ln \frac{R_2}{R_1}}\right] + \frac{4k(R_2^{\,2} - R_1^{\,2})}{R_2^{\,2}}\right]\right) \tag{14}$$

Which is sketched bellow using MATLAB in Figure 3.

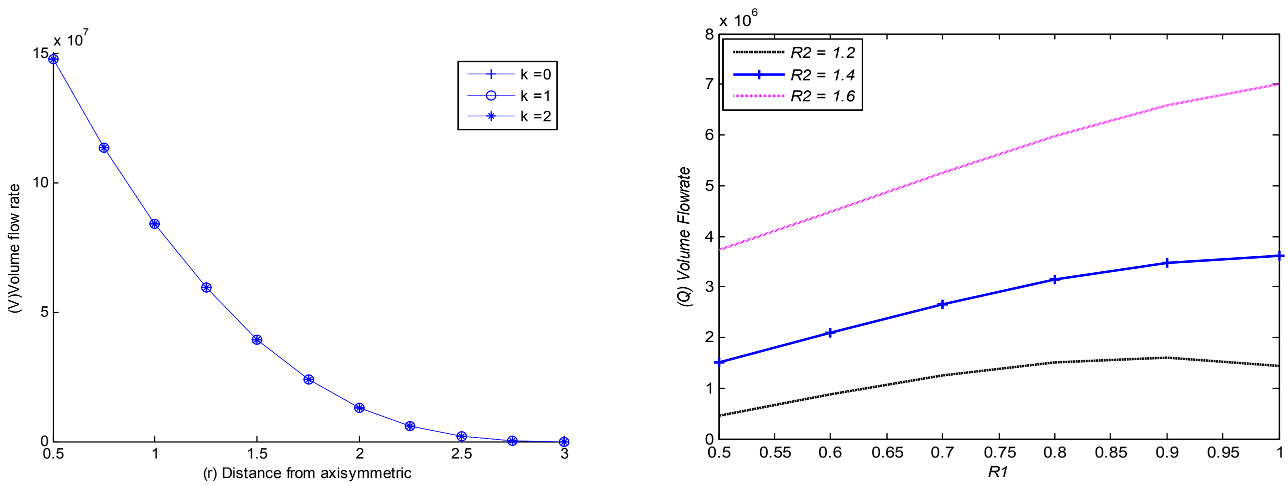

Fig. 3. volume flow rate profiles

From Fig.3we can observe that the volume flow rate is affected by both the distance between the cylinders r and slip length k. As the values of r between concentric cylinders approaches either to the radius of the inner cylinder or the radius of the outer cylinder volume flow rate of fluid decreases. It can also be seen from the figure thatthe volume flow rate is maximum at the centerline of fluid than near to both slip boundary condition. Moreover the figure illustrates as the gap between the cylinders increases the effect of friction coefficient on volume flow rate decreases.

Steady state velocity analysis in the second case

Suppose that there is no fluid slip (no-slip) at the surface of the inner cylinder and fluid slip at the surface of the outer cylinder and both cylinders are fixed. Thus the two constants may be evaluated by applying zero velocity at the surface of the inner cylinder and non-zero velocity at the surface of outer cylinder,

$$\begin{cases} r = R_1 : \boldsymbol{v}_Z = 0 \\ r = R_2 : \boldsymbol{v}_Z = U \end{cases} \quad (15)$$

Eqn. (12) and eqn. (15) gives

$$\mathrm{A} = \frac{\left(U 4\mu + (\mathrm{p}+\rho \mathrm{g}_z)\right)(R_2^{\,2} - R_1^{\,2})}{4\mu \ln \dfrac{R_2}{R_1}} \quad , \mathrm{B} = \mathrm{U} - \frac{U \ln R_2}{\ln \dfrac{R_2}{R_1}} + \frac{\left(\mathrm{p}+\rho \mathrm{g}_z\right) R_2^{\,2}}{4\mu} - \frac{\left(\mathrm{p}+\rho \mathrm{g}_z\right)(R_2^{\,2} - R_1^{\,2})}{4\mu \ln \dfrac{R_2}{R_1}} \ln R_2 ,$$

Substituting these values for constants of integration into eqn. (12) yields the final expression for the velocity profile:

$$\mathbf{V_Z} = \frac{\left(\mathrm{p}+\boldsymbol{\rho} \mathrm{g}_z\right)}{4\boldsymbol{\mu}} \left[\frac{(R_2^{\,2} - R_1^{\,2}) \ln \dfrac{r}{R_2}}{\ln \dfrac{R_2}{R_1}} - (r^2 - R_2^{\,2}) \right] + \left(\mathbf{k}(1 - (\frac{r}{R_2})^2) \right) \frac{\ln \dfrac{r}{R_1}}{\ln \dfrac{R_2}{R_1}} \quad (16)$$

Where $U = k(1 - (\frac{r}{R_2})^2)$, is a constant (or velocity specified) andk is slip length or friction coefficient, which is sketched in fig. 4.

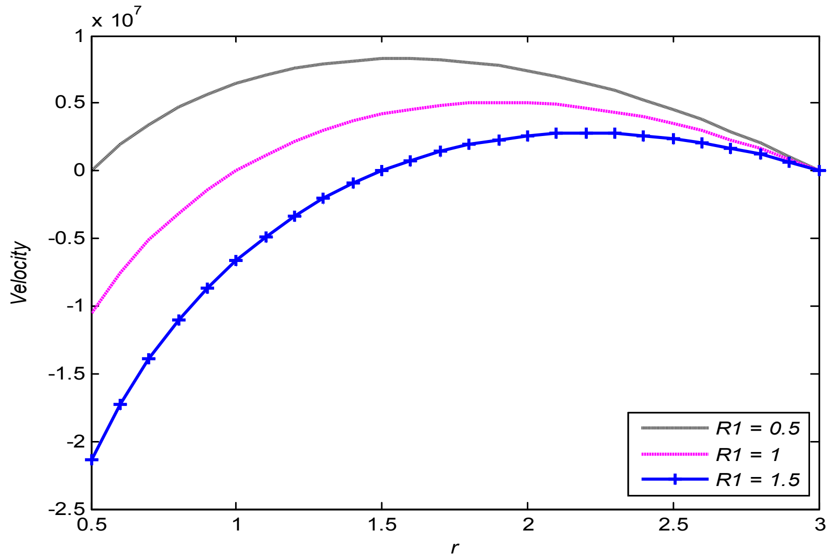

Fig. 4. Velocity profiles

From fig. 4 we observed that as $r \to R_1, r \to R_2, 0 \le k \le 2$ from the centerline of the parallel flow the velocity of the fluid decreases. This illustrates that the velocity of the fluid is affected by both friction coefficient k and the length of the gap between a vertically held concentric cylinders. The graphs also indicate that decreasing a gap between concentric cylinders has a decreasing effect on a fluid velocity. In addition to this it shows that the more close the fluidslip and no-slip boundary condition at the surface of concentric cylinder the less its velocity is affected by the slip length of boundary condition.

Volume flow rate analysis in the second case

The total volume flow rate distribution for this case can be evaluated by using eqn. (13) and eqn. (16) and they yields

$$Q=\int_{R_1}^{R_2}\left[\frac{(\mathrm{p}+\boldsymbol{\rho}\mathrm{g}_z)}{4\boldsymbol{\mu}}\left[\frac{(R_2^{\,2}-R_1^{\,2})\ln\frac{r}{R_2}}{\ln\frac{R_2}{R_1}}-\left(r^2-R_2^{\,2}\right)\right]+\mathbf{k}\,(1-(\frac{r}{R_2})^2)\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}\right]2\pi r dr$$

$$=\left(\frac{(\pi)(R_2^{\,2}-R_1^{\,2})}{8}\right)\left(\left(\frac{(\mathrm{p}+\rho\mathrm{g}_z)}{\mu}\right)\left[\frac{\left(R_2^{\,2}-R_1^{\,2}\right)}{\ln\frac{R_2}{R_1}}+R_2^{\,2}+R_1^{\,2}\right]+\left[k\frac{3R_2^{\,2}-R_1^{\,2})}{R_2^{\,2}\ln\frac{R_2}{R_1}}\right]\right)+\frac{KR_2^{\,2}\pi}{2} \qquad (18)$$

Which is sketched in Figure (5).

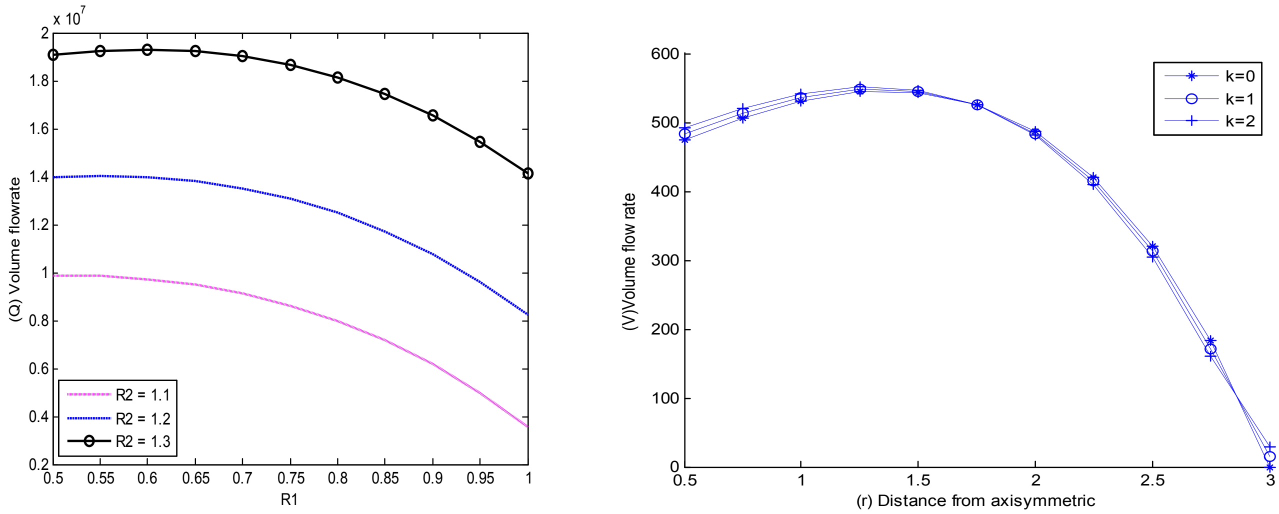

Fig. 5. Volume flow rate of profiles

From Figure 5 it can be observed that the graph increases as the values of r between concentric cylinders increases to midpoint of the gap and the graph decreases when the value of r is either approaches to R_1 or R_2. This indicates the volume flow rate of the fluid is high when the parallel flow of fluid is at the centerline than near to no-slip and slip boundary surfaces of concentric cylinders. From the figure we can also observe that the friction coefficient affects negatively the volume flow rate of the fluid.

Steady state velocity analysis in the second case

In this case let as evaluate the two constants of integration by applying the boundary condition of non-zero velocity at the inner cylinder and the zero velocity at the outer cylinder;

$$\begin{cases} r=R_1: \boldsymbol{v}_Z=U \\ r=R_2: \boldsymbol{v}_Z=0 \end{cases} \qquad (19)$$

Eqn. (12) and eqn. (19) gives

$$\mathrm{A}=\frac{-(U4\mu-(\mathrm{p}+\rho\mathrm{g}_z)(R_2^{\,2}-R_1^{\,2})}{4\mu\ln\frac{R_2}{R_1}}, \qquad \mathrm{B}=U+\frac{(\mathrm{p}+\rho\mathrm{g}_z)R_1^{\,2}}{4\mu}+\frac{U4\mu-(\mathrm{p}+\rho\mathrm{g}_z)(R_2^{\,2}-R_1^{\,2})}{4\mu\ln\frac{R_2}{R_1}}\ln R_1$$

Substituting these values of the constants into equation (12), yields the final expression for the velocity profile:

$$\mathbf{V_Z} = \frac{\left(p+\rho g_z\right)}{4\mu}\left[\frac{(R_2^{\,2}-R_1^{\,2})\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}-(r^2-R_1^{\,2})\right]-\frac{\boldsymbol{K}(R_1^{\,2}-r^2)}{R_1^{\,2}}\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} \quad (20)$$

Where $U = k(1-(\frac{r}{R_1})^2)$, is a constant or velocity specified and **k** is slip length or friction coefficient, which is sketched in Figure (6).

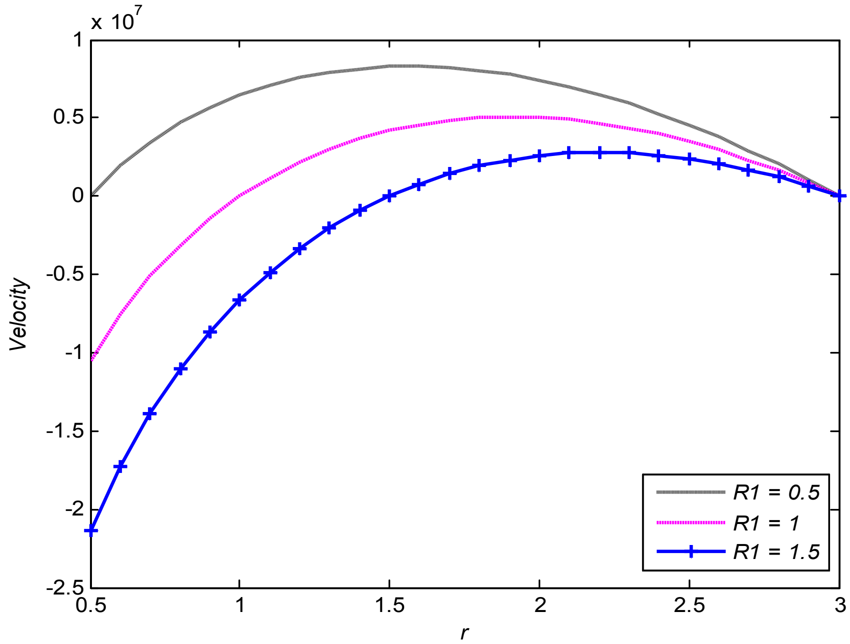

Fig. 6. Velocity profiles

Figure 6shows the velocity of the fluid flowing through the concentric cylinder decreases as $r \to R_1$ *and* $r \to R_2$ where $0 \le k \le 2$ and the maximum value of the velocity in the situation occurs at the centerline of the parallel flow.This in turn implies the velocity of the fluid decreases near to slip boundary condition and no-slip boundary condition of the inner and outer surface of concentric cylinders. The figure also indicates that, decreasing the gap between concentric cylinders has decreases effect on fluid velocity.

Volume flow rate analysis in the third case

From eqn. (13) and eqn. (20), we can obtain a total volume flow rate distribution between the two fixed concentric circular cylinders;

$$Q = \int_{R_1}^{R_2}\left[\frac{\left(p+\rho g_z\right)}{4\mu}\left[\frac{(R_2^{\,2}-R_1^{\,2})\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}-\left(r^2-R_1^{\,2}\right)\right]-\frac{\boldsymbol{K}\left(R_1^{\,2}-r^2\right)}{R_1^{\,2}}\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}\right]2\pi r dr$$

$$= \frac{\pi\left(R_2^{\,2}-R_1^{\,2}\right)}{8}\left[\frac{\left(p+\rho g_z\right)}{\mu}\left[R_2^{\,2}+R_1^{\,2}-\frac{\left(R_2^{\,2}-R_1^{\,2}\right)}{\ln\frac{R_2}{R_1}}\right]+(k\frac{(3R_1^{\,2}-R_2^{\,2})}{R_1^{\,2}\ln\frac{R_2}{R_1}}\right]-\frac{KR_2^{\,2}\pi}{2} \quad (21)$$

Which is sketched in Figure 7.

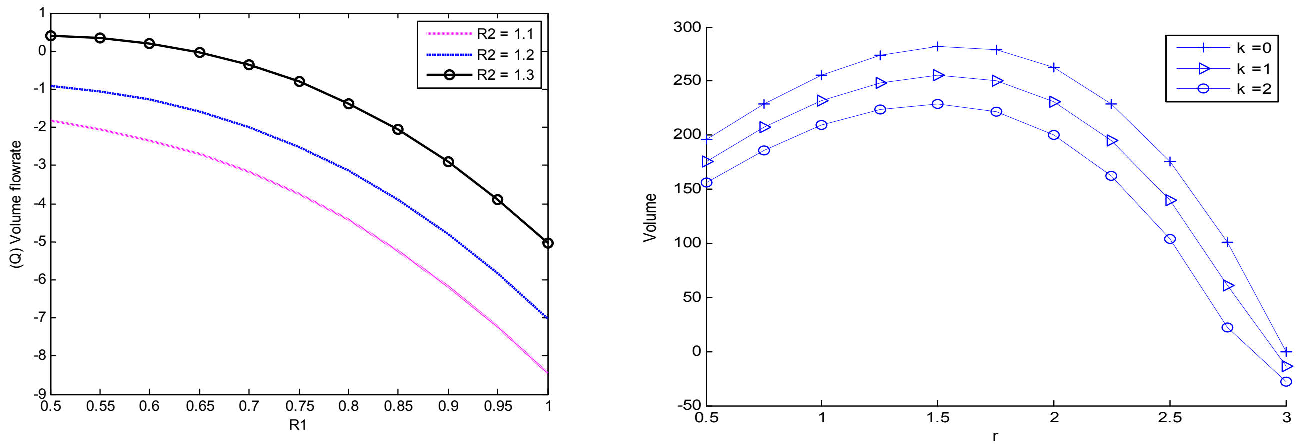

Fig. 7. Volume flow rate profiles

Figure 7 shows that for all values of $k \in [0,2]$, the volume flow rate of a fluid decreases when the value of r decreases from the middle value from R_1 *to* R_2 or when the values of r increases from the middle value of R_1 *and* R_2 to R_2 where $R_1 < R_2$. The figure also illustrates that the volume flow rate of a fluid is maximum when r is at near to the center line of the parallel flow of fluid that is near to the mid-point of R_1 *and* R_2 or when $r \in [1,2]$.

Conclusion

In this paper three distinct cases have been considered to establish analytical solution of a Poiseuille flow of incompressible fluid between two vertically held fixed concentric cylinders subject to slip boundary conditions. In all cases, the governing equations give rise to the Navier-Stokes differential equations and they are solved analytically based on realistic simplifying assumptions and boundary conditions. In each case the velocity and volume flow rate of the fluid have been analyzed by considering different slip conditions and slip length. Moreover, the effect of the distance between the two cylinders on flow velocity and volume flow rate of the fluid has been discussed. The result illustrates that:

- The velocity field involves only one non- zero components, v_z and its magnitude is a function of the axial coordinate and depends on the slip length k and the distance between the two concentric cylinders.
- The flow velocity of the fluid is high at the centerline of the parallel flow when compared to the velocity of fluid near slip boundary condition and no- slip boundary or near to the surface of concentric cylinders.
- Decreasing slip length of boundary condition in motion of parallel flow of fluid has a decreasing effect on the velocity.
- The slip and no-slip boundary condition have an effect on volume flow rate of the fluid in the concentric circular cylinders. This implies the volume flow rate of the fluid changes depending on the value of r and the value of slip length k.

Generally, as it has been discussed the result of this work shows both flow velocityand volume flow rate depends on slip length and the radius between the two concentric cylinders.

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