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RESEARCH ARTICLE

DOUBLE-FRAMED FUZZY SOFT VERSION OF G-MODULUR STRUCTURES

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ABSTRACT

In this paper, we apply the notion of double-framed fuzzy soft set to module theory. We introduce the concept of double-framed fuzzy soft G-modules, fuzzy soft d-ideals of modules and investigate several properties. We give relations between a double-framed fuzzy soft G-modules [DFFSGM] and bipolar fuzzy soft d-ideal [DFFSDI]. We provide a condition for double-framed fuzzy soft G-modules to be a double-framed fuzzy soft d-ideal. We also give characterizations of double-framed fuzzy soft ideal. We consider the concept of strongest double-framed fuzzy relations on double-framed fuzzy soft d-ideals of a module and discuss some related properties.

1. INTRODUCTION

Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [4] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. K.Hayat et.al [7] defined applications of double-framed soft ideals in BE-algebra. Jun *et al* [[9],[10]] introduced the notion of double-framed soft sets (briefly, DFS-sets), and applied it to BCK/BCI- algebras. They discussed double-framed soft algebras (briefly, DFS-algebras) and investigated related properties. A.R.Hadipour [4] defined Double-framed soft BF-algebras and Yongukchoet.al [15] studied on double-framed soft Near-rings. In this paper, we apply the notion of double-framed fuzzy soft set to module theory. We introduce the concept of double-framed fuzzy soft G-modules , fuzzy soft d-ideals of modules and investigate several properties. We give relations between a double-framed fuzzy soft G-modules and bipolar fuzzy soft d-ideal. We provide a condition for double-framed fuzzy soft G-modules to be a double-framed fuzzy soft d-ideal. We also give characterizations of double-framed fuzzy soft ideal. We consider the concept of strongest double-framed fuzzy relations on double-framed fuzzy soft d-ideals of a module and discuss some related properties.

2.**Preliminaries**

2.1 Definition: Let 'S' be a set. A fuzzy set in S is a function $\mu : S \rightarrow [0,1]$.

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2.Preliminaries: In this section as a beginning, the concepts of G-module soft sets introduced by Molodsov and the notions of fuzzy soft set introduced by Maji et al. have been presented.

2.1 Definition [4]: Let G be a finite group. A vector space M over a field K (a subfield of C) is called a G-module if for every g $\in$ G and m $\subset$ M, there exists a product (called the right action of G on M) m.g $\subset$ M which satisfies the following axioms.

1. $m.1_G = m$ for all m $\in$ M (1_G being the identify of G)
2. m. (g. h) = (m.g). h, m $\in$ M, g, h $\in$ G
3. $(k_1 m_1 + k_2 m_2)$. G = $k_1 (m_1. g) + k_2(m_2. g)$, $k_1, k_2 \subset K$, $m_1, m_2 \subset M$ & g $\subset$ G. In a similar manner the left action of G on M can be defined.

2.2. Definition [4]: Let M and M* be G-modules. A mapping Ø: M→M* is a G-module homomorphism if

1. $Ø(k_1 m_1 + k_2m_2) = k_1 Ø (m_1) + k_2 Ø (m_2)$
2. Ø(gm) = g Ø (m), $k_1, k_2 \in K$, m, $m_1,m_2 \in M$ & g $\in$ G.

2.3. Definition [4]: Let M be a G-module. A subspace N of M is a G - sub module if N is also a G-module under the action of G.

Let U be a universe set, E be a set of parameters, P(U) be the power set of U and A ⊆ E.

2.4.Definition[29]: A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11,17,18,26,29,7].

2.5. Definition[6] :The relative complement of the soft set F_A over U is denoted by F^r_A, where $F^r_A : A \to P(U)$ is a mapping given as $F^r_A(a) = U \setminus F_A(a)$, for all a ∈ A.

2.6.Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B$ =(H,C), where

C = A∩B and for all c ∈ C, H(c) = F(c)∩G(c).

2.7. Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C),where C = A∩B and for all c ∈ C, H(c) = F(c)∪G(c).

2.8. Definition[12]: Let F_A and G_B be soft sets over the common universe U and ψbe a function from A to B. Then we can define the soft set ψ (F_A) over U, where ψ (F_A) : B→P(U) is a set valued function defined by $\psi (F_A)(b) = \cup\{F(a) \mid a \in A$ and $\psi (a) = b\}$, if$\psi^{-1}(b) \neq \emptyset$, = 0 otherwise for all b ∈ B. Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \to P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi (a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.9. Definition[13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A)$: B→P(U) is a set-valued function defined by $\psi^\star(F_A)(b)=\cap\{F(a) \mid a \in A$ and $\psi (a) = b\}$, if $\psi^{-1}(b) \neq \emptyset, =0$ otherwise for all b ∈ B. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.1. Theorem [13]: Let F_H and T_K be soft sets over U, F^r_H, T^r_K be their relative soft sets, respectively and ψ be a function from H to K. then, i) $\psi^{-1}(T^r_K) = (\psi^{-1}(T_K))^r$,
ii) $\psi (F^r_H) = (\psi^\star(F_H))^r$ and $\psi^\star(F^r_H) = (\psi (F_H))^r$.

2.10. Definition[14]: Let F_A be a soft set over U and a be a subset of U. Then upper α-inclusion of F_A, denoted by $F_A^{\supseteq\alpha}$, is defined as $F_A^{\supseteq\alpha}$ = {x$\in$A/F(x) $\supseteq \alpha$}. Similarly,

$F_A^{\subseteq\alpha}$ = {x ∈ A | F(x) ⊆α}is called the lower α -inclusion of F_A. A nonempty subset U of a vector space V is called a subspace of V if U is a vector space on F. From now on,V denotes a vector space over F and if U is a subspace of V, then it is denoted by U < V.

2.11. Definition [8]: A double-framed pair $\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$ is called a double-framed fuzzy soft set (briefly DFFS-set) over U where $\bar{\alpha}$ and $\bar{\lambda}$ are mapping from A to P(U).

For a DFS-set $\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$ over U and two subsets γ and δ of U, the γ-inclusive set and the δ-exclusive set of $\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$, denoted by i_A ($\bar{\alpha}$; γ) and $e_A(\bar{\lambda}$, δ) respectively, are defined as follows.

i_A ($\bar{\alpha}$; γ) = { x ∈ A / $\gamma \subseteq \bar{\alpha}$ (x) } and $e_A(\bar{\lambda}$, δ) = { x ∈ A / $\delta \subseteq \bar{\lambda}$(x) } respectively. The set $DF_A(\bar{\alpha},\bar{\lambda})_{(\gamma,\delta)}$ = { x ∈ A / $\gamma \subseteq \bar{\alpha}$ (x), $\delta \subseteq \bar{\lambda}$(x) } is called a double framed including set of < $(\bar{\alpha},\bar{\lambda})$: G > . It is clear that $DF_A(\bar{\alpha},\bar{\lambda})_{(\gamma,\delta)}$ = i_A ($\bar{\alpha}$; γ) ∩ $e_A(\bar{\lambda}$, δ) .

Example: Let U = { c_1,c_2,c_3,c_4 } be the set of four cars under consideration and E = { e_1 =costly, e_2=beautiful, e_3 = fuel efficient, e_4 = modern technology } be the set of parameters and A = {e_1,e_2,e_3} is subset of E. Then

$$(F,A) = \left\{ \begin{array}{l} F(e_1) = \{ (c_1, 0.3, 0.4), (c_2, 0.3, 0.5), (c_3,0.1,0.2), (c_4, 0.7, 0.6)\} \\ F(e_2) = \{(c_1, 0.2, 0.6), (c_2, 0.1, 0.7), (c_3,0.3,0.7), (c_4, 0.5, 0.6)\} \\ F(e_3) = \{(c_1, 0.1, 0.3), (c_2, 0.3, 0.5), (c_3,0.7,0.2), (c_4, 0.3, 0.7)\} \end{array} \right\}$$

From now on, we will take G, as set of parameters, which is a group unless otherwise specified.

Note: 2.5 Let λ_S = ($\overline{\alpha_S}$, $\overline{\beta_S}$,E) be a double framed fuzzy soft set over U. We will say that λ_S(e)= ($\overline{\alpha_S}$(e) , $\overline{\beta_S}$(e)) is image of parameter e ∈ E.

2.12. Definition [8]: Let λ_A and $\lambda_B \in DFS_E$(U) then,

I. If α_A(e) = ∅ and β_A(e)=U for all e ∈ E , λ_A is said to be a null double-framed fuzzy soft set, denoted by □□ = (∅,U,E).
II. If α_A(e) = U and β_A(e)= □ for all e ∈ E , λ_A is said to be an absolute double-framed fuzzy soft set, denoted by □□ = (U, □,E).
III. λ_A is double-framed fuzzy soft subset of λ_B, denoted by $\lambda_A \subseteq \lambda_B$, if $\alpha_A(e) \subseteq \alpha_B(e)$ and $\beta_A(e) \supseteq \beta_B(e)$for all e ∈ E.
IV. Double – framed fuzzy soft union and intersection of λ_A and λ_B, denoted by ($\alpha_A \cup \alpha_B$) : A∪B → P(U) such that ($\alpha_A \cup \alpha_B$)(e) = $\alpha_A(e) \cup \alpha_B(e)$ and ($\beta_A \cap \beta_B$)(e) = $\beta_A(e) \cap \beta_B(e)$ for all e ∈ E. Also ($\alpha_A \cap \alpha_B$) : A∩B → P(U) such that ($\alpha_A \cap \alpha_B$)(e) = $\alpha_A(e) \cap \alpha_B(e)$ and ($\beta_A \cup \beta_B$)(e) = $\beta_A(e) \cup \beta_B(e)$ for all e ∈ E.
V. (v) Double – framed soft complement of λ_A is denoted by $\lambda_A^{□}$ and defined by $\lambda_A^{□}$: E→ P(U) X P(U) such that $\lambda_A^{□}$(e) = {(e, α_A(e), β_A(e)): e ∈ E }.

2.13 Definition [23]: Let U be a universe and E a set of attributes. Then, (U,E) is the collection of all double-framed fuzzy soft sets on U with attributes from E and is said to be double-framed fuzzy soft class.

2.14 Definition [23]: A double-framed fuzzy soft set (F,A) is said to be a null double-framed fuzzy soft set denoted by empty set Φ, if for all e ε A , F(e) = Φ.

2.15 Definition[23]: A double-framed fuzzy soft set (F,A) is said to be an absolute double-framed fuzzy soft set, if for all e ε A , F(e) = DFFU.

2.16 Definition[23]: The complement of a double-framed fuzzy soft set (F,A) is denoted $(F,A)^c$ and is denoted by $(F,A)^c$ = { (x, 1-μ_A^+(x), 1- μ_A^-(x) ; x ε U}.

3. Double-framed fuzzy soft G-modules and Ideals

3.1 Definition: A double-framed fuzzy soft set A (μ_A, ν_A) of S is called a double-framed fuzzy soft G-modules (DFFSGM) of S provided that for all x,y,z,a,b ε S;

(DFFSGM-1) $\mu_A(ax+by) \geq \min \{ \mu_A(x), \mu_A(y) \}$, $\nu_A(ax+by) \leq \max \{ \nu_A(x), \nu_A(y) \}$,
(DFFSGM-2)) $\mu_A(\alpha x) \geq \mu_A(x)$, , $\nu_A(\alpha x) \leq \nu_A(y)$ }

3.2 Definition: A double-framed fuzzy set 'A' in X is called a double-framed fuzzy soft d-ideal (DFFSDI) of X if it satisfies;
(DFFSDI$_1$) $\mu_A(x) \geq T\{ \mu_A(ax+by), \mu_A(y)\}$

(DFFSDI$_2$) $\nu_A(x) \leq S\{ \mu_A(ax+by), \mu_A(y)\}$
(DFFSDI$_3$) $\mu_A(e) \geq \nu_A(x)$ and $\mu_A(e) \geq \nu_A(x)$ and for all x,y ε X.

3.3 Definition: Let λ and μ be two fuzzy subsets in X. The Cartesian Product of $\lambda\times\mu$: $X \times X \rightarrow [0,1]$ is defined by $\lambda\times\mu(x,y) = T\{\lambda(x), \mu(y)\}$ and $\lambda\times\mu : X \times X \rightarrow [0,1]$ is defined by $\lambda\times\mu\,(x,y) = S\{ \lambda(x), \mu(y)\}$ for all x,y ε X.

3.4 Definition: Let f : X → Y be a mapping of modules and 'μ' be a double-framed fuzzy soft set of y. The map μ^f is the pre image of μ_1 and μ_2 under f. so $\mu_1{}^f(x) = \mu^f(x)$, $\mu_2{}^f(x) = \mu^f(x)$

3.5 Definition: Let 'A' be a double-framed fuzzy soft set in a X, the strongest (ψ, χ)- double-framed fuzzy soft relation on X that is fuzzy relation on A is μ_A given by,

$\mu_A(x,y) \cap \psi = T\{A(x), A(y)\} \vee \chi$

$\nu_A(x,y) \cap \psi = S\{A(x), A(y)\} \vee \chi$ for all x,y ε X.

4. MAIN RESULTS

4.1 Proposition : If ϕ is a (ψ, χ)- double-framed fuzzy group of X, then $\mu_\varphi(e) \cap \psi \geq \mu_\varphi(x) \vee \chi$ and $\mu_\varphi(e) \cap \psi \leq \mu_\varphi(x) \vee \chi$ for all x ε X.

Proof: Let x ε X, then

$\mu_\varphi(e) \cap \psi = \mu_\varphi(x\, x^{-1}) \cap \psi \geq T\{ \mu_\varphi(x), \mu_\varphi(x^{-1})\} \vee \chi \geq T\{ \mu_\varphi(x), \mu_\varphi(x)\} \vee \chi \geq \mu_\varphi(x) \vee \chi$and $\mu_\varphi(e) \cap \psi = \mu_\varphi(x\, x^{-1}) \cap \psi \leq S\{ \mu_\varphi(x), \mu_\varphi(x^{-1})\} \vee \chi \leq S\{ \mu_\varphi(x), \mu_\varphi^-(x)\} \vee \chi \leq \mu_\varphi(x) \vee \chi$

This completes the proof.

4.2. Proposition: Let 'ϕ' be a (ψ, χ)- double-framed fuzzy group of X, then the following assertions are valid.

(i) $(\forall\, \alpha\, \varepsilon\, [0,1]\ (\phi_\alpha \neq \phi \Rightarrow \phi_t$ is a group of X)
(ii) $(\forall\, \beta\, \varepsilon\, [1,0]\ (\phi_\beta \neq \phi \Rightarrow \phi_\beta$ is a group of X)

Proof: Let t ε [0,1] be such that $\phi_t \neq \phi$. If x,y ε ϕ_t, then $\mu_\phi(x) \cap \psi \geq t \vee \chi$ and $\mu_\phi{}^+(y) \cap \psi \geq t \vee \chi$. It follows that $\mu_\phi(xy)\ ) \cap \psi \geq T\{ \mu_\phi(x), \mu_\phi(y)\} \vee \chi \geq t \vee \chi$

4.3 Corollary: If ϕ is a(ψ, χ) - double-framed fuzzy group of X, then the sets $\phi_{\mu\varphi(e)}$ and $\phi\, \mu_\varphi(e)$ are group of X.

Proof: Straight forward.

4.4 Proposition: Let $\phi = (X, \mu_\phi, \mu_\phi)$ be a (ψ, χ)- double-framed fuzzy d-ideal of X. If the inequality xy ≤ z holds in X, then $\mu_\phi(x) \cap \psi \geq T\{ \mu_\phi(y), \mu_\phi(z) \} \vee \chi\ \mu_\phi(x) \cap \psi \leq S\{ \mu_\phi(y), \mu_\phi(z) \} \vee \chi$

Proof: Let x, y, z ε X be such that xy ≤ z, then (xy)z = 0, and so $\mu_\phi(x) \cap \psi \geq T\{ \mu_\phi(xy), \mu_\phi(y)\} \vee \chi \geq T\{T\{ \mu_\phi(xy)z, \mu_\phi(z)\}, \mu_\phi(y) \} \vee \chi = T\{T\{ \mu_\phi(e), \mu_\phi(z)\}, \mu_\phi(y)\} \vee \chi = T\{ \mu_\phi(y), \mu_\phi(z) \} \vee \chi$ and $\mu_\phi(x) \cap \psi \leq S\{ \mu_\phi(xy), \mu_\phi(y)\} \vee \chi \leq S\{S\{ \mu_\phi(xy)z, \mu_\phi(z)\}, \mu_\phi(y) \} \vee \chi$

$= S\{S\{ \mu_\phi(e), \mu_\phi(z)\}, \mu_\phi(y)\} \vee \chi = S\{ \mu_\phi(y), \mu_\phi(z) \} \vee \chi$

This completes the proof.

4.5 Proposition: Let ϕ be a(ψ, χ) - double-framed fuzzy d-ideal of X. If the inequality x≤y holds in X, then $\mu_\phi(x) \cap \psi \geq \mu_\phi(y) \vee \chi$ and $\mu_\phi(x) \cap \psi \leq \mu_\phi(y) \vee \chi$.

Proof: Let x, y ε X be such that x ≤ y, then $\mu_\phi{}^+(x) \cap \psi \geq T\{ \mu_\phi{}^+(xy), \mu_\phi{}^+(y)\} \vee \chi = T\{ \mu_\phi(e), \mu_\phi(y)\} \vee \chi = \mu_\phi(y) \vee \chi$ $\mu_\phi(x) \cap \psi \leq S\{ \mu_\phi(xy), \mu_\phi(y)\} \vee \chi$

$= T\{ \mu_\phi(e), \mu_\phi(y)\} \vee \chi = \mu_\phi(y) \vee \chi$

This completes the proof.

4.6 Proposition: In a group X, every (ψ, χ)- double-framed fuzzy d-ideal of X is (ψ, χ)- double-framed fuzzy group of X.

Proof: Let 'ϕ' be a(ψ, χ) double-framed fuzzy d-ideal of a group X. Since $xy \leq x$ for all $x,y \varepsilon X$, it follows from Proposition 4.5 that $\mu_\phi(xy) \cap \psi \geq T\{\mu_\phi(x)$ and $\mu_\phi(x) \cap \psi \leq \mu_\phi(x) \vee \chi$, so from Proposition 3.1

(DFFSG$_1$) $\mu_\phi(xy) \cap \psi \geq T\{\mu(x) \vee \chi \geq T\{\mu_\phi(xy),\mu_\phi(y)\} \vee \chi = T\{\mu_\phi(x), \mu_\phi(y)\} \vee \chi$ and (DFFSG2) $\mu_\phi(xy) \cap \psi \leq \mu_\phi(x) \vee \chi \leq S\{\mu_\phi(xy), \mu_\phi(y)\} \vee \chi \leq S\{\mu_\phi(x), \mu_\phi(y)\} \vee \chi$

$\mu_\phi(x^{-1}) \cap \psi \geq T\{\mu_\phi(xy), \mu_\phi(x)\} \vee \chi = T\{\mu_\phi(e), \mu_\phi(y)\} \vee \chi \geq \mu_\phi(x) \vee \chi$ $\mu_\phi(x^{-1}) \cap \psi \leq S\{\mu_\phi(xy), \mu_\phi(y)\} \vee \chi \leq S\{\mu_\phi(e), \mu_\phi(y)\} \vee \chi \leq \mu_\phi(x) \vee \chi$. Hence ϕ is (ψ, χ)- double-framed fuzzy soft group. The converse of the theorem is not true in general.

4.7. Proposition: Let 'ϕ' be a (ψ, χ) - double-framed fuzzy soft group of a group X such that Proposition 4.2 holds for all x, y, z ε X satisfying the inequality $xy \varepsilon z$ then ϕ is a (ψ, χ)- double-framed fuzzy d-ideal of X.

Proof: Recall from Proposition 4.1; that $\mu_\varphi(e) \cap \psi \geq \mu_\varphi(x) \vee \chi$ and $\mu_\varphi(e) \cap \psi \leq \mu_\varphi(x) \vee \chi$ for all x ε X. Since $x(xy) \leq y$ for all x, y ε X, it follows that Proposition 4.2,

$\mu_\phi(x) \cap \psi \geq T\{\mu_\phi(xy), \mu_\phi(y)\} \vee \chi$ and
$\mu_\phi(x) \cap \psi \leq S\{\mu_\phi(xy), \mu_\phi(y)\} \vee \chi$

Hence ϕ is a (ψ, χ)- double-framed fuzzy soft d-ideal of X.

4.8. Proposition: Let λ and μ be (ψ, χ)- double-framed fuzzy soft d-ideal of X, then λ×μ is also (ψ, χ)- double-framed fuzzy soft d-ideal of X.

Proof: For any $(x_1, x_2), (y_1, y_2) \varepsilon X \times X$, we have (BFd$_1$) $(\lambda \times \mu)(x_1, x_2) \cap \psi = T\{\lambda(x_1), \mu(x_2)\} \cap \psi$

$\geq T\{T\{\lambda(x_1, y_1), \lambda(y_1)\}, T\{\mu(x_2, y_2), \mu(y_2)\}\} \vee \chi$
$= T\{T\{\lambda(x_1, y_1), \mu(x_2, y_2)\}, T\{\lambda(y_1), \mu(y_2)\}\} \vee \chi$
$= T\{(\lambda \times \mu)((x_1, x_2), (y_1, y_2)\} \vee \chi$
$(\lambda\times\mu)(x_1, x_2) \cap \psi = S\{\lambda(x_1), \mu(x_2)\} \cap \psi$
$\leq S\{S\{\lambda(x_1,y_1), \lambda(y_1)\}, S\{\mu(x_2,y_2), \mu(y_2)\} \vee \chi$
$= S\{S\{\lambda(x_1,y_1), \mu(x_2,y_2)\}, S\{\lambda(y_1), \mu(y_2)\} \vee \chi$
$= S\{(\lambda\times\mu)(x_1, x_2)(y_1, y_2), (\lambda\times\mu)(y_1, y_2)\} \vee \chi$
$(\lambda\times\mu)(x_1^{-1}, x_2^{-1}) \cap \psi$
$= T\{\lambda(x_1^{-1}), \mu(x_2^{-1})\} \cap \psi \geq T\{T\{\lambda(x_1,y_1), \lambda(y_1)\}, T\{\mu(x_2,y_2), \mu(y_2)\} \vee \chi$
$= T\{T\{\lambda(x_1,y_1), \mu(x_2,y_2)\}, T\{\lambda(y_1), \mu(y_2)\} \vee \chi$
$= T\{(\lambda\times\mu)(x_1, x_2)(y_1, y_2), (\lambda\times\mu)(y_1, y_2)\} \vee \chi$
$(\lambda\times\mu)(x_1^{-1}, x_2^{-1}) \cap \psi = S\{\lambda(x_1^{-1}), \lambda(x_2^{-1})\} \cap \psi$
$\leq S\{S\{\lambda(x_1,y_1), \lambda(y_1)\}, S\{\mu(x_2,y_2), \mu(y_2)\} \vee \chi$
$= S\{S\{\lambda(x_1,y_1), \mu(x_2,y_2)\}, S\{\lambda(y_1), \mu(y_2)\} \vee \chi$
$\leq S\{(\lambda\times\mu)(x_1, x_2, y_1, y_2), (\lambda\times\mu)(y_1, y_2)\} \vee \chi$

Hence λ×μ is (ψ, χ)- double-framed fuzzy soft d-ideal of X.

4.9 Proposition: Let f : X → Y be a homomorphism of groups. If 'μ' is a (ψ,χ)- double-framed fuzzy softd-ideal of y, then μ^f is (ψ, χ)- double-framed fuzzy soft d-ideal of X.

Proof: For any x ε X, we have

$\mu^f(x) \cap \psi = \mu(f(x)) \cap \psi \geq \mu(e) \vee \chi = \mu(f(e)) \vee \chi = \mu^f(e) \vee \chi$
$\mu^f(x) \cap \psi = \mu(f(x)) \cap \psi \leq \mu(e) \vee \chi = \mu(f(e)) \vee \chi = \mu^f(e) \vee \chi$
Let x, y ε X

$T\{\mu^f(xy), \mu^f(y)\} \cap \psi = T\{\mu(f(xy), \mu(f(y)\} \cap \psi = T\{\mu(f(x).f(y)), \mu(f(y)\} \cap \psi \leq \mu f(x) \vee \chi = \mu^f(x). \vee \chi$
$S\{\mu^f(xy), \mu^f(y)\} \cap \psi = S\{\mu f(xy), \mu(f(x)\} \cap \psi = S\{\mu(f(x),f(x)), \mu(f(x)\} \cap \psi \geq \mu(f(x) \vee \chi = \mu^f(x) \vee \chi$

Hence μ^f is(ψ,χ)- double-framed fuzzy soft d-ideal of X.

4.10. Proposition: Let f : X → Y be an epimorphism of groups. If μ^f is (ψ, χ)- double-framed fuzzy soft d-ideal of X, then $\mu(\psi, \chi)$ - double-framed fuzzy soft d-ideal of Y.

Proof: Let y ε Y, there exists x ε X such that f(x) = y, then

$\mu(y) \cap \psi = \mu(f(x)) \cap \psi = \mu^f(x) \cap \psi \leq \mu^f(e) \vee \chi = \mu(f(e) \vee \chi = \mu(e) \vee \chi$

$\mu(y) \cap \psi = \mu(f(x)) \cap \psi = \mu^f(x) \cap \psi \geq \mu^f(e) \vee \chi = \mu(f(e) \vee \chi = \mu(e) \vee \chi$
Let x, y ε Y, then there exists a, b ε X, such that f(a) = x and f(b) = y. It follows that
$\mu(x) \cap \psi = \mu(f(a) \cap \psi = \mu^f(a) \vee \chi$ and $\mu(x) \cap \psi = \mu(f(a) \cap \psi = \mu^f(a) \vee \chi$
$\geq T\{ \mu^f(ab), \mu^f(b)\} \vee \chi = T\{ \mu(f(ab), \mu(f(b)\} \vee \chi = T\{ \mu(f(a).f(b)), \mu(f(b)\} \vee \chi$
$= T\{ \mu(xy), \mu(y)\} \vee \chi$

Also

$\leq S\{ \mu^f(ab), \mu^f(b)\} \vee \chi = S\{ \mu(f(ab), \mu(f(b)\} \vee \chi = S\{ \mu(f(a).f(b)), \mu(f(b)\} \vee \chi$
$= S\{ \mu(xy), \mu(y)\} \vee \chi$

Hence μ is a(ψ,χ) - double-framed fuzzy soft d-ideal of y.

4.11 Proposition: Let 'A' be a double-framed fuzzy soft set in a group X and μ_A be the strongest (ψ, χ)- double-framed fuzzy soft relation on X, then A is a (ψ, χ)- double-framed fuzzy soft d-ideal of X if and only if μ_A is a (ψ, χ)- double-framed fuzzy soft d-ideal of X × X.

Proof: Suppose that 'A' is a(ψ, χ) - double-framed fuzzy soft d-ideal of X, then

$\mu_A(e, e) \cap \psi = T\{ A(e), A(e)\} \cap \psi$

$\geq T\{ A^+(x), A^+(y)\} \vee \chi = \mu_A{}^+(x, y) \vee \chi$ for all (x, y) ε X × X.

$\mu_A(e, e) \cap \psi = S\{ A(e), A(e)\} \cap \psi \leq S\{ A(x), A(y)\} \vee \chi = \mu_A(x, y) \vee \chi$ for all (x, y) ε X × X.
For any $x = (x_1, x_2)$ and

$y = (y_1, y_2)$ ε X × X.

$\mu_A(x) \cap \psi = \mu_A(x_1, x_2) \cap \psi$
$= T\{ A(x_1), A(x_2)\} \cap \psi \geq T\{T\{A(x_1,y_1), A(y_1)\}, T\{A(x_2, y_2), A(y_2)\} \} \vee \chi$
$= T\{ T\{A(x_1, y_1), A(x_2, y_2)\}, T\{A(y_1), A(y_2) \} \} \vee \chi$
$= T\{ \mu_A(x_1, y_1), (x_2, y_2)), \mu_A(y_1, y_2)\} \vee \chi = T\{ \mu_A(xy), \mu_A(y)\} \vee \chi$

$\mu_A{}^-(x) \cap \psi = \mu_A{}^-(x_1, x_2) \cap \psi$

$= S\{ A(x_1), A(x_2)\} \cap \psi \leq S\{S\{A(x_1, y_1), A(y_1)\}, S\{A(x_2, y_2), A(y_2)\} \} \vee \chi$
$= S\{S\{A(x_1, y_1), A(x_2, y_2)\}, S\{A(y_1), A(y_2) \} \} \vee \chi$
$= S\{ \mu_A(x_1, y_1), (x_2, y_2)), \mu_A(y_1, y_2)\} \vee \chi = S\{ \mu_A(xy), \mu_A(y)\} \vee \chi$

Hence μ_A is a (ψ, χ)- double-framed fuzzy soft d-ideal of X × X. Conversely, suppose that μ_A is a (ψ, χ)- double-framed r fuzzy soft d-ideal of X × X. Then,

$T\{A^+(e), A^+(e)\} \cap \psi = \mu_A{}^+(e, e) \cap \psi$
$\geq \mu_A(x, y) \vee \chi = T\{A(x), A(y)\} \vee \chi\ \forall(x, y)$ ε X × X.
$S\{ A(e), A(e)\} \cap \psi = \mu_A(e, e) \cap \leq \mu_A(x, y) \vee \chi = S\{ A(x), A(y)\} \vee \chi$
for any $x = (x_1, y_1)$ and

$y = (y_1, y_2)$ ε X × X., we have

$T\{A(x_1), A(x_2)\} \cap \psi = \mu_A(x_1, x_2) \cap \geq T\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\} \vee \chi$
$= T\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} \vee \chi = T\{ T\{A(x_1, y_1), A(x_2, y_2)\}, T\{A(y_1), A(y_2)\} \vee \chi$
$= T\{ T\{ A(x_1, y_1), A(y_1), T\{A(x_2, y_2), A(y_2)\} \vee \chi$

Putting $x_1 = x_2 = 0$, we have

$\mu_A(x_1) \cap \psi \geq T\{\mu_A(x_1, y_1), \mu_A(y_1)\} \vee \chi$
Likewise, $\mu_A(x_1y_1) \geq T\{\mu_A(x_1), \mu_A(x_2)\}$
$S\{A(x_1), A(x_2)\} \cap \psi = \mu_A(x_1, x_2) \vee \chi \leq S\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\} \vee \chi$
$= S\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} \vee \chi = S\{ S\{A(x_1, y_1), A(x_2, y_2)\}, S\{A(y_1), A(y_2)\} \vee \chi$
$= S\{ S\{ A(x_1, y_1), A(y_1), S\{A(x_2, y_2), A(y_2)\} \vee \chi$
Putting $x_1 = x_2 = 0$, we have
$\mu_A(x_1) \cap \psi \leq S\{\mu_A(x_1, y_1), \mu_A(y_1)\} \vee \chi$
Likewise, $\mu_A(x_1y_1) \cap \psi \leq S\{\mu_A(x_1), \mu_A(x_2)\} \vee \chi$.

Hence A is a (ψ, χ)- double-framed fuzzy soft d-ideal of X.

4.12 Proposition: Let ϕ be a double-framed fuzzy soft set in X, then ϕ is a(ψ, χ) - double-framed fuzzy soft d-ideal of X if and only if it satisfies the following assertations.
($\forall$ α ε [0,1] ($\phi_t \neq \phi \Rightarrow \phi_t$ is an ideal of X)
($\forall$ β ε [1,0] ($\phi_s \neq \phi \Rightarrow \phi_\beta$ is an ideal of X)

Proof: Assume that ϕ is a(ψ, χ)- double-framed fuzzy soft d-ideal of X. Let (s,t) ε [1, 0] ε [0,1] be such that $\phi_t \neq \phi$ and $\phi_s^- \neq \phi$. Obviously, e ε $\phi_t^+ \cap \phi_s^-$.

Let x, y ε X be such that xy ε ϕ_t and y ε ϕ_t , and
Let a, b ε X be such that ab$\varepsilon\phi_s$ and b ε ϕ_s , then

$\mu_\phi(xy) \cap \psi \geq t \vee \chi$, $\mu_\phi(y) \cap \psi \geq t \vee \chi$, $\mu_\phi(ab) \cap \psi \leq s \vee \chi$ and $\mu_\phi(b) \cap \psi \leq s \vee \chi$.
It follows from Proposition 3.1

$\mu_\phi(x) \cap \psi \geq T \{ \mu_\phi(xy), \mu_\phi(y)\} \geq t \vee \chi$ and

$\mu_\phi(a) \cap \psi \leq S \{ \mu_\phi(ab), \mu_\phi(b) \leq s \vee \chi$ so that x ε ϕ_t^+ and a ε ϕ_s^-. Therefore ϕ_t^+ and ϕ_s^- are ideals of X.

Conversely, suppose that the condition (corollary) is valid. For any x ε X, let $\mu_\varphi(x) \cap \psi = t \vee \chi$ and $\mu_\varphi(x) \cap \psi = s \vee \chi$, then x ε $\phi_t \cap \phi_s$, and so ϕ_t and ϕ_s are non-empty. Since ϕ_t and ϕ_s are ideal of X, e ε $\phi_t \cap \phi_s^-$. Hence $\mu_\phi(e) \cap \psi \geq t \vee \chi = \mu_\phi(x) \vee \chi$ and $\mu_\phi(e) \cap \psi \leq s \vee \chi = \mu_\phi(x) \vee \chi$ for all x ε X.

If there exists x^1, y^1, a^1, b^1 ε X such that $\mu_\phi(x^1) \cap \psi \leq T\{ \mu_\phi(x^1y^1), \mu_\phi(y^1)\} \vee \chi$

and $\mu_\phi(a^1) \cap \psi \geq S\{ \mu_\phi(a^1b^1), \mu_\phi(b^1)\} \vee \chi$ then by taking

$t_0 = ½ \{ \mu_\phi(x^1) + T\{ \mu_\phi(x^1y^1), \mu_\phi(y^1)\}$
$S_0 = ½ \{ \mu_\phi(a^1) + S\{ \mu_\phi(a^1b^1), \mu_\phi(b^1)\}$

We have,

$\mu_\phi(x^1) \cap \psi < t_0 \leq T\{ \mu_\phi(x^1y^1), \mu_\phi(y^1)\} \vee \chi$
$\mu_\phi(a^1) \cap \psi < s_0 \leq S\{\mu_\phi(a^1b^1), \mu_\phi(b^1)\} \vee \chi$

Hence $x^1 \notin \phi_{t0}$, x^1, y^1 ε ϕ_{t0}, y^1 ε ϕ_{t0}, $a^1 \notin \phi_{s0}$ and b^1 ε ϕ_{s0}. This is a contradiction and thus ϕ is a(ψ, χ)- double-framed fuzzy soft d-ideal of X.

Conclusion

This paper is devoted to discussion of combination of soft set theory, set theory and G-module theory. Based on the definition, we have introduced the concepts of double-framed soft G-modules and double-framed soft d-ideals with illustrative examples. Also we analyse strongest double-framed fuzzy relations on double-framed fuzzy soft d-ideals of a module and discuss some related properties.

REFERENCES

[1] Acar U., Koyuncu F., Tanay B., Soft sets and soft rings, *Comput. Math. Appl.,* 59(2010), 3458-3463.
[2] Aktas. H., C. agman N., Soft sets and soft groups, *Inform. Sci.,* 177(2007), 2726-2735.
[3] Ali M.I.,Feng F., Liu X., Min W.K., Shabir M., On some new operations in soft set theory, *Comput. Math. Appl.,* 57(2009), 1547-1553.
[4] Atagun A.O., Sezgin A., Soft substructures of rings, fields and modules, *Comput. Math. Appl.,* 61(3)(2011), 592-601.
[5] Babitha K.V., Sunil J.J., Soft set relations and functions, *Comput. Math. Appl.,* 60(7)(2010), 1840-1849.
[6] Cagman N., Enginoglu S.,Soft matrix theory and its decision making, *Comput. Math. Appl.,* 59(2010), 3308-3314.
[7] Cagman N, Engino ˘ glu S., Soft set theory and uni-int decision making, *Eur. J. Oper. Res.,* 207(2010), 848-855.
[8] Cagman N., C¸ ıtak F., Aktas¸ H., Soft int-groups and its applications to group theory, *Neural Comput. Appl.,* DOI: 10.1007/s00521-011-0752-x.
[9] Cagman N., Sezgin A., Atagun A.O., Soft uni-groups and its applications to group theory, (submitted).
[10] Cagman N., Sezgin A., Atagun A.O., α-inclusions and their applications to group theory, (submitted).
[11] Feng F., Jun Y.B., Zhao X., Soft semirings, *Comput. Math. Appl.*, 56(2008), 2621–2628.
[12] Feng F., Liu X.Y., Leoreanu-Fotea V., Jun Y.B., Soft sets and soft rough sets, *Inform. Sci.,* 181(6)(2011), 1125-1137.
[13] Feng F., Li C., Davvaz B., Ali M.I., Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Comput.,* 14(6)(2010), 899-911.

[14] A.R.Hadipour, Double framed soft BF-algebras- *Indian Journal of Science and Technology*, Vol.7, No.4(2014), 491-496
[15] Y.B.Jun.et.al, Ideal theory of BCK/BCI-algebras based on double-framed soft sets, *Applied Mathematics informatics science,* Vol.7, No.5(2013), (1879-1888).
[16]Y.D.Jun and S.S.Ahn, Double-framed soft sets with applications in BCK/BCI-algebras, *Journal of Applied Mathematics,* Volume:2012, Article ID 178159, 15 Pages.
[17] Jun Y.B., Lee K.J., Zhan J., Soft p-ideals of soft BCI-algebras, *Comput. Math. Appl.,* 58(2009), 2060-2068.
[18] Jun Y.B., Lee K.J., Park C.H., Soft set theory applied to ideals in d-algebras, Comput. Math. Appl., 57(3)(2009), 367-378.
[19] Kazancı O., Yılmaz S¸., Yamak S., Soft sets and soft BCH-algebras, *Hacet. J. Math. Stat.,* 39(2)(2010), 205-217.
[20] Khizar Hayat et.al, Applications of double-framed soft ideals in BE-algebra, *New trends in Mathematical Sciences,*Vol.4(2), 2016, 285-295.
[21] Maji P.K., Biswas R., Roy A.R., Soft set theory, *Comput. Math. Appl.,* 45(2003), 555-562. [20] Maji P.K., Roy A.R., Biswas R., An application of soft sets in a decision making problem, *Comput. Math. Appl.,* 44(2002), 1077-1083.
[22] Majumdar P., Samanta S.K., On soft mappings, *Comput. Math. Appl.,* 60 (9)(2010), 2666-2672.
[23] Molodtsov D., Soft set theory-first results, *Comput. Math. Appl.,* 37(1999), 19-31.
[24] Molodtsov D.A., Leonov V.Yu., Kovkov D.V., Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya, 1(1)(2006), 8-39.
[25] Pilz G., Near-rings, North Holland Publishing Company, Amsterdam-New York-Oxford, 1983.
[26] Sezgin A., Atagun A.O., Aygun E., A note on soft near-rings and idealistic soft near-rings, Filomat., 25(1)(2011), 53-68.
[27] Sezgin A., Atagun A.O., On operations of soft sets, *Comput. Math. Appl.,* 61(5)(2011), 1457-1467.
[28] Zhan J., Jun Y.B., Soft BL-algebras based on fuzzy sets, *Comput. Math. Appl.,* 59(6)(2010), 2037-2046.
[29] Zou Y., Xiao Z., Data analysis approaches of soft sets under incomplete information, *Knowl-Based Syst.,* 21(2008), 941-945.

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