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RESEARCH ARTICLE

EFFECT OF COUPLE STRESS IN PERISTALTIC TRANSPORT OF BLOOD FLOW BY HOMOTOPY ANALYSIS METHOD

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ABSTRACT

The present study deals towards investigating the physiological flow of blood in micro-circulatory system. The effects of various parameters on velocity, pressure rise and frictional force of blood have been computed when the Reynolds number is small and wavelength is large by using Homotopy Analysis Method. The computational results are presented in graphical form. The results are found to be good with those of Shapiro et.al [21] that carried out without consideration of couple stress effect.

INTRODUCTION

The phenomena of peristaltic are of great importance in many engineering and biological system including the study of humans. It is relevant for the study of the ureter, the oviducts and the intestine among others. The word peristalsis itself comes from a Greek word 'Peristaltikes' that means clasping and compressing. It is thus useful for the description of the progressive wave contraction along a channel or tube, the cross-section area of which varies consequently. Physiologically one can say that a peristaltic action in a neuromuscular property of any smooth muscle structure. This phenomenon is used by the body to propel the tube like contents, for example in the ureters, the gastrointestinal tracts and other glandular ducts. The peristaltic pumping is also used in roller and finger pumps. Such devices are used to pump blood, slurries, corrosive fluids and foods when the desire is to prevent the transported fluid from coming into contract with the mechanical parts of the pumps. Through compression mechanism the tube is occluded completely and the positive displacement pumps the fluid through the tube. On other hand, the various forces can be used for effective pumping even if the lumen of the tube is not occluded. In such situation the flow rate depends upon pressure head. Jaffrin and Shapiro [1] give the detailed discussion on the basic mechanism of peristaltic pumping. The study of peristalsis is not only motivated due to the physiological.

Regardless of physiological application, some authors made useful contribution to the peristaltic pumping. It also plays a vital role in the application of particle fluid mixture theory e.g. in the particulate suspension theory of blood [Bedford and Drumheller [2], Hill and Bedford [3], Marble [4], Oka [5], Soo [6], Srivastava and Srivastava [7] and Trowbridge [8]]. Most of the analytical studies use of perturbation series in a small parameter such as the Reynolds number or a dimensionless wave number. This, unfortunately, limits the range of validity of the results. However, a perturbation method does provide explicit information about the physical effects of that parameter. Agrawal and Anwaruddin [9] studies the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation. Mekheimer [10] studied peristaltic flow of blood under effect of magnetic field in non-uniform channels. He observed thet pressure rise for a couple stress fluids (as a blood model) is greater than for a Newtonian fluid and is smaller for a magneto hydrodynamic fluid than for a fluid without an effect of a magnetic field. Hayat et al. [11] have investigated peristaltic transport of a third order fluid under the effect of a magnetic field. Hayat et al. [12] have studied the peristaltic flow of conducting fourth grade fluid in a planer channel. Hayat et al. [13] have investigated the effects on an endoscope and magnetic field on the peristaltic flow of a Jeffrey fluid.

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The study of a couple stress fluid is very useful in understanding various physical problems, because it possesses the mechanism to describe rheologically complex fluids such as liquid crystals, liquids containing long-chain molecules as polymeric suspensions, animal and human blood and lubrication. Experimental studies on blood flow [14,15] indicate that under a certain flow conditions blood flow may deviate markedly from Newtonian flow behaviour. Valanis and Sun [16] studied the Poiseuille flow of a fluid with couple stress with application to blood flow, by taking into account the deformable nature of suspended particles (red cells). Popel et al. [17] have also used couple stress theory to explain observed rheological anomalies of blood including the Fohraeus-Lindqvist and Segre-Silberberg effects. Moreover, the couple stress fluid as a true representation for blood flow has been investigated by many authors [18,19]. These theoretical studies of blood flow indicate that some of the non-Newtonian flow properties of blood may be explained by assuming the blood to be a fluid with couple stress. Being motivated by the observations reported in the above mentioned studies, we have undertaken here a study that concerns peristaltic flow of blood. With an aim to take account of the particle size effect, blood has been modelled as a couple-stress fluid. The problem has been analysed by using lubrication theory [20]. The results of the present study will serve as a reasonably good estimate of various fluid mechanical parameters for peristaltic transport of blood in small blood vessels in a pathological state. Since flow behaviour in an axi-symmetric vessel resembles that in a channel, we have studied here two-dimensional channel flow of the fluid. The results of this study will be applicable to blood vessels in the micro-circulatory system without any restriction.

Mathematical Modelling and Analysis

Let us consider the peristaltic motion of blood on a porous channel. Blood is treated as a viscous couple stress fluid (non-Newtonian). We take (X, Y) as Cartesian coordinates of a point, X being measured in the direction of wave propagation and Y in the direction normal to the mean position of the corresponding small blood vessel. In the analysis that follows we consider channel flow. The constitutive equations and equations of motion for couple stress fluid flow in the absence of body moment and body couple can be put as [19,18,14,15]

$$\tau_{ji,j} = \rho \frac{dv_i}{dt} \qquad \text{.......................................(1)}$$

$$e_{ijk} T_{jk}^{A} + M_{ji,j} = 0 \qquad \text{.......................................(2)}$$

$$\tau_{ij} = -P\delta_{ij} + 2\mu\, d_{ij} \qquad \text{.......................................(3)}$$

$$\mu_{ij} = 4\eta\omega_{j,i} + 4\eta'\omega_{i,j} \qquad \text{.......................................(4)}$$

in which ρ is the fluid density, τ_{ij} and T_{ij}^{A} designate the symmetrical and antisymmetrical parts of the stress tensor T_{ij}, v_i the velocity vector, M_{ij} the couple stress tensor μ_{ij} designate the deviatoric part of M_{ij}, ω_i stands for the vorticity vector, d_{ij} is the symmetric part of the velocity gradient, η and η' are constants associated with the couple stress effect, P the pressure. The geometry of the wall surface (cf. Fig. 1) is defined as

$$\bar{h}(\bar{x},\bar{t}) = a(\bar{x}) + b \sin\frac{2\pi}{\lambda}(\bar{x} - c\bar{t}) \qquad \text{.......................................(5)}$$

with

$$a(\bar{x}) = a_0 + k\bar{x}, \qquad \text{.......................................(6)}$$

Thus the equations that govern the flow of the couple stress fluid

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad \text{.......................................(7)}$$

$$\rho\left\{\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right\} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu\nabla^2\bar{u} - \eta\nabla^4\bar{u}, \qquad \text{.......................................(8)}$$

and

$$\rho\left\{\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right\} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu\nabla^2\bar{v} - \eta\nabla^4\bar{v}, \qquad \text{.......................................(9)}$$

Where

$\nabla^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}$, $\nabla^4 = \nabla^2\nabla^2$, and are the velocity components in the corresponding coordinates. Since it is presumed that the couple stress is caused by the presence of the suspending particles, obviously the clear fluid cannot support couple stress at the boundary, hence we have tacitly assumed that, the components of the couple stress tensor at the wall vanish. Hence the boundary conditions are:

$$\frac{\partial \bar{u}}{\partial \bar{y}} = 0,\ \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} = 0, \qquad \text{at } \bar{y} = 0, \qquad \text{……………………….(10a)}$$

$$\left.\begin{array}{c} \bar{u} = 0, \qquad \bar{v} = c\frac{\partial \bar{h}}{\partial \bar{x}}, \\ -\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{x}}\right)\frac{\partial \bar{h}}{\partial \bar{x}} + \frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} - \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = 0 \end{array}\right\} \text{at } \bar{y} = \bar{h}(\bar{x}, \bar{t}) \qquad \text{………………………..(10b)}$$

Introducing the non-dimensional terms to equations (7 -10)

$$x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{a_0},\ u = \frac{\bar{u}}{c},\ v = \frac{\lambda \bar{v}}{a_0 c}, p = \frac{a_0^2}{\lambda \mu c}\bar{p}(\bar{x}), t = \frac{c\bar{t}}{\lambda}, \qquad \text{………………………(11)}$$

$$Re = \frac{\rho c a_0}{\mu}, \qquad \delta = \frac{a_0}{\lambda},\ h = \frac{\bar{h}}{a_0} = 1 + \frac{\lambda k x}{a_0} + \Phi sin2\pi(x - t).$$

Where $\Phi(amplituderatio) = {}^{b}/_{a_0} < 1$

Equations of motion and boundary conditions in the dimensionless form become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad \text{…………………………(12)}$$

$$Re\delta\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right\} = -\frac{\partial p}{\partial x} + \nabla^2 u - \frac{1}{\gamma^2}\nabla^4 u, \qquad \text{……………..…………(13)}$$

and

$$Re\delta^3\left\{\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right\} = -\frac{\partial p}{\partial y} + \delta^2\nabla^2 v - \frac{\delta^2}{\gamma^2}\nabla^4 v, \qquad \text{……………………..…….(14)}$$

Where $\gamma = \sqrt{\mu/\mu}\, a_0$ the couple-stress fluid parameter ($\sqrt{\eta/\mu}$ has the dimensions of length and can be identified with a property which depends on the size of the molecules).

The boundary conditions are;

$$\frac{\partial u}{\partial y} = 0,\ \frac{\partial^3 u}{\partial y^3} = 0, \text{ at } y = 0, \qquad \text{…………..……………..(15a)}$$

$$\left.\begin{array}{c} u = 0,\ v = \frac{\partial h}{\partial x}, \\ -\left(\delta^4\frac{\partial^2 v}{\partial x^2} - \delta^2\frac{\partial^2 u}{\partial y\partial x}\right)\frac{\partial h}{\partial x} + \delta\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2} = 0 \end{array}\right\} \text{at } y = h(x,t) \qquad \text{…………………….………(15b)}$$

Using the long wavelength approximation and neglecting the inertia term δ, it follows from Eqs (12)-(15) that the appropriate equations describing the flow in the laboratory frame of reference are

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} - \frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}, \qquad \text{…………………………..(16)}$$

$$\frac{\partial p}{\partial y} = 0, \qquad \text{……………….…………..(17)}$$

with dimensionless boundary conditions

$$\frac{\partial u}{\partial y} = 0,\ \ \frac{\partial^3 u}{\partial y^3} = 0, \quad \text{at } y = 0, \qquad \text{……………..……………(18)}$$

$$u = 0\ ,\ \ \frac{\partial^2 u}{\partial y^2} = 0\ , \quad \text{at} \quad y = \text{at } y = h(x,t) = 1 + \frac{\lambda k x}{a_0} + \Phi sin2\pi(x - t)$$

To solve equation (16) by means of the Homotopy Analysis Method.

We define the non-linear operator as

$$N[\Phi(x, y; q)] = \frac{1}{\gamma^2}\frac{\partial^4 \Phi(x,y;q)}{\partial y^4} - \frac{\partial^2 \Phi(x,y;q)}{\partial y^2} + \frac{\partial p}{\partial x} \qquad \text{……………….………….(19)}$$

Solve the above equation under the boundary conditions. then we get the series solution:

$$u(x,y) = u_0(x,y) + \sum_{m=1}^{\infty} u_m(x,y) \quad \text{....(20)}$$

We choose the initial guesses for the Homotopic solutions as:

$$u_0(x,y) = \frac{\partial p}{\partial x}\left\{\gamma^2\left(\frac{5h^4}{24} + \frac{h^2y^2}{4} - \frac{y^4}{24}\right)\right\} \quad \text{....(21)}$$

Using Homotopy Analysis Method, were cursively obtain

$$u_1(x,y) = \frac{\partial p}{\partial x}\left\{\gamma^2\left(\frac{y^6}{720} - \frac{h^2y^4}{48}\right)\right\} \quad \text{....(22)}$$

$$u_2(x,y) = \frac{\partial p}{\partial x}\left\{\frac{y^4}{48}(2-\gamma^2h^2-h^2) + \frac{y^6}{1440}(2+2\gamma^2-h^2) - \frac{y^8}{40320}(\gamma^2)\right\} \quad \text{....(23)}$$

$$u_3(x,y) = \frac{\partial p}{\partial x}\left\{\frac{y^4}{48\gamma^2}(2\gamma^2-\gamma^4h^2-2\gamma^2h^2+2-h^2) + \frac{y^6}{1440}(4\gamma^2+\gamma^2h^2+2-h^2) - \frac{y^8}{40320}(3\gamma^2+1-h^2) - \frac{y^{10}}{36\ 28800}(\gamma^2)\right\} \quad \text{....(24)}$$

We obtain the solution as follows

$$u(x,y,t) = \frac{\partial p}{\partial x}\left\{\frac{5\gamma^2h^4}{24} + \frac{y^2}{4}(h^2) + \frac{y^4}{48\gamma^2}(2\gamma^2-3\gamma^4h^2-3\gamma^2h^2+2-h^2) + \frac{y^6}{1440}(6\gamma^2+\gamma^2h^2+4-2h^2) - \frac{y^8}{40320}(4\gamma^2+1-h^2) - \frac{y^{10}}{36\ 28800}(\gamma^2)\right\} \quad \text{....(25)}$$

The instantaneous volume rate is given by

$$Q(x,t) = \int_0^h u\,dy = \frac{\partial p}{\partial x}\left\{\frac{h^5}{120\gamma^2}(25\gamma^4+11\gamma^2+1) - \frac{h^7}{5040\gamma^2}(60\gamma^4+61\gamma^2+21) - \frac{h^9}{36\ 2880}(-40\gamma^2+73) - \frac{h^{11}}{39916\ 800}(\gamma^2+110)\right\} \quad \text{....(26)}$$

Or

$$\frac{dp}{dx} = \frac{Q(x,t)}{\left\{\frac{h^5}{120\gamma^2}(25\gamma^4+11\gamma^2+1) - \frac{h^7}{5040\gamma^2}(6\ 0\gamma^4+6\ 1\gamma^2+21) - \frac{h^9}{362880}(-40\gamma^2+73) - \frac{h^{11}}{39916800}(\gamma^2+110)\right\}} \quad \text{....(27)}$$

The pressure rise $\Delta p_L(t)$ and the friction force $F_L(t)$ (at the wall) in the channel of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t) = \int_0^{\frac{L}{\lambda}}\left(\frac{dp}{dx}\right)dx, \quad \text{....(28)}$$

$$\Delta F_L(t) = \int_0^{\frac{L}{\lambda}} h\left(\frac{-dp}{dx}\right)dx, \quad \text{....(29)}$$

Use of Eqs(18) and Eq.(26) in Eqs (27) and (28) yields

$$\Delta p_L(t) = \int_0^{\frac{L}{\lambda}} \frac{Q(x,t)}{\left\{\frac{h^5}{120\gamma^2}(25\gamma^4+11\gamma^2+1) - \frac{h^7}{5040\gamma^2}(6\ 0\gamma^4+6\ 1\gamma^2+21) - \frac{h^9}{362880}(-40\gamma^2+73) - \frac{h^{11}}{39916800}(\gamma^2+110)\right\}}dx \quad \text{....(30)}$$

$$\Delta F_L(t) = \int_0^{\frac{L}{\lambda}} \frac{-hQ(x,t)}{\left\{\frac{h^5}{120\gamma^2}(25\gamma^4+11\gamma^2+1) - \frac{h^7}{5040\gamma^2}(6\ 0\gamma^4+6\ 1\gamma^2+21) - \frac{h^9}{362880}(-40\gamma^2+73) - \frac{h^{11}}{39916800}(\gamma^2+110)\right\}}dx, \quad \text{....(31)}$$

NUMERICAL RESULTS AND DISCUSSION

In order to discuss the results obtained above quantitatively, we assume the form of instantaneous volume flow rate $Q(x,t)$, periodic in $(x-t)$ as previously done [11,23]:

$$Q(x,t) = \overline{Q} + \Phi sin2\pi(x-t), \quad \text{....(32)}$$

where $\overline{Q}$ is the time-average of the flow over one period of the wave. This form of $Q(x,t)$ has been assumed in view of the fact that the constant value of $Q(x,t)$ gives Δp_L, always negative, and hence there will be no pumping action. Using this form of $Q(y,t)$, we now compute the dimensionless pressure rise $\Delta p_L$and friction force F_L over channel length for various values of the dimensionless flow average $\overline{Q}$, amplitude ratioΦ, and couple stress parameter γ. The average rise in pressure $\Delta\overline{p}_L$, and friction $\overline{F_L}$ are then evaluated by averaging $\Delta p_L(t)$ and $F_L(t)$ over one period of wave. Using the following values of the parameters[24]:

$a_0 = 0.01$ cm, $L = \lambda = 10$ cm, $\quad k = \frac{0.5a_0}{L} = 0.0005,$

The integral in Eqs (30) and (31) are numerically evaluated by using the software MATHEMATICA. It is noted that the theory of long wavelength in the present investigation remains applicable here as the radius of the channel at the inlet $1a_0$ = 0.01 cm is small compared to the wavelength λ= 10 cm. This means that $\delta = a_0/\lambda << 1$. Figures show that the result is computed and the couple stress parameter perfectly matches with the result reported by Shapiro et.al [1]. The average pressure rise versus flow rate $\overline{Q}$ plotted in the fig (1)$\Phi = 0.4, \gamma = 1, 2$, respectively. As expected the average pressure rise decreases as the flow rate increases and it achieves its maximum value at zero flow rate. Furthermore, the average pressure rise decreases as the couple stress parameter increases. From the same figure, we further derive the information that for a Newtonian fluid the magnitude of the pressure rise is less than that for the couple stress fluid both in pumping and co-pumping regions.

Friction force is plotted versus time in figures (2.1to 2.3) for $\overline{Q}$=0, $\overline{Q}$= 0.25, $\overline{Q}$=0.5 respectively. it is clear that the friction force has opposite behaviour as compared with the pressure rise. Figure (3) represent the variation in the pressure rise versus time at $\overline{Q} = 0.0, 0.25, 0.5$ $\Phi = 0.4$ and at , $\gamma = 1, 2, 3$.It is noted that the pressure rise decreases as the couple stress parameter increases. In other words, the pressure rise decreases as the size of the suspended particle decreases. The idea of the distribution of axial velocity can be shown in figure (4) for values of couple stress and amplitude ratio. Since velocity and height of channel change with time, the velocity has been studied at the particular time t. The influence of change of geometry and the effect of the couple stress parameter γ on the velocity is shown in figure (5). It is clear that the velocity increases as γ increases.

In figure (6) the maximum pressure rise which is obtained by putting $\overline{Q} = 0$ is plotted versus amplitude ratio Φ for $\gamma = 1, 2, 3$. It is shown that the maximum pressure rise increases as the amplitude ratio increases. Further we noticed that pressure rise decreases as the γ increases. Figure (7) shows the pressure rise versus time for different values of amplitude ratio Φ.It is clear from the figure that as amplitude ratio increases pressure rise increases for $\overline{Q}$=0.Figure(8) is obtained by plotting the frictional force versus flow rate for different amplitude ratio Φ. From figure it is clear that frictional force increases as the flow rate increases.

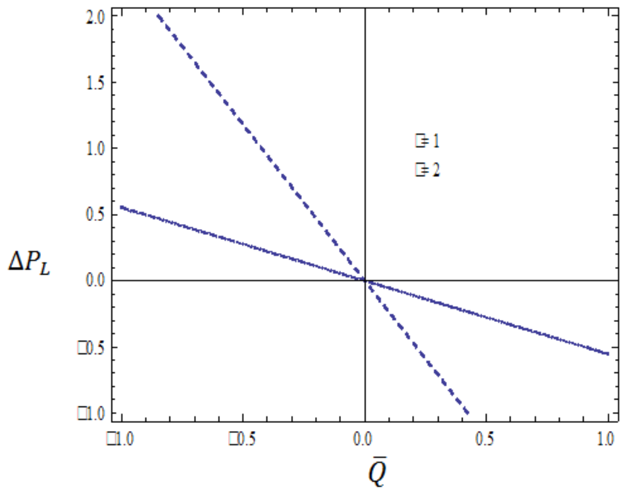

Fig. 1. Pressure versus flow rate for different value of γ, $\varphi = 0.4$,

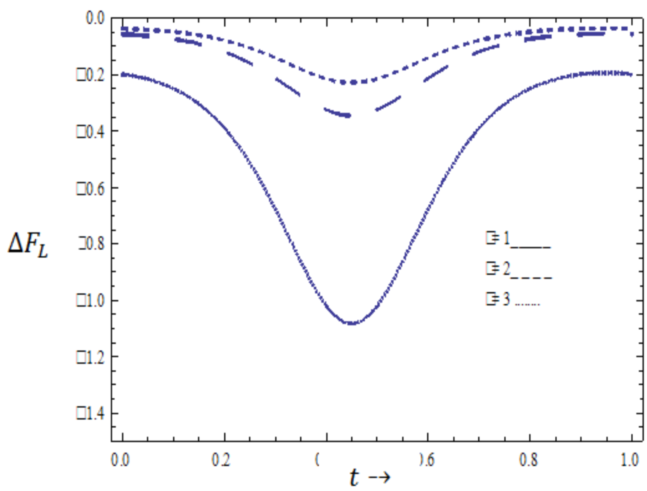

Fig. 2.1. Frictional force versus time for different values of γ when $\overline{Q}$=0 , $\varphi = 0.4$,

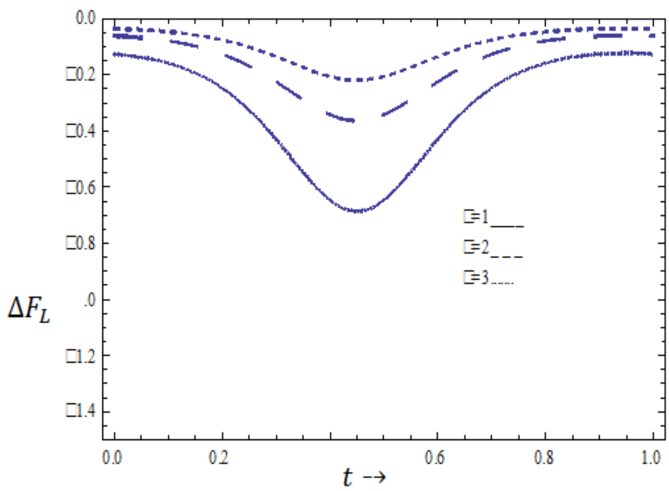

Fig. 2.2. Frictional force versus time for different values of γ when $\overline{Q}$=0.25 , $\varphi = 0.4$

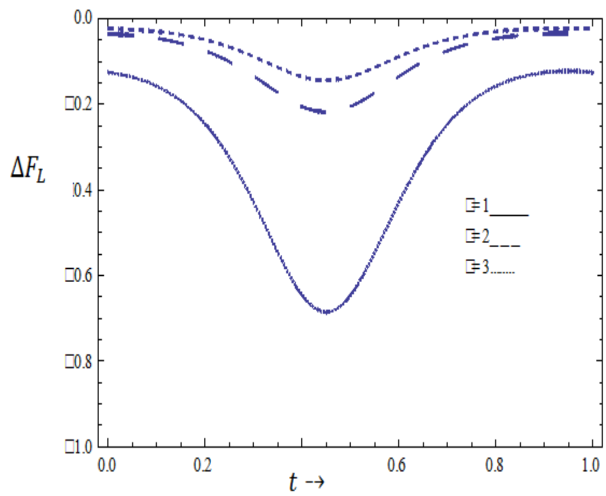

Fig. 2.3. Frictional force versus time for different values of γ when $\overline{Q}$=0.25 , $\varphi = 0.4$

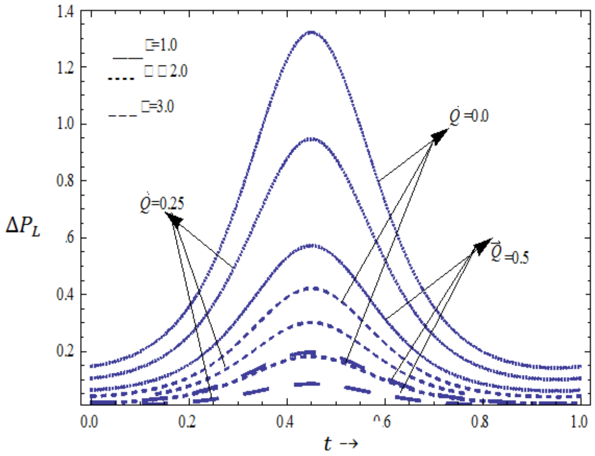

Fig. 3. Pressure versus time t for different value of $\gamma$and flow rate $\overline{Q}$ $\varphi = 0.4$,

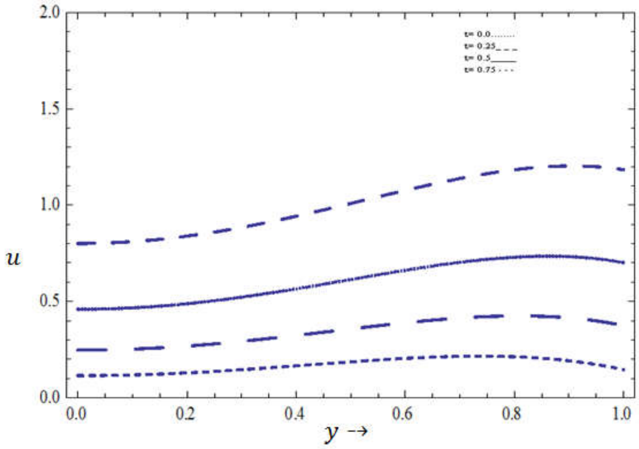

Fig. 4. Velocity versus y at different value of t at $\gamma = 1.0, \varphi = 0.4$,

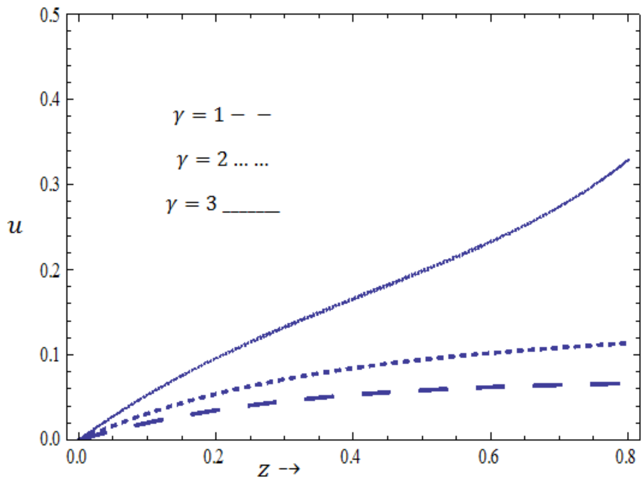

Fig. 5. Velocity versus z different values of $\gamma, \varphi = 0.4$,

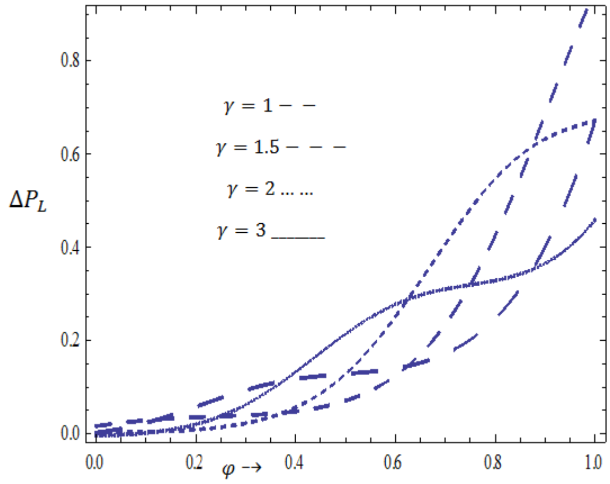

Fig. 6. Pressure rise versus amplitude ratio φ for different value of γ

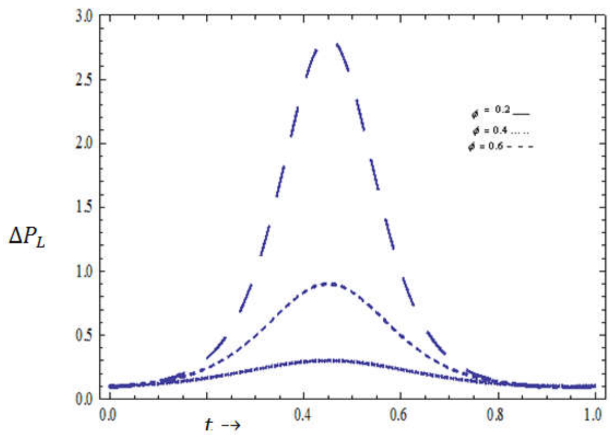

Fig. 7. Pressure rise versus t for different value of amplitude ratio φ at $\gamma = 1.0$

F [redacted]

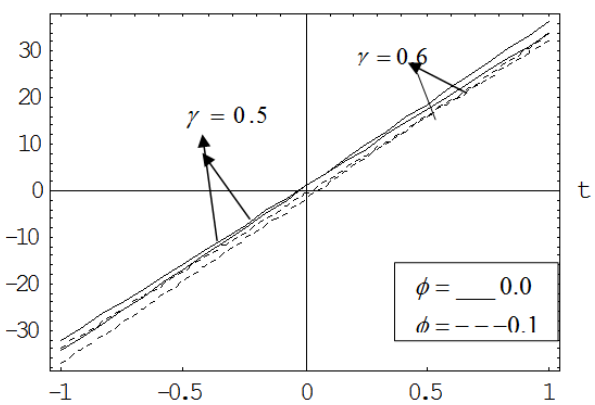

Fig. 8. The Friction force on the versus flow rat e for $\gamma = 0.5, \gamma = 0.6$

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