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RESEARCH ARTICLE

# MATRIX REPRESENTATION OF DOUBLE LAYERED COMPLETE FUZZY GRAPH

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## ABSTRACT

In this paper, we have defined a new matrix representation using edge membership values of the complete fuzzy graph. Uncertainties in a problem are represented as fuzzy matrices using fuzzy principles. Recent days fuzzy matrices have become very famous. In this paper unlike the usual matrix representation of a fuzzy graph with respect to vertices, a new matrix representation with edge membership values as rows and columns is introduced. The relationship between the double layered complete fuzzy graph and the given fuzzy graph whose crisp graph is a complete are analyzed.



## 1. INTRODUCTION

The concept of fuzzy set theory was introduced by Zadeh in 1965. Fuzzy graph theory was introduced by Rosenfeld in 1975 (Rosenfeld *et al*., 1975). It is well known that matrices play a major role in various field such as mathematics, physics, statistics, engineering etc. Matrices with entries and matrix operation defined by fuzzy operations are fuzzy matrices. Fuzzy matrices play a fundamental role in fuzzy set theory. They provide us with a logical framework within which many problems of practical applications can be formulated. Fuzzy matrices can be used when fuzzy uncertainty occurs in a problem. Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory (Overhinnikov, 1981). Fuzzy matrices were introduced by Thomson (Thomson, 1977). Two new operations in fuzzy graphs were introduced by Shayamal and Pal (2004). The determinant and adjoint of a square fuzzy matrix are introduced by Ragab and Emam (1995). Pathinathan and Jesintha Rosline had defined the double layered fuzzy graph (Pathinathan and Jesintha Rosline, 2014). In this paper, the relationship between the matrix representations of double layered complete fuzzy graph using vertices and given fuzzy graph whose crisp graph is find to be a strong graph is examined.

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### 2. Preliminaries

### 2.1 Definition

A fuzzy graph G is a pair of functions G: (σ, μ) where a fuzzy subset of a non-empty set S and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ, μ) is denoted by $G^*$ :( $\sigma^*$, $\mu^*$).

### 2.2 Definition

Let G: (σ, μ) be a fuzzy graph, the order of G is defined as O (G) = $\sum_{u\in V} \sigma(u)$.

### 2.3 Definition

Let G: (σ, μ) be a fuzzy graph, the size of G is defined as S (G) = $\sum_{u,v\in V} \mu(u, v)$.

### 2.4 Definition

Let $\sigma_{DL}$: V→ [0, 1] be a subset of V and $\mu_{DL}$: V× V→ [0, 1] be a symmetric fuzzy relation on $\sigma_{DL}$. Any two vertex of the double layered complete fuzzy graph is adjacent. The vertex set of complete double layered fuzzy graph be σ U μ and it's denoted by $K_{\sigma \cup \mu}$.

**2.5 Definition**

Transitivity: Let $x, y, z \in E$ then $\forall\ (x,y),(y,z),(x,z) \in E \times E$.

$\mu_R(x,z) \geq MAX[\ MIN(\mu_R(x,y), \mu_R(y,z))]$

**2.6 Definition**

Let G: (σ, μ) be a fuzzy graph, the degree of a vertex u in G is defined as $d_G(u) = \sum_{\substack{v \neq u \\ v \in V}} \mu\,(u,v)$ and is denoted as $d_G(u)$.

**2.7 Definition**

A fuzzy graph G: (σ,μ) is said to be strong fuzzy graph if μ(u, v) = σ(u) ∧ σ(v) (u , v) in $\mu^*$.

**2.8 Definition**

Let G be a fuzzy graph, the μ-compliment of G is denoted as $G^\mu$: ($\sigma^\mu$, $\mu^\mu$) where σ* U μ*and

$$\mu^\mu(u,v) = \begin{cases} \sigma(u) \wedge \sigma(v) - \mu(u,v) \ if\ \mu(u,v) > 0 \\ 0\ if\ \mu(u,v) = 0 \end{cases}$$

**3. Matrix representation of DLCFG**

**Definition 3.1**

A complete fuzzy graph G: (σ,μ) with the fuzzy relation μ to be reflexive and symmetric is completely determined by the fuzzy matrix $M_K$,

Where

$$(M_K)_{ij} = \begin{cases} \mu(v_i, v_j) & if\ i \neq j \\ \sigma(v_i) & if\ i = j \end{cases}$$

If $\sigma^*$ has n elements then $M_K$ has n × n elements.

*Example*

Consider the complete fuzzy graph with vertex 3 ($K_3$)

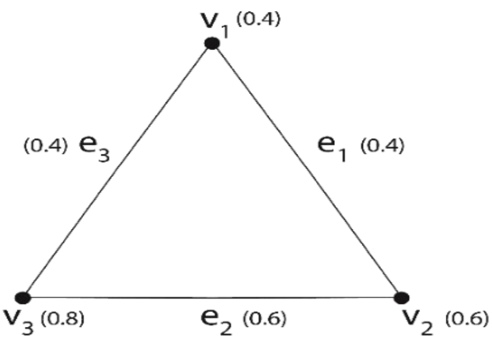


**Figure 1. A complete fuzzy graph ($K_3$)**

The matrix representation with respect to vertices for the complete fuzzy graph G is given by

$$M_{(K_\sigma)} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix} \end{array}$$

The Double Layered Complete Fuzzy Graph for K3 is given by

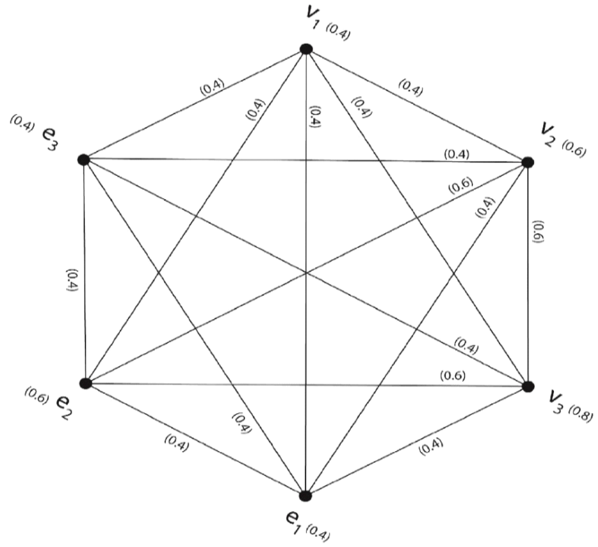


**Figure 2. DLCFG of $K_3$**

The matrix representation of double layered complete fuzzy graph is

$$M_{DL(K_{\sigma \cup \mu})} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & e_1 & e_2 & e_3 \end{array} \\ \begin{pmatrix} 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 & 0.4 & 0.6 & 0.4 \\ 0.4 & 0.6 & 0.8 & 0.4 & 0.6 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 & 0.4 & 0.6 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \end{pmatrix} \end{array}$$

**Remark 3.1**

In this paper, we named the above matrix as Matrix representation of a complete fuzzy graph with respect to vertices and is denoted as $M_{K_\sigma}$

**3. 2 Edge Matrix representation of complete fuzzy graph**

For a complete fuzzy graph $G = (\sigma, \mu)$ with the fuzzy relation to be a reflexive, symmetric and transitive, the edge matrix $M_{K_\mu}$ is defined as follows,

$$(M_{K_\mu})_{ij} = \begin{cases} \min\{\mu(e_i), \mu(e_j)\}\ if\ v_i\ is\ the\ common\ vertex\ between\ e_i\ and\ e_j \\ \mu(e_i)\ if\ i = j \\ 0\ otherwise \end{cases}$$

If contains 'n' elements then $M_{K_\mu}$ is a square matrix of order n.

***Example***

For figure 1 the edge matrix representation is given by

$e_1 \quad e_2 \quad e_3$

$$M_{K_\mu} = \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.4 \\ 0.4 & 0.4 & 0.4 \end{pmatrix}$$

Thus for figure 2, the matrix representation becomes

$$M_{DL(K_{\sigma\cup\mu})} = \begin{bmatrix} M_{K_\sigma} & D_{K_\mu} \\ M_{K_\mu} & M_{K_\mu} \end{bmatrix}$$

## 4. Theoretical concepts

**Theorem: 4.1**

$M_{DL(K_\sigma)}$ is a symmetric matrix.

**Proof:**

$$(M_{DL(K_\sigma)})_{i,j} = \mu(v_i, v_j)$$
$$= \mu(v_j, v_i)$$

(Therefore μ is a symmetric matrix)

$$= (M_{DL(K_\sigma)})_{j,i}$$

$$M_{DL(K_\sigma)} = M^T_{DL(K_\sigma)}$$

**Theorem 4.2**

The Trace of the double layered complete fuzzy graph is equal to the sum of the order and size of the complete fuzzy graph. (i.e.) Trace $M_{DL(K_\sigma)} = \mathrm{Order}(K_\sigma) + \mathrm{Size}(K_\sigma)$.

**Proof:**

$\mathrm{Trace}(M_{DL(K_\sigma)})$ = Sum of the diagonal entries in $M_{DL(K_\sigma)}$

$$= \sum_{i=1}^{n} \mu_{DL(G)} (v_i, v_i)$$
$$= \sum_{v_i \in \sigma^*_{DL}} \sigma_{DL(G)}(v_i)$$
$$= \sum_{\substack{v_i \in \sigma^* \\ e_i \in \mu^*}} \sigma(v_i) + \mu(e_i)$$

By the definition of node set in DLCFG.

$$= \sum_{v_i \in \sigma^*} \sigma(v_i) + \sum_{e_i \in \mu^*} \mu(e_i)$$
$$= Order\ (G) + Size\ (G)$$
$$\mathrm{Trace}(M_{DL(K_\sigma)}) = Order\ (G) + Size\ (G)$$

**Theorem 4.3**

$M_{K_{(\sigma)}}$ is a symmetric iff $M_{K_{(\sigma\cup\mu)}}$ is also symmetric.

**Proof:**

**Case i:**

Let $M_{K_{(\sigma\cup\mu)}}$ is a symmetric.

$M_{(K_\sigma)}$ is a symmetric matrix is obvious.

**Case ii:**

Let $M_{(K_\sigma)}$ is a symmetric.

$$(M_{K_\mu})_{i,j} = \mu(e_i, e_j)$$
$$= \mu(e_j, e_i)$$
$$= (M_{K_\mu})_{j.i}$$

$$(M_{K_\mu})_{i,j} = (M_{K_\mu})_{j,i}$$

$$M_{K_\mu} = M^T_{K_\mu}$$

$$M_{K_{(\sigma\cup\mu)}} = \begin{bmatrix} M_{K_\sigma} & D_{K_\mu} \\ D_{K_\mu} & M_{K_\mu} \end{bmatrix}$$

$$M_{K_{(\sigma\cup\mu)}} = M^T_{K_{(\sigma\cup\mu)}}$$

$M_{K_{(\sigma\cup\mu)}}$ is symmetric.

**Theorem: 4.4**

Every matrix representation of complete fuzzy graph $M_{(K_\sigma)}$ is transitive.

**Proof**

We can illustrate the proof of this theorem with the following example. Consider the complete fuzzy graph with vertex 3 ($K_3$)

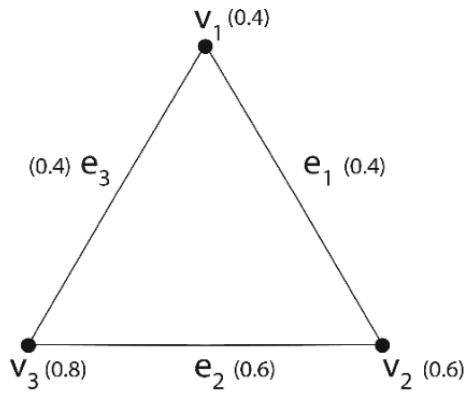


**Figure 3. A complete fuzzy graph ($K_3$)**

The matrix representation with respect to vertices for the complete fuzzy graph G is given by

$$M_{(K_3)} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix} \end{array}$$

By the definition of transitivity,

Let $x, y, z \in E$ then $\forall\ (x, y), (y, z), (x, z) \in E \times E$.
$\mu_R(x, z) \geq MAX[\ MIN(\mu_R(x, y), \mu_R(y, z))]$

The matrix representation with respect to vertices for the complete fuzzy graph G is given by

$$M_{(K_3)} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix} \end{array}$$

Arc ($v_1$, $v_1$)

$\mu(v_1, v_1) \wedge \mu(v_1, v_1) = (0.4) \wedge (0.4) = 0.4$
$\mu(v_1, v_2) \wedge \mu(v_2, v_1) = (0.4) \wedge (0.4) = 0.4$
$\mu(v_1, v_3) \wedge \mu(v_3, v_1) = (0.4) \wedge (0.4) = 0.4$

Max [0.4, 0.4, 0.4] = 0.4
$\mu(v_1, v_1) = 0.4$
Arc ($v_1$, $v_2$)
$\mu(v_1, v_1) \wedge \mu(v_1, v_2) = (0.4) \wedge (0.4) = 0.4$
$\mu(v_1, v_2) \wedge \mu(v_2, v_2) = (0.4) \wedge (0.6) = 0.4$
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$\mu(v_1, v_3) \wedge \mu(v_3, v_3) = (0.4) \wedge (0.6) = 0.4$
Max [0.4, 0.4, 0.4] = 0.4
$\mu(v_1, v_3) = 0.4$

Similarly, Arc ($v_2$, $v_1$) = 0.4, Arc ($v_2$, $v_2$) = 0.6, Arc ($v_2$, $v_3$) = 0.6,

Arc ($v_3$, $v_1$) = 0.4, Arc ($v_3$, $v_2$) = 0.6, Arc ($v_3$, $v_3$) = 0.6
$M_{(K_3)}$ is satisfies the transitivity.

**Remark 4.1**

Every matrix representation of the double layered complete fuzzy graph is transitive respectively.

**5. Conclusion**

In this paper, we have defined a new matrix representation using edge membership values of the complete fuzzy graph. The relationship between the matrix representations of double layered complete fuzzy graph using vertices and given fuzzy graph whose crisp graph is found to be a strong graph is examined. Numerical example is given to verify the results. Further analysis will lead to application of DLCFG in different networks.

## REFERENCES


Jon Arockiaraj J. and V. Chandrasekaran, 2017. *"Double layered complete fuzzy graph", Global Journal of Pure and Applied Mathematics*, 13(9) 6633-6646.

Madhumangal Pal, 2013. "Intersection Graphs: An Introduction", *Annals of Pure and Applied Mathematics*, 4(1) 43-91.

Mini Tom and M.S. Sunitha, 2014. "Sum Distance in Fuzzy Graphs", *Annals of Pure and Applied Mathematics*, 7(2) 73-89.

Mordeson J.N. and P.S. 2001. Nair, "Fuzzy Graphs and Fuzzy Hypergraphs", Physicaverlag Publication, Heidelberg 1998, second edition.

Mordeson, J.N. 1993. "Fuzzy line graphs", Pattern Recognition Letter, 14: 381 – 384.

Nagoorgani A. and J. Malarvizhi, 2010. "Some aspects of neighbourhood fuzzy graph", *International Journal of Pure and Applied Sciences*, 29E, 2: 327 – 333.

Nagoorgani A. and K.Radha, 2009. "The degree of a vertex in some fuzzy graphs", *International Journal of Algorithms, Computing and Mathematics*, 2(3) 107 - 116.

Nagoorgani A. and M. Basheed Ahamed, 2003. "Order and size in fuzzy graphs", Bulletin of Pure and Applied Sciences, 22E(1) 145 – 148.

Overhinnikov, S.V. 1981. "Structure of fuzzy relations", Fuzzy Sets and Systems, 6: 169 – 195.

Pathinathan T. and J. Jesintha Rosline, 2014. "Double layered fuzzy graph", *Annals of Pure and Applied Mathematics*, 8(1) 135-143.

Pathinathan T. and J. Jesintha Rosline, 2014. "Matrix representation of Double layered fuzzy graph and its properties", *Pure and Applied Mathematics*, 8(2) 51-58.

Ragab M.Z. and E.G. Emam, 1995. "The determinant and adjoint of a square fuzzy matrix", *An International Journal of Information Sciences-Intelligent Systems*, 84: 209-220.

Rosenfeld, A. 1975. Fuzzy graphs, in: L.A.Zadeh, K.S.Fu, K.Tanaka and M.Shimura, (editors), "Fuzzy sets and its application to cognitive and decision process", Academic press, New York, 77 – 95.

Samanta S. and M. Pal, 2011. "Fuzzy tolerance graphs", *International Journal of Latest Trends in Mathematics*, 1(2) 57-67.

Shayamal A.K. and M.Pal, 2004. "Two new operations on fuzzy matrices", *Journal of Applied Mathematics and Computing*, 15: 91 – 107.

Sunitha M.S. and A.Vijayakumar, 2002. "Complement of a fuzzy graph", *Indian Journal of Pure and Applied Mathematics*, 33(9) 1451-1464.

Sunitha M.S. and Sunil Mathew, 2013. "Fuzzy Graph Theory: A Survey", *Annals of Pure and Applied Mathematics*, 4(1) 92-110.

Thomson, M.G. 1977. "Convergence of powers of a fuzzy matrix", *Journal of Mathematical Analysis and Application*, 57: 476 – 480.

Yeh R.T. and S.Y. Bang, 1975. "Fuzzy relations, fuzzy graphs and their applications to clustering analysis", in: L.A.Zadeh, K.S.Fu, K.Tanaka and M. Shimura, (editors), Fuzzy sets and its application to cognitive and decision process, Academic press, New York, 125 – 149.


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