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ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN: 0976-3376

Asian Journal of Science and Technology
*Vol. 08, Issue, 09, pp.5834-5839, September, 2017*

**RESEARCH ARTICLE**

# GLOBAL WEIGHT DOMINATING VERTEX SET ON S - VALUED GRAPHS

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## ARTICLE INFO

*Article History:*
Received 18th June, 2017
Received in revised form
19th July, 2017
Accepted 17th August, 2017
Published online 30th September, 2017

*Key words:*

S−valued graphs,
Vertex domination,
Vertex domination number,
Total weight domination on
S−valued graphs.

## ABSTRACT

In [3], we studied the weight dominating vertex set on S− valued graphs. In this paper, we introduce the notion of Global and Total Global weight dominating vertex set on S− valued graphs.



## INTRODUCTION

The concept of domination number was introduced by E.J.Cockayne and S.T.Hedetniemi (1977). The concept of total dominating set and total domination number of a graph was introduced in E.J.Cockayne ,R.M.Dawes and S.T.Hedetniemi (1980). The concept of global dominating set and global domination number was introduced by E.Sampathkumar (Jeyalakshmi, 2016). The concept of total global domination number was discussed in (Kulli, 1996).

In (Jonathan Golan Semirings, ?) Chandramouleeswaran et.al. introduced the concept of Semiring- valued graphs and discussed several properties. In our paper we introduced the concept of weight dominating vertex set and weight dominating vertex number for the S−valued graph $G^S$ (Jeyalakshmi, 2016). In (Jeyalakshmi, 2017), we have introduced the concept of total weight dominating vertex set and total weight dominating vertex number for a S−valued graph $G^S$. In this paper, we introduce the concept of Global weight dominatingvertex set and Total Global weight dominating vertex set. We also define the terms Global weight dominating vertex number and total Global weight dominating vertex number for $G^S$, and obtain some bounds for total Global weight dominating vertex number of $G^S$.

### 1. Preliminaries

In this section, we recall some basic definitions that are needed for our work.

**Definition 2.1 :**[5]A semiring $(S,+,\cdot)$ is an algebraic system with a non-empty set S together with two binary operations + and $\cdot$ such that

1. $(S, +, 0)$ is a monoid.
2. $(S, \cdot)$ is a semigroup.
3. for all $a,b,c\in S$, $a\cdot(b+c) = a\cdot b + a\cdot c$ and $(a + b)\cdot c = a\cdot c + b\cdot c$.
4. $0\cdot x = x\cdot 0 = 0\ \forall\ x \in S$.

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**Definition 2.2:** Let (S, +, ·) be a semiring. A canonical Pre-order $\lesssim$ in S is defined as follows:

for a ,b ∈ S , a $\lesssim$ b if and only if, there exist a c ∈ S such that a + c = b.

**Definition 2.3:** [7] Let G =(V,E) be a given graph with V,E ≠∅. For any semiring (S, +, ·), a semiring-valued graph (or a S – valued graph), $G^s$, is defined to be the graph $G^S$ = (V, E, σ, ψ) where σ : V → S and ψ :E→S is defined to

be $\psi(x,y) = \begin{cases} \min(\sigma(x), \sigma(y)) & \text{if } \sigma(x) \lesssim \sigma(y) \text{ or } \sigma(y) \lesssim \sigma(x) \\ 0 & \text{otherwise} \end{cases}$ for every unordered pair (x,y)of E ⊂ V× V. We call σ, a S–vertex set and ψ, a S edge set of $G^S$.

**Definition: 2.4:** Consider the S-valued graph $G^S$ = (V, E, σ, ψ) with |V| = n and |E| = m. The complement of $G^S$denoted by $\overline{G^S}$= (V, $E_1$, σ, $\psi_1$)where σ : V → S and$\psi_1$ :$E_1$→S and $E_1$is the set of edges joining those vertices of G which are not adjacent and $\psi_1(e_1)$∈ S where $e_1 \in E_1$.

**Note 2.5:** We presume that both $G^S$and the $\overline{G^S}$have non-empty edge set where $\overline{G^S}$is the complement of $G^S$.

**Definition 2.6 : [3]** A vertex υ in $G^S$ is said to be a weight dominating vertex if $\sigma(u) \lesssim \sigma(v)$, ∀ u ∈ $N_S$ [v].

**Definition 2.7 : [3]** A subset D ⊆ V is said to be a weight dominating vertex set of $G^S$ if for each v ∈D , $\sigma(u) \lesssim \sigma(v)$, ∀ u ∈ $N_S$ [v].

**Definition 2.8 :[3]** If D is a weight dominating vertex set of $G^S$ , then the scalar cardinality of D, denoted by $|D|_S$, is defined by $|D|_S = \sum_{v\in D}\sigma(v)$.

**Definition 2.9 :[3]** The cardinality of the minimal weight dominating vertex set D ⊆ V is called the weight dominating vertex number of $G^S$ . It is denoted by $\gamma_V^S(G^S)$.

That is $\gamma_V^S(G^S)$ = ($|D|_S$, |D|).

**Notation :** Consider the S-valued graph$G^S$ = (V, E, σ, ψ). Denote $V_S = \{(\sigma(v_i), v_i) / \text{where } i = 1,2,3,\ldots n \text{ and } \sigma(v_i) \in S\}$.

**Definition 2.10:[4]**Consider the S-valued graph$G^S$ = (V, E, σ, ψ). Let $T_D^S$ ⊆ V. The open neighbourhood of $T_D^S$, denoted by $N_S(T_D^S)$ , is defined by $N_S(T_D^S) = \cup_{v\in T_D^S} N_S(v)$.

**Definition 2.11:[4]**Consider the S-valued graph$G^S$ = (V, E, σ, ψ). Let $T_D^S$ ⊆ V. The closed neighbourhood of $T_D^S$, denoted by $N_S[T_D^S]$, is defined by $N_S[T_D^S] = \cup_{v\in T_D^S} N_S[v]$

**Definition 2.12 :** Consider the S-valued graph$G^S$ = (V, E, σ, ψ). Let $T_D^S$ ⊆ V. For any v∈$T_D^S$ The private neighbour set of v with respect to $T_D^S$, denoted by $PN_S[v, T_D^S]$, is defined by $PN_S[v, T_D^S] = N_S[v] - N_S[T_D^S - \{v\}]$.

**Definition 2.13:[4]** Let$T_D^S$⊆ Vbe a weight dominating vertex set. If $N_S[T_D^S] = V_S$, then $T_D^S$ is called a total weight dominating vertex set of $G^S$.

## 3. Global weight dominating vertex set on S−valued graphs

In this section, we introduce the notion of Global weight dominating vertex set on S−valued graphs and prove some simple properties.

**Definition 3.1:** Consider the S−valued graph $G^S$ = (V,E,σ,ψ). A weight dominating vertex set D ⊆ V of $G^S$ is a global weight dominating vertex set of the S−valued graph $G^S$ if D ⊆ V is also a weight dominating vertex set of $\overline{G^S}$. It is denoted by $GD^S$.

**Example 3.2:** Let (S = {0,a,b,c},+,·) be a semiring with the following Cayley tables:

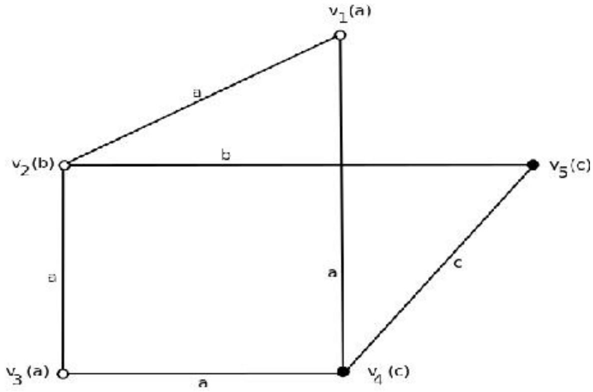
| + | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | a | b | c |
| a | a | b | c | c |
| b | b | c | c | c |
| c | c | c | c | c |

| · | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | b | c |
| b | 0 | b | c | c |
| c | 0 | c | c | c |

Let $\lesssim$ be a canonical pre-order in S, given by

0 $\lesssim$ 0, 0 $\lesssim$ a,0$\lesssim$ b, 0 $\lesssim$ c, a $\lesssim$ a, b $\lesssim$ b,b $\lesssim$ a, c $\lesssim$ c, c $\lesssim$ a, c $\lesssim$ b.

Consider the S-valued graph $G^S$:



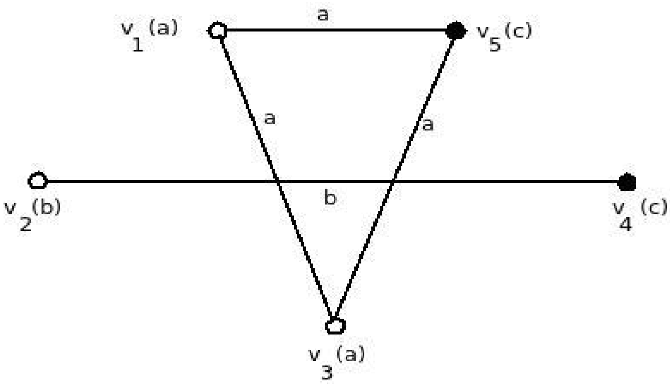

where σ : V → S is defined by

$\sigma(v_1) = \sigma(v_3) = a$ ;$\sigma(v_2) = b$ and$\sigma(v_4) = \sigma(v_5) = c$and ψ : E → S by

$\psi(v_2,v_5) = b$ ; $\psi(v_4,v_5) = c$ and$\psi(v_1,v_2) = \psi(v_2,v_3) = \psi(v_1,v_4) = \psi(v_3,v_4) = \psi(v_3,v_4) = a$

Here D = {$v_4$(c),$v_5$(c)} is a weight dominating vertex set of $G^S$.

Consider the complement of $G^S$ that is $\overline{G^S}$:




$\overline{G}^S$

Here D = {$v_4$(c),$v_5$(c)} is also a weight dominating vertex set of $\overline{G^S}$.

Hence D = {$v_4$(c),$v_5$(c)} is a global weight dominating vertex set of $G^S$.

**Definition 3.3:** If $GD^S$ is a global weight dominating vertex set of $G^S$, then the scalar cardinality of $GD^S$, denoted by $|GD^S|_S$, is defined by$|GD^S|_S = \sum_{v \square G_D^S} \sigma$ V .

**Definition 3.4:** A subset $GD^S$ □ V is said to be a minimal global weight dominating vertex set of $G^S$ if

1. $GD^S$ is a global weight dominating vertex set.
2. No proper subset of $GD^S$ is a global weight dominating vertex set.

**Definition 3.5:** The cardinality of the minimal global weight dominating vertex set $GD^S$ □ V is called the global weight domination vertex number of $G^S$. It is denoted by. That is,

$\gamma^S_{GD^S}(G^S) = |GD^S|_S, |GD^S|$

where$GD^S$ is a minimal total weight dominating vertex set of $G^S$.

From the above example $\gamma^S_{GD^S}(G^S) = |GD^S|_S, |GD^S|$ ,= (c + c,2) = (c,2).

**Theorem 3.6:** A dominating set D of $G^S$ is global weight dominating vertex set if and only if for each u □ V − D, there exist a v □ D such that v is not adjacent to u in $G^S$.

**Proof:** Consider the S−valued graph $G^S = (V,E,\sigma,\psi)$.

Let D □ V be a weight dominating vertex set of $G^S$.

That is, for each $v \in D$, $\sigma\ u \precsim \sigma\ v$, $\forall u \in N_S[\upsilon]$.

If D is a global weight dominating vertex set then D dominates the S−valued graph $\overline{G^S}$.

Assume that, for each u □ V − D there exist a v □ D such that u and v are adjacent in $G^S$.

Then u and v are not adjacent in $\overline{G^S}$. Hence v cannot dominate u in $\overline{G^S}$. Thus D cannot be a weight dominating vertex set for $\overline{G^S}$. This contradicts the assumption that D is a global weight dominating vertex set of $G^S$. Hence for each u □ V − D, there exist a v □ D such that v is not adjacent to u in $G^S$. Retracing the steps we get the converse.

**Theorem 3.7:** For any S−valued graph $G^S$ with p vertices $\gamma^S_{GD^S}(G^S) = (\sum_{v□V} \sigma\ v, p)$ if and only if $G^S = k^S_P$ or $k^{\overline{S}}_P$.

**Proof:**

Consider $G^S = k^S_P$.

Let $GD^S$ be a minimal global weight dominating vertex set.

□ $GD^S$ is a minimal weight dominating vertex set for both $G^S$ and $G^S$.

Suppose $\gamma^S_{GD^S}(G^S) = (\sum_{v□V} \sigma\ v, p)$ and $G^S$ is neither $k^S_P$ or $k^{\overline{S}}_P$.

Since $G^S = k^S_P$, any vertex in $G^S$ will dominate all other vertices in $G^S$.

That is $GD^S = \{(v,\sigma(v)\}$. However $GD^S$ is a minimal weight dominating vertex set for $\overline{G^S}$ which has p isolated vertices in $E_{\overline{G^S}} = \varphi$. Therefore $GD^S$ will include all the p vertices. Thus, $\gamma^S_{GD^S}(G^S) = (\sum_{v□V} \sigma\ v, p)$ for $\sigma\ v$ □ $S$

For $G^S = k^{\overline{S}}_P$ the proof is analogous.

On the otherhand, if $\gamma^S_{GD^S}(G^S) = (\sum_{v□V} \sigma\ v, p)$ ,trivially $\overline{G^S} = k^S_P$. Otherwise $G^S$ contains $k^S_P$. As a subgraph, then minimum weight domination number is p and there may be other vertices not in $k^S_P$ which has to be included in the weight dominating vertex set.

Contradicting the value of $\gamma^S_{GD^S}(G^S)$.

Then $G^S$ has atleast one edge uv and a vertex w not adjacent to v.

Then $V_S - \{(v,\sigma(v))\}$ is a global weight dominating vertex set of $G^S$ and $\gamma^S_{GD^S}(G^S) \precsim (\sum_{v□V} \sigma\ v, p-1)$ which is a contradiction. Hence $G^S = k^S_P$ or $k^{\overline{S}}_P$.

**Theorem 3.8:** Let $G^S$ be a S−valued graph such that neither $G^S$ nor $\overline{G^S}$ have an S− isolated vertex then $\gamma^S_V(G^S) = \gamma^S_{GD^S}(G^S)$

**Proof :**

Consider the S−valued graph $G^S = (V,E,\sigma,\psi)$.

Let $GD^S$ □ V be a minimal global weight dominating vertex set of $G^S$.

$\gamma^S_{GD^S}(G^S) = |GD^S|_S, |GD^S|$ ……………………(1)

Then □ $GD^S$ is a minimal weight dominating vertex set for $G^S$ and $\overline{G^S}$. Since $GD^S$ is a minimal weight dominating vertex set of $G^S$, then

$\gamma^S_V(G^S) = |GD^S|_S, |GD^S|$ ……………………(2)

Comparing (1) and (2) we get, $\gamma_V^S(G^S) = \gamma_{GD^S}^S(G^S)$.

**4. Total Global weight dominating vertex number on S−valued graphs**

In this section, we introduce the notion of Total global weight dominating vertex set and total global weight dominating vertex number. By an example we prove that a graph may have a total weight dominating vertex set but not a total global weight dominating vertex set.

**Definition 4.1:** Consider the S−valued graph $G^S = (V,E,\sigma,\psi)$. A total weight dominating vertex set $T_D^S \subseteq V$ of $G^S$ is a total global weight dominating vertex set of a S−valued graph $G^S$ if $T_D^S \subseteq V$ is also a total weight dominating vertex set of $\overline{G}^S$. It is denoted by TGD.

**Definition 4.2:** If $TGD^S$ is a total global weight dominating vertex set of $G^S$, then the scalar cardinality of $TGD^S$, denoted by $|TGD^S|_S$, is defined by $|TGD^S|_S = \sum_{v \in TGD^S} \sigma(v)$.

**Definition 4.3:** A subset $TGD^S \subseteq V$ is said to be a minimal global weight dominating vertex set of $G^S$ if

1. $TGD^S$ is a total global weight dominating vertex set.
2. No proper subset of $TGD^S$ is a total global weight dominating vertex set.

**Definition 4.4:** The cardinality of the minimal total global weight dominating vertex set $TGD^S \subseteq V$ is called the total global weight domination vertex number of $G^S$. Itis denoted by $\gamma_{TGD^S}^S(G^S)$.

That is,

$$\gamma_{TGD^S}^S(G^S) = (|TGD^S|_S, |TGD^S|)$$

**Note :** We note that $\gamma_V^S(G^S)$ and $\gamma_{GD^S}^S(G^S)$ are defined for any S−valued graph $G^S$ while $\gamma_{TGD^S}^S(G^S)$ is only defined for $G^S$ with $\delta_S(G^S) \geq 1$ and $\gamma_{TGD^S}^S(G^S)$ is only defined for $G^S$ with $\delta_S(G^S) \geq 1$ and $\delta_S\overline{(G)}^S \geq 1$, where $\delta_S(G^S)$ is the minimum degree of $G^S$.

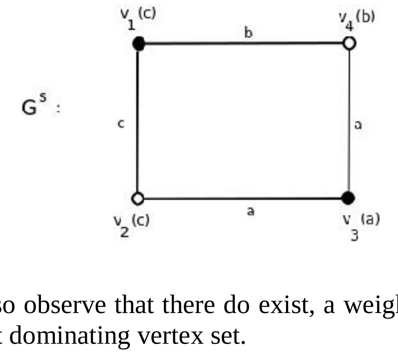
**Remark 4.5:** A weight dominating vertex set in $G^S$ need not be, in general, a global weight dominating vertex set of $G^S$ ,also a total weight dominating vertex set need not be, in general, a total global weight dominating vertex set are as seen in the following example.

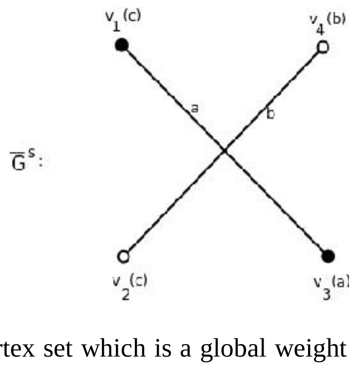
Consider the semiring with canonical pre-order as given in Example 3.2.

Consider the following S−valued graph $G^S$ and $\overline{G}^S$.

In $G^S$ and $\overline{G}^S$, $D = \{v_1(c), v_3(c)\}$ is a weight dominating vertex set of $G^S$ but not for $\overline{G}^S$.

Hence D is not a global weight dominating vertex set of $G^S$. Also $D = \{v_1(c), v_3(c)\}$ is a total wight dominating vertex set of $G^S$ but not for $\overline{G}^S$. Hence D is not a total global weight dominating vertex set of $G^S$.

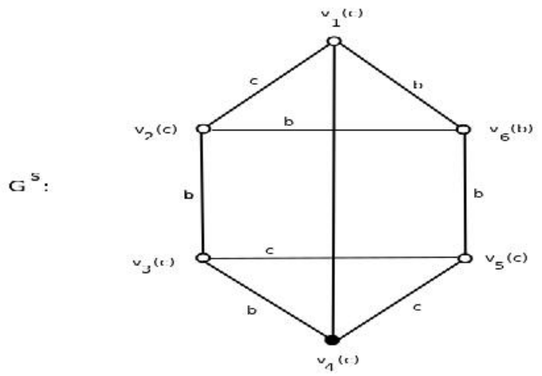


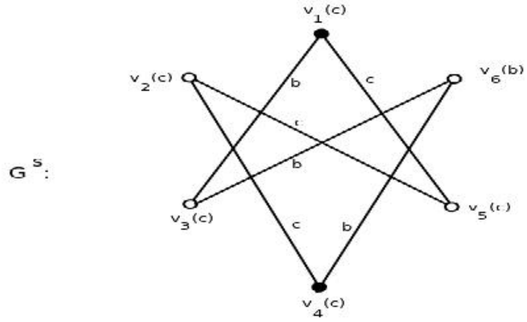




**Remark 4.6:** We also observe that there do exist, a weight dominating vertex set which is a global weight dominating vertex set but not a total weight dominating vertex set.

**Remark 4.7:** There do exist a total weight dominating vertex set which is both weight dominating vertex set as well as global weight dominating vertex set but not a total global weight dominating vertex set of $G^S$.

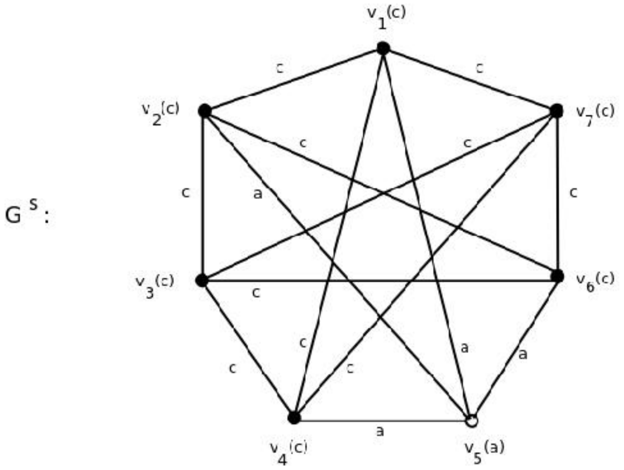
For example, Consider the S−valued graph $G^S$ and the semiring with canonical pre-order as given in Example 3.2 as follows.

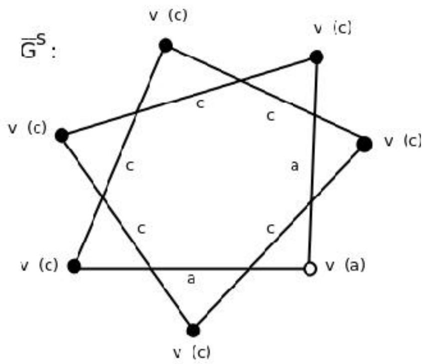



Here D = {$v_1$(c),$v_4$(c)} is a weight dominating vertex set which is also a global weight dominating vertex set but not a total weight dominating vertex set and $T_D^S$ = {$v_1$(c),$v_2$(c),$v_3$(c),$v_4$(c),$v_5$(c)} is total weight dominating vertex set which is both weight dominating vertex set and global weight dominating vertex set.

**Remark 4.8:** A total weight dominating vertex set which is also a total global weight dominating vertex set. However we observe that to obtain a total global weight dominating vertex set we need to add few more vertices to a minimal total weight dominating vertex set of $G^S$ as seen in the following example.




Consider $T_D^S$ = {$v_1$(c),$v_2$(c),$v_3$(c),$v_4$(c),$v_6$(c),$v_7$(c)} is a total weight dominating vertex set which is also a total global weight dominating vertex set of $G^S$.

## REFERENCES


Coakayne, E.J. and S.T. Heditniemi. 1977. Networks 7, 247-61.

Coakayne, E.J., Dawes, R.M. and Heditniemi, S.T. 1980. Networks 10, 211-19.

Jeyalakshmi, S. and Chandramouleeswaran, M. 2017. Total weight dominating vertex set on S− valued Graphs, *Mathematical Sciences International Research Journal* Vol. 6 , Issue 2., 147-150.

Jeyalakshmi. S. and Chandramouleeswaran. M., 2016. Vertex domination on S− valued Graphs, *IOSR Journal of Mathematics*(IOSR-JM), Vol. 12 (III), 08 - 12.

Jonathan Golan Semirings and Their Applications, Kluwer Academic Publishers, London.

Kulli, V. R. and Janakiram, B. 1996. The total gloal domination number of a graph, *Indian J. Pure appl. Math.*, 27(6), 537-542, June 1996.

Rajkumar, M., Jeyalakshmi, S and Chandramouleeswaran, M. 2015. Semiring-valued Graphs, *International Journal of Math. Sci.and Engg. Appls.*, Vol. 9 (III), 141-152.

Sampathkumar, E. 1989. The global domination of a graph, *Math.Phys. Sci*. 23.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