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RESEARCH ARTICLE

NUMERICAL SOLUTION OF HYDROMAGNETIC INCOMPRESSIBLE LAMINAR FLOW OVER NONLINEAR STRETCHING SHEET

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ABSTRACT

In this thesis, the problem of the magnetohydrodynamics boundary layer flow of an incompressible viscous fluid over a non-linear stretching sheet is studied. Using a suitable similarity transformation, the governing partial differential equations are transformed into a system of nonlinear higher order ordinary differential equations. The resulting equations are solved numerically using implicit finite difference scheme known as Keller box method by implementing in matlab. The effects of flow parameters like magnetic field parameter M, thermal radiation R, the suction/injection parameter s, Prandtl number Pr, Eckert number Ec and power index parameter n are demonstrated graphically for velocity and temperature profiles, skin friction coefficient and surface heat transfer rate are presented numerically as well. Numerical results obtained on the values of surface heat transfer rate are compared with previously reported cases available from literature and they are found to be in a very good agreement.

INTRODUCTION

The study on hydromagnetic boundary layer flow over a stretching sheet is a subject of great interest due to its various applications in designing cooling system which includes liquid metals, MHD generators, accelerators, pumps and so on. Furthermore, the continuous surface heat and mass transfer problems have many practical applications in electro-chemistry and polymer processing (Mabood *et al*., 2015). Many chemical engineering processes, like metallurgical and polymer extrusion involve cooling of a molten liquid being stretched into a cooling system. The fluid mechanical properties of the penultimate product depend mainly on the process of stretching and on the rate of cooling. A fluid is a substance that can flow in an enclosure, in a pipe, in a channel or over a plate. Fluid flow in the presence of magnetic field is called hydromagnetic flow and the study of hydromagnetic flows is called magnetohydrodynamics (MHD) (Kimeu *et al*., 2014). Hydromagnetic behavior of boundary layer flow over a moving surface in the presence of transverse magnetic field is a basic and important problem in magnetohydrodynamic (MHD). MHD flows of Newtonian fluids were investigated by (Pavlov, 1974). The MHD flow and heat transfer for a viscous fluid over a stretching sheet has enormous applications in many engineering problems such as plasma studies, petroleum industries, geothermal energy extractions, the boundary layer flow control in the field of aerodynamic and many others.

The flow of an incompressible viscous fluid over stretching surface has important applications in polymer industry. In view of these applications, (Sakiadis, 1961) initiated the study of boundary layer flow over continuous solid surface moving with constant speed and he presented its numerical result. The two dimensional laminar boundary layer flow of an incompressible viscous fluid over stretching sheet studied by (Crane, 1970) which moves in its own plane with a velocity varying linearly with distance from a fixed point, because he noted that in the polymer industry it is sometimes necessary to consider a stretching plastic sheet. (Gupta and Gupta, 1977) examined the heat and mass transfer in the flow over a stretching surface (with suction or blowing) issuing from a thin slit. A non-isothermal moving sheet was dealt with and the temperature and concentration distribution profiles for that situation were obtained. The heat transfer over a stretching sheet of a hydromagnetic flow has been studied by (Chakrabarthi and Gupta, 1979).

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The flow over a stretching surface in three dimensional was discussed by (Wang, 1984). The MHD flow characteristics over a stretching sheet of a visco elastic fluid was demonstrated by (Andersson, 1992). Later his work was extended by (Char, 1994) with mass transfer. The boundary layer flow and heat transfer of a nanofluid over a permeable stretching sheet with the effects of magnetic field, slip boundary condition and thermal radiation have been analyzed by (Wubshet and Bandari, 2013). The numerical analysis of magnetic field on Eyring-Powell fluid flow towards a stretching sheet has been discussed by (Akbar and Ebaid, 2015). All the above investigations are for the flow over a linearly stretching sheet. Also there were many investigations for the flow over nonlinear stretching sheet. (Vajravelu, 2001) analyzed the viscous flow over a nonlinearly stretching sheet. The heat transfer characteristics of viscous fluid over a nonlinear stretching sheet were later introduced by (Cortell, 2007). A great number of studies for the boundary layer flow over a nonlinear stretching sheet under different aspects of heat and mass transfer, slip and convective boundary conditions and so on, are presented by (Abbas and Hayat, 2011, Hayat, 2011, Rana and Bhargava, 2012). Motivated by the above investigations, the objective of the present study is to analyze the numerical solution of hydromagnetic incompressible laminar flow over nonlinear stretching sheet. In line with this, we study the effect of power index parameter n, magnetic field parameter M, Prandtl number Pr, thermal radiation R, Eckert number Ec, and suction/injection parameter s on hydromagnetic incompressible fluid flow over nonlinear stretching sheet in the presence of heat source/sink. In addition to this, the effect of some parameters on skin friction coefficient and surface heat transfer rate will also be investigated. The boundary layer equations governed by the partial differential equations are first transformed into a system of higher order nonlinear ordinary differential equations using appropriate similarity transformation before being solved numerically using Keller box method (Cebeci and Bradshaw, 1984).

Description of the Method

Mathematical Formulation

We consider a steady, two dimensional, hydromagnetic boundary layer flow of a viscous, incompressible, electrically conducting fluid over nonlinear stretching sheet subjected to suction/injection in the presence of uniform transverse magnetic field. The velocity components u and v are taken in x and y directions respectively. It is assumed that the velocity of stretching sheet is $U_w(x) = cx^n$ and constant mass transfer velocity is $V = V_w(x)$ with $V_w(x) < 0$ for suction and $V_w(x) > 0$ for injection respectively. The x-axis is taken along the stretching sheet and y-axis is perpendicular into the fluid. The fluid is electrically conducting and the magnetic field $B(x)$ is assumed to be applied in the y-direction. The magnetic Reynolds number is taken to be small so that the induced magnetic field is negligible. The temperature of the surface maintained at a constant temperature T_w and far away from the sheet temperature is T_∞, where $T_w > T_\infty$

Under boundary layer approximation, the continuity, momentum and energy equations are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathcal{V}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u \qquad (2)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y} + Q_O(T - T_\infty) \qquad (3)$$

where u and v are the components of velocity respectively in the x and y directions, T is the temperature, T_w is the surface temperature, T_∞ is the temperature far from the sheet, ρ is the fluid density (assumed to be constant), σ is the electrical conductivity of the fluid, μ is the dynamic viscosity, $(\mathcal{V} = \frac{\mu}{\rho})$ is the kinematic viscosity, k is the thermal conductivity, q_r is the radiative heat flux, Q_O is the heat source/sink, c_p is the specific heat at a constant pressure.

The appropriate boundary conditions for the velocity components and temperature are given by

$$u = U_w(x) = cx^n\ ,\ v = v_w(x) \qquad (4)$$

$$T = T_w\ , \qquad y = 0$$

$$u \to 0\ , \qquad T \to T_\infty\ , \qquad y \to \infty \qquad (5)$$

Where n is power index parameter and c is a constant rate of stretching.

We assume that the external electric field and polarization effects are negligible in Eq. (2) and the magnetic field $B(x)$ is considered in the form (Chaim, 1995)

$$B(x) = B_o(x)x^{\frac{n-1}{2}} \qquad (6)$$

Using Rosseland's approximation the radiative heat flux q_r for radiation (Brewster, 1972) we obtain

$$q_r = \frac{-4\sigma_1}{3\alpha_R}\frac{\partial T^4}{\partial y} \tag{7}$$

Where σ_1 is the Stefan–Boltzmann constant, α_R is the absorption coefficient. The temperature difference within the flow region, namely, the term T^4 is assumed to be a linear function of temperature. The linear approximation for temperature is obtained by expanding T^4 into a Taylor series at infinity and neglecting higher-order terms:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

By using eqs. (8) and (9), eq. (3) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\sigma_1 T_\infty^3}{3\alpha_R \rho c_p}\frac{\partial^2 T}{\partial y} + \frac{Q_o}{\rho c_p}(T - T_\infty) \tag{9}$$

In order to investigate the velocity and temperature distribution of the fluid over nonlinear stretching sheet the following dimensionless similarity variables (Dodda *et al.*, 2016) are introduced to employ similarity transformations:

$$\eta = y\sqrt{\frac{C(n+1)}{2\mathcal{V}}}\,x^{\frac{n-1}{2}} \tag{10}$$

$$\psi(x,y) = \sqrt{\frac{2\mathcal{V}C}{n+1}}\,f(\eta)x^{\frac{n+1}{2}} \tag{11}$$

$$G(\eta) = \frac{T-T_\infty}{T_W-T_\infty} \tag{12}$$

with the velocity components

$$u = \frac{\partial\psi}{\partial y} = cx^n f'(\eta) \quad \text{and}$$

$$v = -\frac{\partial\psi}{\partial x} = -\sqrt{c\mathcal{V}\left(\frac{n+1}{2}\right)}\,x^{\frac{n-1}{2}}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right) \tag{13}$$

where $\psi(x,y)$ is the stream function, $f(\eta)$ is the dimensionless stream function, $f'(\eta)$ is the velocity profile, η is the similarity variable and $G(\eta)$ is the temperature profile.

Using similarity transformations, we get the reduced nonlinear higher ordinary differential equations,

$$f''' + ff'' - \beta f'^2 - Mf' = 0 \tag{14}$$

$$G'' + \frac{R}{R+1}Pr\left(fG' + Ecf''^2 + QG\right) = 0 \tag{15}$$

Where primes denote differentiation with respect to η. The control parameter β, the magnetic field parameter M, the thermal radiation parameter R, the Prandtl number Pr, the Eckert number Ec, heat source/sink parameter Q and the wall mass transfer at the sheet s are given by

$$\beta = \frac{2n}{n+1}, \qquad M = \frac{2\delta B_O^2(x)}{\rho c(n+1)}, \qquad Q = \frac{2Q_o}{\rho c_p c(n+1)}, \qquad Pr = \frac{\mu c_p}{k}$$

$$R = \frac{3\alpha_R k}{16\sigma_1 T_\infty^3}, \qquad Ec = \frac{U_W^2}{c_p(T_W-T_\infty)}, \qquad s = \frac{-V_W}{\sqrt{c\mathcal{V}\left(\frac{n+1}{2}\right)}\,x^{\frac{n-1}{2}}} \quad \text{and} \quad U_w = cx^n$$

Subject to the boundary conditions

$$f'(0) = 1, \qquad f(0) = s, \qquad f'(\infty) = 0 \qquad G(0) = 1, \qquad G(\infty) = 0 \tag{16}$$

The physical quantities of interest in this problem are the skin-friction parameter c_f defined by

$$c_f = \frac{\tau_w}{\rho U_w^2} = \sqrt{\frac{n+1}{2}}(Re)^{\frac{-1}{2}}f''(0) \tag{17}$$

and the local Nusselt number Nu

$$Nu = \frac{xq_w}{k(T_w - T_\infty)} = -(Re)^{\frac{1}{2}}\sqrt{\frac{n+1}{2}}G'(0) \tag{18}$$

Where the wall shear stress τ_w, the local Reynolds number Re and the local rate of heat transfer of the surface q_w are respectively defined by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = c\mu\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{3n-1}{2}}f''(0)\ , \quad Re = \frac{U_w x}{\nu} \quad \text{and}$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k(T_w - T_\infty)\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}G'(0) \tag{19}$$

Method of solution

Equations (14) and (15) subject to the boundary conditions (16) were solved numerically by Keller box method which is implemented in matlab. The Keller box method is an implicit finite difference method that can be used to solve differential equations. In this method the transformed differential equations (14) and (15) are written in terms of first order system (Mitiku and Devaraj, 2017), for that we introduce new dependent variables u, v and g such that

$$\begin{aligned} f' &= u \\ u' &= v \\ G' &= g \end{aligned} \tag{20}$$

$$v' + fv - \beta u^2 - Mu = 0 \tag{21}$$

$$g' + \frac{R}{R+1}Pr(fg + Ecv^2 + QG) = 0 \tag{22}$$

with the new boundary conditions

$$\begin{aligned} &u(0) = 1 \qquad f(0) = s \qquad u(\infty) = 0 \\ &G(0) = 1 \qquad G(\infty) = 0 \end{aligned} \tag{23}$$

We now consider the net rectangle in the $x - \eta$ plane as shown in figure 4b and the net points defined as follows:

$$x_0 = 0\ , \quad x_n = x_{n-1} + k_n\ , \quad n = 1,2,\dots,N$$

$$\eta_0 = 0\ , \quad \eta_j = \eta_{j-1} + h_j\ , \quad j = 1,2,\dots,J\ , \quad \eta_j = \eta_\infty \tag{24}$$

where k_n is the $\Delta\, x$-spacing and h_j is the $\Delta\eta$-spacing. Here n and j are the sequence of numbers that indicate the coordinate location.

A brief description of the method is given below (Cebeci and Bradshaw, 1984).

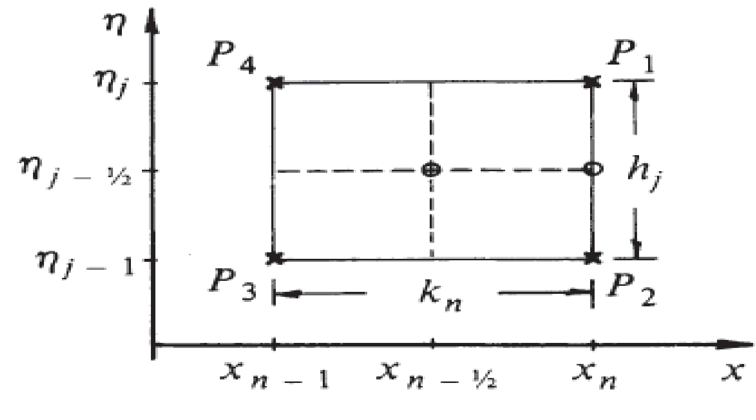

Figure 1. Finite difference grid for the Box Method

Now write the finite difference approximations of the ordinary differential equations (20) for the midpoint $\left(x_n, \eta_{j-\frac{1}{2}}\right)$ of the segment P_1P_2 using centered difference derivatives, this is called centering about $\left(x_n, \eta_{j-\frac{1}{2}}\right)$.

$$\frac{f_j-f_{j-1}}{h_j}=u_{j-\frac{1}{2}}$$

$$\frac{u_j-u_{j-1}}{h_j}=v_{j-\frac{1}{2}} \tag{25}$$

$$\frac{G_j-G_{j-1}}{h_j}=g_{j-\frac{1}{2}}$$

Ordinary differential equations (21) and (22) are approximated by the centering about the midpoint $\left(x_{n-\frac{1}{2}},\ \eta_{j-\frac{1}{2}}\right)$ of the rectangle $P_1P_2P_3P_4$.

$$\frac{v_j-v_{j-1}}{h_j}+f_{j-\frac{1}{2}}v_{j-\frac{1}{2}}-\beta u_{j-\frac{1}{2}}^2-Mu_{j-\frac{1}{2}}=0 \tag{26}$$

$$\frac{g_j-g_{j-1}}{h_j}+Pr\left(\frac{R}{R+1}\right)\left[f_{j-\frac{1}{2}}g_{j-\frac{1}{2}}+Ecv_{j-\frac{1}{2}}^2+QG_{j-\frac{1}{2}}\right]=0 \tag{27}$$

Here $u_{j-\frac{1}{2}}=\frac{u_j+u_{j-1}}{2}$, etc.

Equations (25)-(27) becomes

$$f_j-f_{j-1}-\frac{h_j}{2}\left(u_j+u_{j-1}\right)=0$$
$$u_j-u_{j-1}-\frac{h_j}{2}\left(v_j+v_{j-1}\right)=0 \tag{28}$$

$$G_j-G_{j-1}-\frac{h_j}{2}\left(g_j+g_{j-1}\right)=0$$
$$v_j-v_{j-1}+\frac{h_j}{4}\left(f_j+f_{j-1}\right)\left(v_j+v_{j-1}\right)-\beta\frac{h_j}{4}\left(u_j+u_{j-1}\right)^2-M\frac{h_j}{2}\left(u_j+u_{j-1}\right)=0 \tag{29}$$

$$g_j-g_{j-1}+\frac{R}{4(R+1)}h_jPr\left[\left(f_j+f_{j-1}\right)\left(g_j+g_{j-1}\right)+Ec\left(v_j+v_{j-1}\right)^2\right]$$

$$+\frac{R}{2(R+1)}h_jPrQ\left(G_j+G_{j-1}\right)=0 \tag{30}$$

Now linearize the nonlinear system of equations (28) - (30) using the Newton's linearization scheme, that is, we assume for $(i+1)^{th}$ iterate

$$f_j^{i+1}=f_j^i+\delta f_j^i\text{ , etc.} \tag{33}$$

Substituting Eq. (31) into Eqs. (28) - (29) and dropping the quadratic terms in δf_j^i, δu_j^i, δv_j^i, δG_j^i and δg_j^i (Adhikari and Sanyal, 2013), we obtain a tridiagonal system of algebraic equations

$$\delta f_j-\delta f_{j-1}-\frac{h_j}{2}\left(\delta u_j+\delta u_{j-1}\right)=(r_1)_j$$
$$\delta u_j-\delta u_{j-1}-\frac{h_j}{2}\left(\delta v_j+\delta v_{j-1}\right)=(r_2)_j \tag{32}$$

$$\delta G_j-\delta G_{j-1}-\frac{h_j}{2}\left(\delta g_j+\delta g_{j-1}\right)=(r_3)_j$$
$$(a_1)_j\delta v_j+(a_2)_j\delta v_{j-1}+(a_3)_j\delta f_j+(a_4)_j\delta f_{j-1}+(a_5)_j\delta u_j+(a_6)_j\delta u_{j-1}=(r_4)_j \tag{33}$$

$$(b_1)_j\delta g_j+(b_2)_j\delta g_{j-1}+(b_3)_j\delta f_j+(b_4)_j\delta f_{j-1}+(b_5)_j\delta v_j+(b_6)_j\delta v_{j-1}+(b_7)_j\delta G_j$$
$$+(b_8)_j\delta G_{j-1}=(r_5)_j \tag{34}$$

Where, $(a_1)_j=1+\frac{h_j}{4}\left(f_j+f_{j-1}\right)$, $(a_3)_j=(a_4)_j=\frac{h_j}{4}\left(v_j+v_{j-1}\right)$
$(a_2)_j=-1+\frac{h_j}{4}\left(f_j+f_{j-1}\right)$, $(a_5)_j=(a_6)_j=-\frac{\beta h_j}{2}\left(u_j+u_{j-1}\right)-\frac{h_j}{2}M$
$(b_1)_j=1+\frac{R}{4(R+1)}h_jPr\left(f_j+f_{j-1}\right)$, $(b_3)_j=(b_4)_j=\frac{R}{4(R+1)}h_jPr\left(g_j+g_{j-1}\right)$

$(b_2)_j = -1 + \frac{R}{4(R+1)} h_j Pr(f_j + f_{j-1}), \qquad (b_5)_j = (b_6)_j = \frac{R}{2(R+1)} h_j PrEc(v_j + v_{j-1})$

$(b_7)_j = (b_8)_j = \dfrac{R}{2(R+1)} h_j PrQ$

and $(r_1)_j = f_{j-1} - f_j + \frac{h_j}{2}(u_j + u_{j-1}), \qquad (r_3)_j = G_{j-1} - G_j + \frac{h_j}{2}(g_j + g_{j-1})$

$(r_2)_j = u_{j-1} - u_j + \dfrac{h_j}{2}(v_j + v_{j-1})$

$(r_4)_j = v_{j-1} - v_j - \dfrac{h_j}{4}\left[(f_j + f_{j-1})(v_j + v_{j-1})\right] + \dfrac{h_j}{4}\beta(u_j + u_{j-1})^2 + \dfrac{Mh_j}{2}(u_j + u_{j-1})$

$(r_5)_j = g_{j-1} - g_j - \dfrac{R}{4(R+1)} h_j Pr\left[(f_j + f_{j-1})(g_j + g_{j-1}) + Ec(v_j + v_{j-1})^2\right]$

$-\dfrac{R}{2(R+1)} h_j PrQ(G_j + G_{j-1})$

with the boundary conditions

$\delta u_o = 1, \quad \delta f_o = s, \quad \delta u_J = 0, \quad \delta G_o = 1, \quad \delta G_J = 0$

Hence, the linearized system (32) - (34) can be written in the matrix form as

$$A\delta = r \tag{35}$$

Where, $A = \begin{bmatrix} [A_1] & [C_1] & & & & & \\ [B_2] & [A_2] & [C_2] & & & & \\ & [B_3] & [A_3] & [C_3] & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & & & [B_J] & [A_J] \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \cdot \\ \cdot \\ \cdot \\ \delta_J \end{bmatrix}, \quad r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \cdot \\ \cdot \\ \cdot \\ r_J \end{bmatrix}$

The elements of the matrix A are block of order 5×5.

$[A_1] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ -d_1 & 0 & 0 & -d_1 & 0 \\ 0 & -d_1 & 0 & 0 & -d_1 \\ (a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\ (b_6)_1 & (b_2)_1 & (b_3)_1 & (b_5)_1 & (b_1)_1 \end{bmatrix}, \quad [A_j] = \begin{bmatrix} -d_j & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -d_j & 0 \\ 0 & -1 & 0 & 0 & -d_j \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ 0 & (b_8)_j & (b_3)_j & (b_5)_j & (b_1)_j \end{bmatrix}, \quad 2 \le j \le J;$

$[B_j] = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -d_j & 0 \\ 0 & 0 & 0 & 0 & -d_j \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & (b_6)_j & (b_2)_j \end{bmatrix} \quad 2 \le j \le J; \qquad [C_j] = \begin{bmatrix} -d_j & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ 0 & (b_7)_j & 0 & 0 & 0 \end{bmatrix}, \quad 1 \le j \le J-1;$

Here, $d_j = \frac{h_j}{2}$ and $\delta_1 = \begin{bmatrix} \delta v_o \\ \delta g_o \\ \delta f_1 \\ \delta v_1 \\ \delta g_1 \end{bmatrix}, \quad \delta_j = \begin{bmatrix} \delta u_{j-1} \\ \delta G_{j-1} \\ \delta f_j \\ \delta v_j \\ \delta g_j \end{bmatrix}, \quad 2 \le j \le J, \quad r_j = \begin{bmatrix} (r_1)_j \\ (r_2)_j \\ (r_3)_j \\ (r_4)_j \\ (r_5)_j \end{bmatrix}, \quad 1 \le j \le J$

The solution of equation (35) can be obtained using block elimination method, which consist of forward and backward sweeps

Forward sweep

To solve equation (35), we use LU factorization for decomposing matrix A into a product of a lower triangular matrix L and an upper triangular matrix U as follows (Adhikari and Sanyal, 2013),

$$\mathrm{A} = \mathrm{LU} \tag{36}$$

where

$$L = \begin{bmatrix} [\alpha_1] & & & & \\ [\beta_2] & [\alpha_2] & & & \\ & \ddots & \ddots & & \\ & & [\beta_{J-1}] & [\alpha_{J-1}] & \\ & & & [\beta_J] & [\alpha_J] \end{bmatrix}, \quad U = \begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I] & [\Gamma_2] & & \\ & & \ddots & \ddots & \\ & & & [I] & [\Gamma_{J-1}] \\ & & & & [I] \end{bmatrix}$$

$[I]$ is the identity matrix of order 5×5 and $[\alpha_i]$ and $[\Gamma_1]$ are 5×5 matrices which elements are determined by the following equations:

$$[\alpha_1] = [A_1][A_1][\Gamma_1] = [C_1]$$

$$[\alpha_j] = [A_j] - [B_j][\Gamma_{j-1}] \qquad j = 2,3,\dots,J \tag{37}$$

$$[\alpha_j][\Gamma_j] = [C_j] \qquad j = 2,3,\dots,J-1$$

Backward sweep

Equation (36) can be substituted into equation (35), and so we get

$$\mathrm{LU\delta} = \mathrm{r} \tag{38}$$

If we define, $\mathrm{U\delta} = \mathrm{W}$ (39)

Then equation (40) becomes

$$\mathrm{LW} = \mathrm{r} \tag{40}$$

Where

$$W = \begin{bmatrix} w_1 & w_2 & \dots & w_{j-1} & w_J \end{bmatrix}^T$$

w_j are the 5×1 column matrices. The elements W can be found by solving Eq. (40)

$$[\alpha_1][w_1] = [r_1]$$

$$[\alpha_j][w_j] = [r_j] - [\beta_j][w_{j-1}], \qquad j \le 2 \le J \tag{41}$$

Once the elements of W are found, we can find the solution of Eq. (39) using the recurrent relations

$$[\delta_J] = [w_J]$$
$$[\delta_j] = [w_j] - [\Gamma_j][\delta_{j+1}], \qquad 1 \le j \le J-1 \tag{42}$$

These calculations are repeated until convergence criterion is satisfied and calculations are stopped when

$$\left|\delta v_o^{(i)}\right| < \varepsilon$$

where ε is the desired level of accuracy. In this study, the value of $\varepsilon = 10^{-4}$.

NUMERICAL RESULTS

Table 1. Comparison for the values of $-G'(0)$ with those of (Cortell, 2007), (Zaimi *et al*., 2014) and (Ranna & Bhargava, 2012), and taking $Ec = 0$, $f(0) = 0$ and $f'(0) = 1$

Pr	n	(Cortell,2007)	(Ranna &Bhargava,2012)	(Zaimi *et al*.,2014)	Present result
	0.2	0.610262	0.6113	0.61131	0.611310.
	0.5	0.595277	0.5967	0.59667	0.596688
	1.00	-	-	-	0.583866
	1.50	0.574537	0.5768	0.57686	0.576870
1.00	2.00	-	-	0.57245	0.572456
	3.00	0.564472	0.5672	0.56719	0.567191
	4.00	-	-	0.56415	0.564156
	8.00	-	-	0.55897	0.562182
	10.00	0.554960	0.5578	0.55783	0.558974
	0.1	-	-	1.61805	1.618083
	0.2	1.607175	1.5910	1.60757	1.607599
	0.3	-	-	1.59919	1.599215
	0.4	-	-	-	1.592345
	0.5	1.586744	1.5839	1.58658	1.586607
	0.8	-	-	1.57389	1.573911
5.00	1.00	-	-	1.56787	1.567891
	1.50	1.557463	1.5496	1.55751	1.557539
	2.00	-	-	1.55093	1.550951
	2.50	-	-	1.54636	1.546386
	3.00	1.542337	1.5372	1.54271	1.543034
	10.00	1.528573	1.5260	1.52877	1.528791

Table 2. Skin friction coefficients and surface heat transfer rates for different values of some parameters

		0	0.465294	0.389629
	-0.3	0.50	0.746278	0.357697
		1.00	0.861806	0.345395
		5.00	1.062580	0.325373
		10.00	1.103957	0.321470
		0	0.628317	0.499160
0.72		0.50	0.890104	0.473009
	0.00	1.00	1.000484	0.462573
		5.00	1.194853	0.445156
		10.00	1.235218	0.441698
		0	0.818142	0.625810
	0.3	0.50	1.057832	0.605580
		1.00	1.161494	0.597250
		5.00	1.346739	0.583039
		10.00	1.385561	0.580172
		0	0.465294	0.846113
		0.50	0.746278	0.788438
	-0.3	1.00	0.861806	0.764890
		5.00	1.062581	0.724411
		10.00	1.103957	0.716153
		0	0.628317	1.961397
		0.50	0.890104	1.914727
7.00	0.00	1.00	1.000484	1.895278
		5.00	1.194853	1.861477
		10.00	1.2352184	1.854538
		0	0.818142	3.467760
	0.3	0.50	1.057832	3.437271
		1.00	1.161493	3.424323
		5.00	1.346739	3.401583
		10.00	1.385561	3.396885

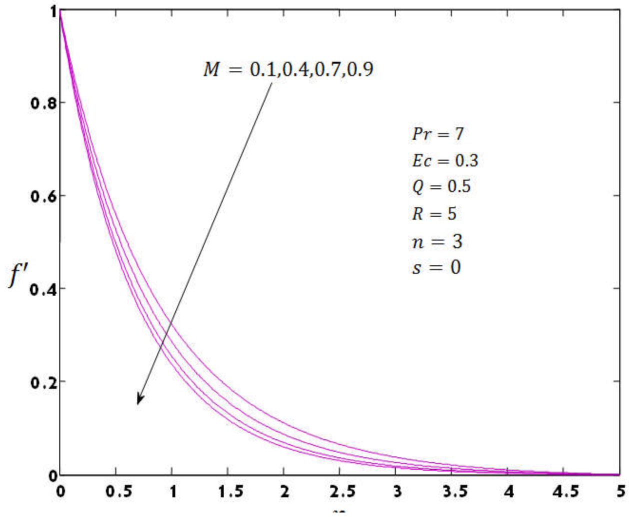

Figure 2. Effect of magnetic field parameter M on velocity profile

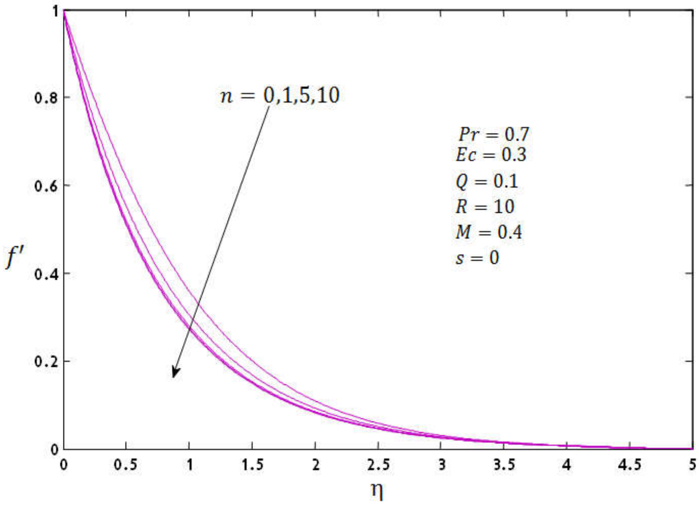

Figure 3. Effect of power index parameter n on velocity profile

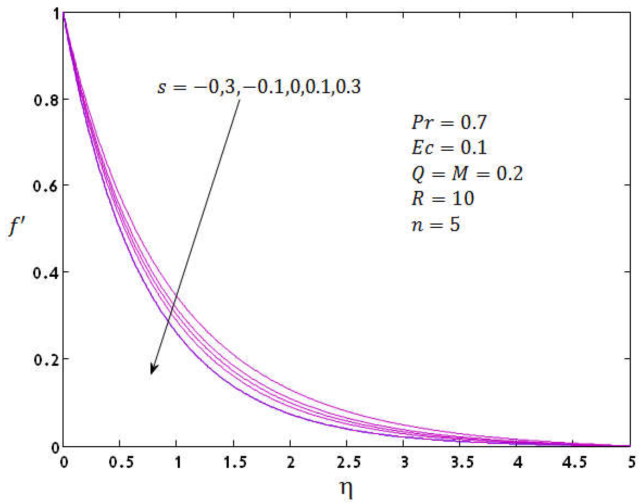

Figure 4. Effect of suction/injection parameter s on velocity of the fluid

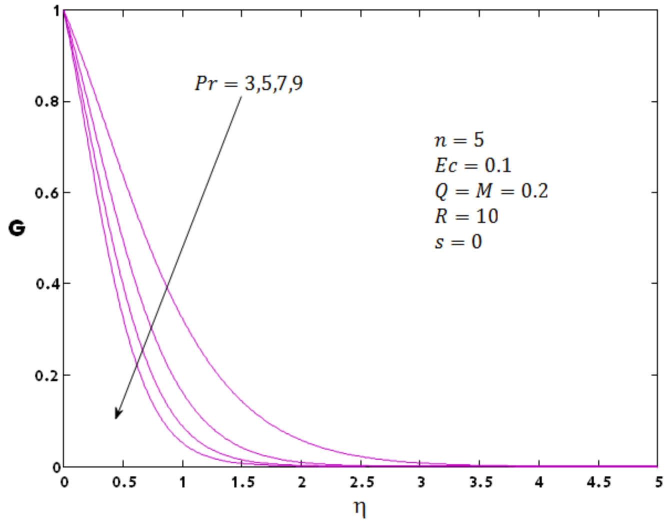

Figure 5. Effect of Prandtl number Pr on temperature

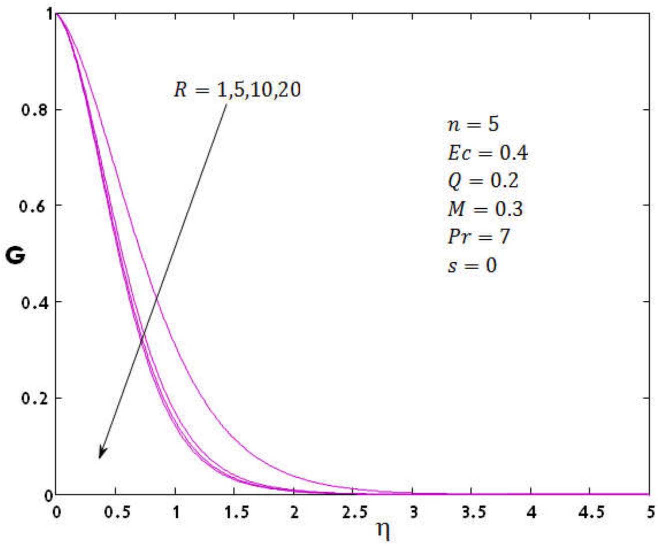

Figure 6. The effect of radiation parameter R on temperature of the fluid

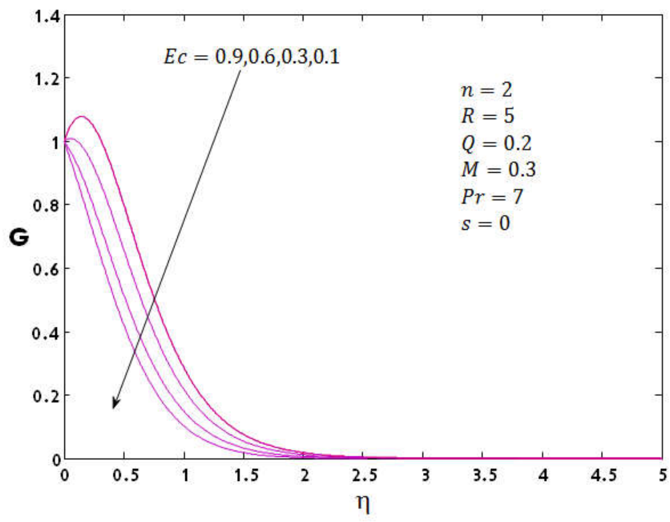

Figure 7. Effect of Eckert number Ec on temperature of the fluid

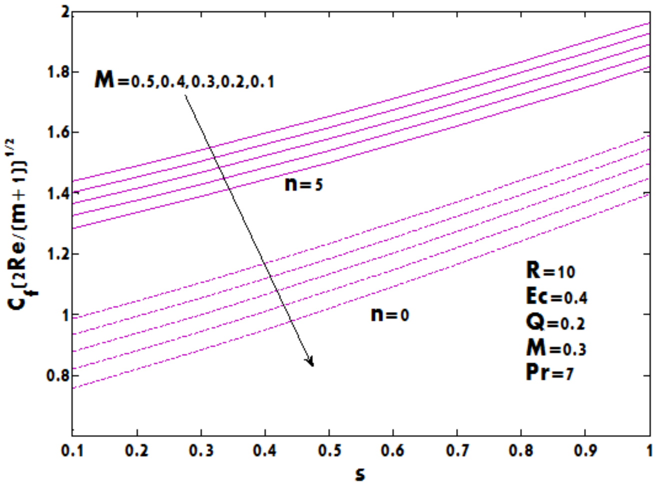

Figure 8. Effects of magnetic field parameter M, power index parameter n and suction/injection parameter s on skin friction coefficient

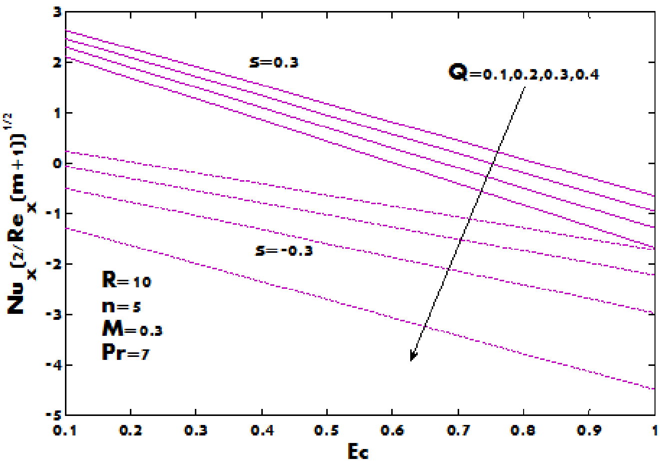

Figure 9. Effects of heat source/sink parameter Q, Eckert number Ec and suction/injection parameter s on surface heat transfer rate

DISCUSSION

The system of nonlinear higher order ordinary differential equations (14) and (15) with the boundary conditions (16) were solved numerically using Keller box method by taking the step size $\Delta\eta = 0.01$ in η and within the interval $[0, \eta_\infty]$, where η_∞ is the boundary layer thickness. The effect of different parameters like power index parameter, magnetic field parameter, Prandtl number, thermal radiation, Eckert number and suction/injection parameter on velocity and temperature profiles, skin friction coefficient and surface heat transfer rate have been analyzed. To verify the accuracy of the present results, comparison has been made with the previous results of (Cortell, 2007), (Zaimi *et al.*, 2014) and (Ranna and Bhargava, 2012) in Table 1. For instance, Figure 2, Figure 3 and Figure 4 shows the effect of magnetic field parameter M, power index parameter n and suction/injection parameter s on velocity profile respectively, while the other parameters are constant. We observed from the figures that increasing magnetic field parameter, power index parameter and suction/injection parameter decreases the velocity of the fluid. Figure 5 and 6 show the effect of Prandtl number Pr and radiation parameter R on temperature profile respectively. The figures reveal that increasing Prandtl number and radiation parameter decreases the temperature of the fluid. Figure 7 indicates the effect of Eckert number Ec on temperature profile. The figure shows that increasing Eckert number increases temperature of the fluid. Figure 8 displays the effects of magnetic field parameter M, power index parameter n and suction/injection parameter s on skin friction coefficient. As we observed from the figure increasing in the magnetic field parameter M, power index parameter n and suction/injection parameter s enhances an increase in skin friction coefficient. Figure 9 depicts the effects of heat source/sink parameter Q, Eckert number Ec and suction/injection parameter s on surface heat transfer rate. The figure reveals that increasing heat source/sink parameter Q and Eckert number E_c decreases surface heat transfer rate. The figure also shows surface heat transfer rate is higher in suction than injection. Table 1 shows, for values of Prandtl number $Pr = 1$ and $= 5$, the values of surface heat transfer rate $-G'(0)$, are found in excellent agreement with previously reported from literature. Table 2 demonstrates the influence of various parameters on skin friction coefficients and surface heat transfer rate. It is evident from the table that with increasing the Prandtl number, power index parameter and suction/injection parameter increases skin friction coefficients and reduces surface heat transfer rate with increasing power index parameter.

Conclusion

In this study, hydromagnetic incompressible laminar flow over a nonlinear stretching sheet is considered and the governing nonlinear partial differential equations are transformed into higher order nonlinear ordinary differential equations and solved numerically by Keller box method. The effects of various governing parameters on velocity and temperature profiles, skin friction coefficient and surface heat transfer rate were analyzed. Briefly the above discussion can be summarized as follows:

- A decrease of surface heat transfer rate results from increasing heat source/sink parameter Q and Eckert number Ec, and surface heat transfer rate is higher in suction than injection.
- The skin friction coefficient increases with increasing magnetic field parameter$M$, power index parameter n and suction/injection parameter s and it decreases the velocity of the fluid.
- The velocity of the fluid increases with decreasing magnetic field parameter M, power index parameter n and suction/injection parameter s.
- The temperature of the fluid increases with increasing Eckert number Ec and decreasing radiation parameter R and Prandtl number Pr.

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