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RESEARCH ARTICLE

ON $\beta^* g^*$-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study $\beta^* g^*$-closed sets and $\beta^* g^* p$-closed sets, $\beta^* g^* s$-closed sets in Topological spaces.

INTRODUCTION

In 1970, Levine [6] first considered the conceot of generalized closed (briefly, g-closed) sets were defined and investigated. Arya and Nour[2] defined generalized semi open (briefly, gs-open) sets using semi open sets. Veerakumar[11], S. Yuksel and Becern [12], A.Acikgoz[1] introduced g^*-closed set, β^*- sets and $\beta^* g$- closed sets respectively. We introduced a new class of sets $\beta^* g^*$-closed sets and study their simple properties.

Throughout this paper (X,τ), (Y,σ) and (Z,η) $(or\ X, Y, Z)$ represents topological spaces on which no seperaxion axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and A^c $(or\ X - A)$ denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition: 1.1 A subset A of a topological space (X,τ) is called:

- pre open [8] $A \subseteq int\big(cl(A)\big)$,
- semi open [5] $A \subseteq cl\big(int(A)\big)$,

The family of all preopen sets (resp. semi open sets) in X will be denoted by $po(X)$ (resp. $so(X)$). A semi closure (resp. pre closure) of a subset A of X denoted by $scl(A)$ (resp. $pcl(A)$) is defined to be the intersection of all semi closed (resp. pre closed) sets containing A. A semi interior (resp. pre interior) of a subset X denoted by $s\ int(A)$ (resp. $p\ int(A)$) is defined to the union of all semi open (resp. pre open) sets contained in A.

Definition: 1.2 A subset A of a topological space (X,τ) is called:

- a generalized closed set (briefly g-closed) [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ),
- a g^*- closed [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in (X,τ),

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- a gp-closed [7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ),
- a gs -closed[2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).

The complements of above sets are called their respective open sets.

Definition:1.3 A subset A of a space (X, τ) is called a β^*-set [12] if$A = U \cap V$, where U is open and $int(V) = cl(int(V))$.
Definition:1.4 A subset A of a space (X, τ) is called a $\beta^* g$-closed set[1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is β^*-set in X.

2. $\beta^* g^*$-closed set

Definition 2.1. A subset A of a space (X, τ) is called $\beta^* g^*$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\beta^* g$-open in X.

Definition 2.2. A subset A of a space (X, τ) is called $\beta^* g^* s$-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\beta^* g$-open in X.

Definition 2.3. A subset A of a space (X, τ) is called $\beta^* g^* p$-closed set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\beta^* g$-open in X.

Theorem 2.4. Let (X, τ) be a topological space. Then we have

- Every closed set is a $\beta^* g^*$-closed set.
- Every $\beta^* g^*$-closed set is a g -closed set.

Proof

- Let A be a closed set in (X, τ) and U be a $\beta^* g$-open set such that $A \subseteq U$. Since A is closed, cl(A) = A, So $cl(A) \subseteq U$. Hence A is $\beta^* g^*$-closed set in (X, τ).
- Let A be a $\beta^* g^*$-closed set in (X, τ) and $A \subseteq U$ where U is $\beta^* g$-open set. Since every open set is a $\beta^* g$-open set, So U is an open set of (X, τ). Since A is a $\beta^* g^*$-closed set, we obtain that $cl(A) \subseteq U$, hence A is a g-closed set of (X, τ).

Remark 2.5. The converse of the above theorem need not be true as seen from the following examples.

Example 2.6. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$. Then the subset $A = \{b, c, d\}$ is a $\beta^* g^*$-closed set, but it is not a closed set.

Example 2.7. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{c\}, X\}$. Then subset $A = \{a\}$ is a g-closed set, but it is not a $\beta^* g^*$-- closed set.

Theorem 2.8. Let (X, τ) be a topological space. Then we have

- Every $\beta^* g^*$- closed set is a $\beta^* g^* p$ - closed set
- Every $\beta^* g^*$- closed set is a $\beta^* g^* s$ - closed set

Proof: (i) Assume that A is a β^* g*- closed set in (X, τ) and $A \subseteq U$ where U is a β^* g- open set. We have $pcl(A) \subseteq cl(A) \subseteq U$. Therefore $pcl(A) \subseteq U$. Hence A is a β^* g* p - closed set in (X, τ)

(ii) Assume that A is a $\beta^* g^*$-closed set in (X, τ) and $A \subseteq U$ where U is a $\beta^* g^*$- ope set. We have $scl(A) \subseteq cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$. Hence A is a β^* g* s - closed set in (X, τ).

Example 2.9. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{b\}$ is a $\beta^* g^* p$-closed set, but it is not a $\beta^* g^*$-closed set.

Example 2.10. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{b, c\}$ is a $\beta^* g^* s$-closed set, but it is not a $\beta^* g^*$-closed set.

Theorem 2.11. Let (X, τ) be a topological space. Then we have

- Every $\beta^* g^* p$- closed set is a gp-closed set.
- Every $\beta^* g^* s$-closed set is a gs-closed set.

Proof. (i) Assume that A is a β^*g^*p - closed set of (X , τ). Let $A \subseteq U$ where U is a β^* g- open set. Since every open set is a β^*g^*-open set. Since A is a β^*g^*p -closed set, Therefore $pcl(A) \subseteq U$. Hence A is a gp-closed set of (X, τ).

(ii) Assume that A is a β^*g^*s - closed set of (X , τ). Let $A \subseteq U$ where U is a β^*g^*- open set. Since every open set is a β^*g^* −open set. Since A is a β^*g^*s -closed set, Therefore $scl(A) \subseteq U$. Hence A is a gs-closed set of (X, τ).

Remark 2.12. The converse of the above theorem need not be true as seen from the following examples.

Example 2.13. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, X\}$. Then the subset $A = \{b, d\}$ is a gp-closed set, but it is not a β^*g^*p -closed set.

Example 2.14. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}X\}$. Then the subset $A = \{c\}$ is a gs-closed set, but it is not a β^*g^*s -closed set.

Theorem 2.15. Let (X , τ) be a topological space. Then we have

- Every β^*g^* −closed set is a gp-closed set
- Every β^*g^* −closed set is a gs-closed set

Proof. (i) Assume that A is a β^*g^*- closed set of (X , τ). Let $A \subseteq U$ where U is a β^*g^*- open set. Since every open set is a β^*g^*–open, we have $pcl(A) \subseteq U$. Hence A is a gp -closed set of (X , τ).

(ii) Assume that A is a β^*g^* −closed set of (X , τ). Let $A \subseteq U$ where U is a β^*g^*- open set. Since every open set is a β^*g^*–open, we have $scl(A) \subseteq U$. Hence A is a gs -closed set of (X , τ).

Remark 2.16. The converse of the above theorem need not be true as seen from the following examples.

Example 2.17. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{b, c, d\}, X\}$. Then the subset $A = \{d\}$ is a gp -closed set, but it is not a β^*g^*-closed set.

Example 2.18. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{b, c, d\}, X\}$. Then the subset $A = \{c\}$ is a gs -closed set, but it is not a β^*g^*-closed set.

Remark 2.19. A β^* - set is independent from β^*g^*-closed set as it can be seen from the next two examples.

Example 2.20. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{a\}$ is a β^*-set, but it is not a β^*g^* -closed set.

Example 2.21. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{a, b, d\}$ is a β^*g^*-closed set, but it is not a β^* -set.

Theorem 2.22. If A and B are β^*g^*-closed, then $A \cup B$ is a β^*g^*-closed set.

Proof. Let A and B are β^*g^*-closed sets in X. Let U be β^*g-open set in X such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are β^*g-closed sets. $cl(\mathrm{A}) \subseteq U$ and $cl(\mathrm{B}) \subseteq U$. Hence $cl(A \cup B) = cl(A) \cup cl(B) \subseteq U$. Therefore $A \cup B$ is β^*g^*-closed set whenever A and B are β^*g^*-closed set.

Remark 2.23. The finite intersection of two β^*g^*-closed sets need not be β^*g^*-closed set.

Example 2.24. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the subset $A = \{a, b, c\}$ and $\{a, b, d\}$ are β^*g^*-closed sets, but $\{a, b, c\} \cap \{a, b, d\} = \{a, b\}$ is not a β^*g^*-closed set.

Theorem 2.25. If $A \subseteq B \subseteq cl(A)$ and A is a β^*g^*-closed subset of (X , τ), then B is also a β^*g^*-closed subset of (X , τ).

Proof. Let U be a β^*g-open subset, such that $A \subseteq B \subseteq U$,Since A is β^*g^*-closed subset of (X , τ). $cl(A) \subseteq U$, by hypothesis $A \subseteq B \subseteq cl(A)$, $cl(\mathrm{A}) = \mathrm{cl(B)}$. Hence $cl(\mathrm{B}) \subseteq \mathrm{U}$ whenever $\mathrm{B} \subseteq \mathrm{U}$, Therefore B is β^*g^*-closed subset of (X , τ).

Theorem 2.26. For any topological space (X , τ), every singleton $\{x\}$ of X is a β^*g-open set.

Proof. Let $x \in X$. Let $\{x\} \in \tau$, then $\{x\}$ is a β^*g-open set. If $\{x\} \notin \tau$, then $int(\{x\}) = \emptyset = cl(int(\{x\}))$, so $\{x\}$ is a β^*g-open set.

Theorem 2.27. A subset A of X is β^*g^*-closed set in X if and only if $cl(\mathrm{A}) - \mathrm{A}$ Contains no nonempty β^*g-closed set in X.

Proof:. Suppose that F is a nonempty β^*g-closed subset of $cl(\mathrm{A}) - \mathrm{A}$. Now $F \subseteq cl(\mathrm{A}) - \mathrm{A}$. $F \subseteq cl(\mathrm{A}) \cap \mathrm{A}^c$. Therefore $F \subseteq cl(\mathrm{A})$ and $F \subseteq \mathrm{A}^c$. Since F^c is β^*g-open such that $A \subseteq F^c$ and A is β^*g^*-closed, $cl(A) \subseteq F^c$, ie $F \subseteq cl(A)^c$. Hence $F \subseteq cl(A) \cap [\mathrm{cl(A)}]^c = \phi$. Ie, $F = \phi$. Thus $cl(\mathrm{A}) - \mathrm{A}$ contains no nonempty β^*g^*-closed set.

Conversely, Assume that $cl(\mathrm{A}) - \mathrm{A}$ Contains no nonempty β^*g-closed set. Let $A \subseteq U$, U is β^*g-open. Suppose that $cl(\mathrm{A})$ is not contained in U. Then $cl(\mathrm{A}) \cap \mathrm{U}^c$ is a nonempty β^*g-closed set and contained $cl(\mathrm{A}) - \mathrm{A}$ which is contradiction. Therefore $cl(\mathrm{A}) \subseteq \mathrm{U}$ and hence A is β^*g-closed set.

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