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REVIEW ARTICLE

STUDY β- $T_{1/2}$ SPACE

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ABSTRACT

In this paper we study β -In this paper we study β - $T_{1/2}$ spaces via the concept of generalized closed sets of type β and also the relationship of tand also the relationship of thisspace to both β – T_0 andβ- T_1.It is showed that the space is located betweenβ - $T_{1/2}$is located between the space β – T_0 and space β - T_1

INTRODUCTION

The notions ofβ-open set and β-closed sets play important role in studding many of topological spaces. These notions are introduced and studied by Abd-Almonsef (Miguel Caldas *et at.,* 2011). Also, they have studied another types of open sets in topological spaces such as semi β-open sets and β-generalized closed set.After that many authors have studied his class of sets by defining their neighborhoods, separation axioms, compactness and functions. The concept of g-closed sets in topological spaces was introduced in 1970 by Levine (Abdel Monsef *et al.,* 2005), a subset A of (X,τ) to be g- closed set; cl(B)⊆U, whenever B⊆U and U is open set. After the work of Levine on g- closed sets, various mathematicians turned their attention to the generalizations of various concepts in topology. Later, in 1994, Maki, Devi and Balachandran (Abdel Monsef *et al.,* 1985) generalized the concept of g-closed sets to β-generalized closed sets By definition a subset of A of (X,τ) issaid to be β - generalized closed set *iff*β *cl* (B) $\subseteq U$; $B \subseteq U$ *and U is open set.* By using this concept we study and discuss a new topological space which is called β _ $T_{1/2}$space. A space (X,τ) is β _ $T_{1/2}$ space if every β-generalized closed subset of (X,τ) is β- closed set. And we have clarified the relation between β _ $T_{1/2}$space,β _ T_0space (Abdel Monsef *et al.,* 1986), and β _ T_1space (Abdel Monsef *et al.,* 1986).

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Definitions and concepts

In this section we recall some definitions and results, which will be used in this sequel. For detail we refer to e.g. (Caldas, 2003; Jafari, 2001; Takashi, 2001).Throughout this paper, the sets $XandY$ are topological spaces with no separation properties assumed unless explicitly stated. All sets are considered to be subsets of topological spaces. The closure and interior of a set A are denoted by $cl(A)$ and $int(A)$, respectively.

Definitions 1

Let X be a topological space. A subset B of X is called:

- β- open set (Abdel Monsef, 1987) if B ⊆cl(int(cl(B))).
- 2 - β- closed set (Abdel Monsef, 1987) ifint(cl(int(B))) ⊆B.
- 3 - The intersection of all β- closed set containing a subset B of X is called β-closure of B and denoted by βcl (B).
- 4-β- interior of a subset B of X is the largest β- open set contained in B, and denoted by β-int (B). We denote the family of β-open sets of (X,τ) by βO (X), and denote the family of all β- closed set of (X,τ) byβC (X)
- 5-semi-β-open set (Reilly, 2001) if there exists β-open subset U of X such that, $U \subset A \subset clU$. The family of all semi-β-open subsets of X is denoted by$\beta SO(X)$, the complement of every semi-β-open set is called, semi-

β-closed subset of X(Reilly, 2001) and denoted by $\beta SC(X)$.

- 6- semi-open set (Levine, 2010) if $A \subset cl(int(A))$. The set of all semi-open sets isdenoted by $SO(X)$. The complement of every semi-open set is called semi-closed subset of X(Ganster and Steiner, 2000).
- 7- pre-open set (Beceren and Noiri, 2008) if$A \subset int(cl(A))$. The set of all pre-open sets is denoted by $PO(X)$. The complement of every pre-open set is called, pre-closed subset of$X$.
- 8- generalized closed set(Levine, 1970) if$cl(A) \subset U$: $U \in \tau, A \subset U$and denoted by,g-closed set. The complement of every g-closed set is called g-open set.
- 9- generalized β-closed set (Reilly, Ivan, 2001) if $\beta cl(A) \subset U$: $U \in \beta O(X), A \subset U$and denote by g$\beta$-closed set. The complement of every gβ-closed set is called βg-open.
- 10- regular open set (Maki *et al.,* 1994) t if $A = int(cl(A))$. The set of all regular open sets is denoted by $RO(X)$.
- 11-β-regular open set (Maki *et al.,* 1994) if A is β-open set and β-closed set in the sametime. The family of all β-regular open sets is denoted by $\beta RO(X)$.

Definitions 2

- A space $(X, \tau)$is said to be β_ T_0 (Abdel Monsef *et al,* 1986) if, for x, $y \in X$, $x \neq y$, there exists
- *β*- open set containing (*xbut not y*) or (*y but not x*) in this the space.
- **2-** A space (X,τ) is said to beβ _ T_1 (Abdel Monsef *et al,* 1986) if, for x, $y \in X$, $x \neq y$, there existsU_1, U_2 are β - open sets such that ($x \in U_1$ *and* $y \notin U_1$) or ($y \in U_2$ *and* $x \notin U_2$in this space.
- 3-- A space $(X, \tau)$is said to beβ _ T_1 (Levine, 1970) if every β -generalized closed set is
- β - closed set in this the space.

RESULTS AND DISCUSSION

Proposition 1

Every closed subset of a topological space (X,τ) is g – closed. (The converse is not true).

Proof

Let $A \subseteq X$ be closed set, and let $A \subseteq U$, where U is open set, since A is closed setthen$(cl)A=A$, hence $cl(A) \subseteq U$, i.e. A is g - closed.

Example 1

Let $X=\{a, b, c\}$, $\tau=\{X,\{a\},\{c\}, \{a,c\}\}$, so,
$\tau^c =\{\varphi, X,\{b,c\},\{a,b\},\{b\}\}$
let $A = \{a\}$, $U = \{a,b,c\}$ open set ,
Now, since $clA \subseteq \{a,b\} \subseteq U$, i.e. $A = \{a\}$is g - closed set,
but it is not closed set .

Proposition 2

Every g – closed subset of a topological space (X,τ) is g – closed. (The converse is not true)

Proof

Let $A \subseteq X$ be g -closed set, and let $A \subseteq U$, where U is open set, since A is βg - closed set, then, $cl(A) \subseteq U$, and henceint $(cl(A)) \subseteq$ int(U), but U is open set so, int($cl(A) \subseteq U$.

Since β cl (A) is the smallestβ- closed set containing A, so, βcl (A) =$A \cup$int(cl (int (A))) $\subseteq A \cup cl(U) \subseteq U$, i.e. A is βg–closed

Example 2

Let$X = \{a,b, c\}$,

$\tau =\{ X,\varphi,\{ a\} ,\{c\} ,\{a,c\}\}$, so, $\tau^c=\{X,\varphi ,\{b, c\},\{a,b\},\{b\}\}$

let $A = \{ c \}$, $U = X$ open set.

Now, since $cl(A)= \{b,c\} \subseteq U$, and int$(cl(A))= \{c\}$,
cl (int(cl (A)))= $\{b,c\} \subseteq U$ i.e.

$A = \{c\}$is βg -closed set, but it is not g –closedset, Since if we take$U = \{a, c\}$, $cl(A) =\{b, c\} \not\subset U$.

Proposition 3

Every closed subset of a topological space (X,τ) is β - closed (The converse is not True)

Proof

Let $A \subseteq X$ be closed set, then $clA = A$, hence int(cl(A))= int(A), but int(A) $\subseteq A$, so
Int(cl(A))$\subseteq A$, and int(cl(int(A)))$\subseteq cl$(A). Then
Int(cl(int(A)))$\subseteq A$,
i.e. A is β –closed.

Example 3

Let A = { a, b, c ,d },$\tau = \{ X,\varphi,\{ a \},\{c \},\{ a,c \}, \{ a, b ,d \} \}$,
S o $\tau^c = \{ X,\varphi$,{, b, c ,d }, { a, b,d } ,{ b, d, } ,{ c} }

A= {a, c} Now, since,int$(A)= \{a,c\} \subseteq U$, and cl(int((A))= $c\}$,
int(cl(int(A)))= $\{c\} \subseteq A$, i.e. $A = \{a,c\}$is β - closed set, but it is not closed set.

Theorem 1

For a space (X, τ), the following are equivalent:

1. (X,τ) isβ _ $T_{1/2}$.
2. For each singleton $\{x\}$ of X, $\{x\}$ is *β*- open set or *β*- closed set

Theorem 2

For a space (X,τ) the following are equivalent:

1. (X,τ) isβ _ $T_{1/2}$.
2. Every subset of X is the intersection of all *β*- open sets and all *β*- closed sets containing it.

Proof

(1)⟹(2), if (X,τ) isβ _ $T_{1/2}$. With $B \subseteq X$, therefore by

Theorem1 then for each singleton $\{x\}$ of X, $\{x\}$ is β- open set or β- closed set.

$B = \cap \{X \setminus \{x\}; x \notin B\}$ is the intersection of all β- open sets and all closed sets containing it.

(2) ⟹(1),for each $x \in X$, *then* $X \setminus \{x\}$ is the intersection of all β- open sets and all β- closed sets containing it, hence $X \setminus \{x\}$ is either β- open set or β– closed Set, therefore by Theorem 1 (X,τ) is β _ $T_{1/2}$

Lemma 1

For a space (X,τ) the following are equivalent:

1. Every subset of X is β- generalized closed set.
2. $\beta O(X) = \beta C(X)$.

Proof

1) ⟹(2),Let is $U \in \beta O(X)$,Then by hypothesis, U is β- generalized closed set Which implies that $\beta cl(U) \subset U$, so, $\beta cl(U) = U$, therefore $U \in \beta C(X)$

$\Rightarrow \beta O(X) \subseteq \beta C(X)$ …………(1)

Let $V \in \beta C(X) \Rightarrow X \setminus V \in \beta O(X)$, Then by hypothesis $X \setminus V$ is β-generalized closed set, and then $X \setminus V \in \beta C(X) \Rightarrow V \in \beta O(X)$,

$\Rightarrow \beta C(X) \subseteq \beta O(X)$ …………(2)

From (1) and (2) $\beta O(X) = \beta C(X)$.

(2) ⟹(1), If B is subset of X such that $B \subseteq U$ where $U \in \beta O(X)$

Then $U \in \beta C(X) \Rightarrow \beta cl(U) = U$

Now, $B \subseteq U \Rightarrow \beta cl(B) \subseteq \beta cl(U) = U$

$\Rightarrow \beta cl(B) \subseteq U$

⟹ B is β- generalized closed set.

Proposition 4

The property of being a β _ $T_{1/2}$space is hereditary.

Proof

If Y is a subspace of β _ $T_{1/2}$space X, and $y \in Y \subseteq X$, then $\{y\}$ is β-open set or β- closed set in X (by Theorem 5.1). Therefore $\{y\}$ is either β- open set or β- closed set in Y. Hence Y is a β _ $T_{1/2}$ space.

Theorem 3

A space X is β -T_1 space if and only if $\{ x \}$ is β- closed$\forall x \in X$.

Proof

Let X be β-T_1 space.

Let $p \in X$, to prove $\{p\}$ is β- closed set.

$x \in \{p\}^c = X \setminus \{p\} \Rightarrow x \neq p$ in X,

Hence there exists an β - open set G such that $x \in G$, $p \notin G$ or $x \notin G$, $p \in G$.

If $x \in G$, $p \notin G \Rightarrow x \in G \subseteq \{p\}^c \Rightarrow \{p\}^c$ isan β -open set

$\Rightarrow \{p\}$ is β- closed set .

Let $\{p\}$ be an β -closed set, $\forall p \in X$, to prove $X$is β-T_1 space. Let $x \neq y$in X,

Hence $\{ x \}$,$\{ y \}$ are β -closed sets $\Rightarrow \{ x \}^c$, $\{ y \}^c$areβ -open sets *and* $y \in \{x\}^c$, $x \notin \{x\}^c$, $x \in \{y\}^c$, $y \notin \{ y \}^c$

Therefore X is β -T_1 space.

Theorem 4

Every β -T_1 is β – $T_{1/2}$space. (The converse is not true).

Proof

Since X is β -$T_1$by using theorem3 then $\{x\}$ isβ- closed set, $\forall x \in X$.

And by using theorem 1 we will get X is β – $T_{1/2}$space

Example 4

Let $X = \{a,b, c\}$,$\tau = \{\varphi, X,\{a\}\}$
$\beta O(X) = \{\varphi, X,\{a\},\{a,b\},\{a,c\}\}$, $\beta c(X) = \{\varphi X,\{b,c\},\{c\},\{b\}\}$

Then (X,τ) is β – $T_{1/2}$but is not β –T_1space.

Theorem 5

Every is β – $T_{1/2}$is β –T_0space (The converse is not true).

Proof

Let $x, y \in X$; $x \neq y$

Since X is β – $T_{1/2}$space, by using theorem 5.1 then $\{x\}$ is either β- open set orβ- closed set, $\forall x \in X$.

(1) If $\{x\}$ isβ- open set, $\forall x \in X$.

Since $x \neq y$, therefore $x \in \{x\}$ and $y \notin \{x\}$
$\Rightarrow X$ is β –T_0space

(2) If $\{x\}$ isβ - closed set, $\forall x \in X$. then $X \setminus \{x\}$ is β - open set,

Therefore $x \notin X \setminus \{x\}$ and $y \in X \setminus \{x\}$
$\Rightarrow X$ is β –T_0space.

Example 5

Let X = {1,2,3}, $\tau(X) = \{\varphi, X ,\{1\},\{1,2\}\}$

$\beta O(X) = \tau(X)$

$\beta cl(X) = \{\varphi, X ,\{2,3\},\{3\}\}$is β –$T_0$space but is not β – $T_{1/2}$ space, because {2} is not β - open set and is not β - closed set .

Conclusions and Recommendations

In this paper we development new concepts in general topology where new spaces Topologically different from the known spaces have emerged. We obtained a relationship between β – $T_{1/2}$space and β –$T_0$and β –T_1 space. We also discovered that the property of being a β – $T_{1/2}$space is hereditary. The study of properties of space β – $T_{1/2}$and the possibility of finding spaces other than the space β – $T_{1/2}$located between β –$T_0$and β –T_1remains an issue and need to be resolved in the future.

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