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RESEARCH ARTICLE

REMARKS ON STRONGLY AND COMPLETELY gs_α**-IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce and investigate new classes of irresolute functions called completely gs_α**-irresolute functions and strongly gs_α**-irresolute functions in topological spaces via gs_α**-closed sets and obtain some of their characterizations. Moreover we examine the relationships of these functions with the other existing functions.

INTRODUCTION

In 1972, Crossley and Hildebrand (Crossley and Hildebrand, 1972) introduced the notion of irresoluteness. Irresolute functions give new path towards research. Many different forms of irresolute functions have been introduced over the course of years whose importance is significant in various branches of Mathematics and related sciences (Erdal Ekici and Saeid Jafari, 2008; Navalagi and Abdul-Jabbar, 2006). In 1974, Arya and Gupta (Arya and Gupta, 1974) introduced the notion of completely continuous functions. The purpose of the present paper is to introduce the concept of completely gs_α**-irresolute functions and strongly gs_α**-irresolute functions via gs_α**-closed sets introduced by Santhini et al. (2017) in topological spaces. Also we investigate their relationships along with their basic properties.

Preliminaries

In this section, we recall some basic definitions and properties used in our paper.

Definition: 2.1. A subset A of a topological space (X,τ) is said to be semi*α-open[10] if $A \subseteq cl(\alpha intA)$

Definition: 2.2. A subset A of a space (X,τ) is called gs_α**-closed set [Santhini and Lakshmi Priya, 2017] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi*α-open in (X,τ).

Definition: 2.3. A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called

(i) Contra-continuous [Dontchev, 1996] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).
(ii) Completely continuous [Arya and Gupta, 1974] if $f^{-1}(V)$ is regular open in (X,τ) for every open set V of (Y,σ).
(iii) R-map [Erdal Ekici and Saeid Jafari, 2008] if $f^{-1}(V)$ is regular open in (X,τ) for every regular open set V of (Y,σ).

Definition: 2.4. A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called pre-semi*α-open (Robert and Pious Missier, 2014) if f(U) is semi*α-open in (Y,σ) for every semi*α-open set U in (X,τ).

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Definition: 2.5. A function f: (X,τ) → (Y,σ) is called semi-closed [Devi et al., 1995] if $f(F)$ is semi-closed in (Y, σ) for any closed set F in (X, τ).

Definition: 2.6. A function f: (X,τ) → (Y,σ) is called a gs_α**-continuous [Santhini and Lakshmi Priya, 2017] if $f^{-1}(V)$ is gs_α**-closed in (X,τ) for every closed set V in (Y,σ).

Definition: 2.7. A function f: X → Y is said to be contra gs_α**-continuous [Santhini and Lakshmi Priya, 2017] if $f^{-1}(V)$ is gs_α**-closed in X for every open set V in Y.

Definition: 2.8. A function f: (X,τ) → (Y,σ) is called a gs_α**-irresolute[Santhini and Lakshmi Priya, 2017] if $f^{-1}(V)$ is gs_α**-closed set in (X,τ) for every gs_α**-closed set V in (Y,σ).

Definition: 2.9. A function f: (X,τ) → (Y,σ) is said to be

(i) Irresolute [Crossley and Hildebrand, 1972] if $f^{-1}(V)$ is semi-closed in (X,τ) for every semi-closed V of (Y,σ).
(ii) ω-irresolute [Veera Kumar, 1999] if f^{-1} (V) is ω-closed in (X, τ) for each ω- closed V of (Y,σ).
(iii) gs-irresolute [Devi et al., 1993] if f^{-1} (V) is gs-closed in (X,τ) for every gs-closed V of (Y,σ).
(iv) gsp-irresolute [Sheik John, 2002] if f^{-1} (V) is gsp-closed in (X,τ) for every gsp-closed V of (Y,σ).
(v) g*s-irresolute [Pushpalatha, 2011] if f^{-1} (V) is g*s-closed in (X,τ) for every g*s-closed V of (Y,σ).
(vi) semi*-irresolute [Pious Missier and Robert, 2014] if f^{-1} (V) is semi*-closed set in (X,τ) for every semi*-closed V of (Y,σ).
(vii) semi*α-irresolute [Robert and Pious Missier, 2014] if f^{-1} (V) is semi*α -closed set in (X,τ) for every semi*α-closed V of (Y,σ).

Definition: 2.10.

(i) A space X is locally indiscrete [Willard, 1970] if every open set in X is closed.
(ii) A space X is called locally gs_α**-indiscrete [Santhini and Lakshmi Priya, 2017] if every gs_α**-open set is closed in X.

Definition: 2.11.

(i) A space (X,τ) is called a ${}_{\alpha}T_{s**}$-space[13] if every gs_α**-closed set in it is closed.
(ii) A space (X,τ) is called a ${}^{\alpha}T_{s**}$-space[13] if every gs-closed set in it is gs_α**-closed.

Definition: 2.12. [Navalagi and Abdul-Jabbar, 2006]

(i) r-T_1 if for every pair of distinct points x and y in X there exists an r-open sets G and H containing x and y respectively such that x ∉ H and y ∉ G.
(ii) r-T_2 if for every pair of distinct points x,y in X there exists disjoint gs_α**-open sets U and V containing x and y respectively.

Definition: 2.13. [Santhini and Lakshmi Priya, 2017]

(i) gs_α**-T_1 if for every pair of distinct points x,y in X there exists a gs_α**-open set U containing x not y and gs_α**-open set V containing y but not x.
(ii) gs_α**-T_2 if for every pair of distinct points x,y in X there exists disjoint gs_α**-open sets U and V containing x and y respectively.

*3 Strongly gs_α**-irresolute functions*

In this section, strongly gs_α**-irresolute functions are introduced and investigated.

Definition: 3.1. A function f: (X,τ) → (Y,σ) is called strongly gs_α**-irresolute if $f^{-1}(V)$ is closed in (X,τ) for every gs_α**-closed set V in (Y,σ).

Example: 3.2. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and σ = {φ, Y, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = f(d) = c is strongly gs_α**-irresolute.

Theorem: 3.3. (i) Every strongly gs_α**-irresolute function is gs_α**-irresolute.

(ii) Every strongly gs_α**-irresolute function is g*s-irresolute.
(iii) Every strongly gs_α**-irresolute function is semi*-irresolute.
(iv) Every strongly gs_α**-irresolute function is irresolute.
(v) Every strongly gs_α**-irresolute function is continuous.
(vi) Every strongly gs_α**-irresolute function is ω-irresolute.

Proof

(i) Let V be a gs_α**-closed set in Y. Since f is strongly gs_α**-irresolute, $f^{-1}(V)$ is closed set in X. By theorem 3.2[13], $f^{-1}(V)$ is gs_α**-closed in X and so f is gs_α**-irresolute.
(ii) → (vi). Similar to the proof of (i).

Remark: 3.4. The converses of the above theorem are need not true as seen from the following examples.

Example: 3.5. Let X = Y = {a,b,c} , τ = {φ,X, {a}, {b}, {a, b}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = c, f(b) = a, f(c) = b is gs_α**-irresolute but not strongly gs_α** -irresolute .

Example: 3.6. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {a, b}, {c, d}, {a, c, d}} and σ = {φ, Y, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = f(d) = c is g*s-irresolute but not strongly gs_α**-irresolute.

Example: 3.7. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}, {a, b, c}} and σ = {φ, Y, {a}, {b}, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = f(b) = a,f(c) = b, f(d) = c is semi*-irresolute but not strongly gs_α**-irresolute.

Example: 3.8. Let X = Y = {a,b,c,d} , τ = {φ,X, {c}, {a, d}, {a, c, d}} and σ = {φ, Y, {a, b}, {a, b, c}, {a, b, d}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = c,f(c) = d, f(d) = a is irresolute but not strongly gs_α** -irresolute.

Example: 3.9. Let X = Y = {a,b,c} , τ = {φ,X, {a}, {a, b}} and σ = {φ, Y, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = c is continuous but not strongly gs_α**-irresolute.

Example: 3.10. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}, {a, b, c}} and σ = {φ, Y, {a}, {b}, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = f(d) = c is ω-irresolute but not strongly gs_α**-irresolute.

Characterizations of strongly gs_α-irresolute functions**

Theorem: 4.1. Let f: (X,τ) → (Y,σ) be a function then the following are equivalent

(1) f is strongly gs_α**-irresolute.
(2) For every gs_α**-open set F of Y, $f^{-1}(F)$ is open in X.

Proof

(1) ⇒ (2). Let F be a gs_α**-open set in Y. Then Y–F is a gs_α**-closed set in Y. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is closed in X which implies $f^{-1}(F)$ is open in X.
(2) ⇒ (1). Similar to the proof of (1) ⇒ (2).

Theorem: 4.2. Let f: (X,τ) → (Y,σ) is strongly gs_α**-irresolute function. Then for each x ∈ X and each gs_α**-open set V of Y containing f(x), there exists an open set U of X containing x such that f(U) ⊆ V.

Proof. Let x ∈ X and V be any gs_α**-open set of Y containing f(x). Since f is strongly gs_α**-irresolute, $f^{-1}(V)$ is open in X and containing x. Let U = $f^{-1}(V)$. Then U is an open subset of X containing x and f(U) ⊆ V.

Theorem: 4.3. If f: (X,τ) → (Y,σ) is a pre-semi*α-open, strongly gs_α**-irresolute function , then f is gs_α**-irresolute.

Proof. Let B be gs_α**-closed in (Y,σ) and U be a semi*α-open set containing $f^{-1}(B)$. Then B ⊆ f(U),where f(U) is pre-semi*α-open in (Y,σ). Since B is gs_α**-closed, scl (B) ⊆ f(U) and hence $f^{-1}(scl(B))$ ⊆ U. Since f is strongly gs_α**-irresolute, $f^{-1}(scl(B))$ is closed in (X,τ). By theorem 3.2 [Santhini and Lakshmi Priya, 2017], $f^{-1}(scl(B))$ is gs_α**-closed in (X,τ). Thus scl($f^{-1}(scl(B))$) ⊆ U. Consequently scl($f^{-1}(B)$) ⊆ scl($f^{-1}(scl(B))$) ⊆ U which shows that $f^{-1}(B)$ is gs_α**-closed in (X,τ).

Theorem: 4.4.

Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any functions. Then

(i) g ∘ f: (X,τ) → (Z,μ) is strongly gs_α**-irresolute and g is strongly gs_α**-irresolute and f is strongly gs_α**-irresolute.
(ii) g ∘ f: (X,τ) → (Z,μ) is gs_α**-irresolute and g is strongly gs_α**-irresolute and f is gs_α**-irresolute.
(iii) g ∘ f: (X,τ) → (Z,μ) is contra gs_α**-continuous and g is contra-continuous and f is strongly gs_α**-irresolute.
(iv) g ∘ f: (X,τ) → (Z,μ) is continuous and g is strongly gs_α**-irresolute and f is continuous.
(v) g ∘ f: (X,τ) → (Z,μ) is irresolute and g is strongly gs_α**-irresolute and f is irresolute.
(vi) g ∘ f: (X,τ) → (Z,μ) is g*s-irresolute and g is strongly gs_α**-irresolute and f is

g*s-irresolute.

(vii) $g \circ f: (X,\tau) \rightarrow (Z,\mu)$ is gs_α**-irresolute and g is strongly gs_α**-irresolute and f is strongly gs_α**-irresolute.

Proof

(i) Let V be a gs_α**-closed set in Z. Since g is strongly gs_α**-irresolute, $g^{-1}(V)$ is closed in Y. By theorem 3.2 [Santhini and Lakshmi Priya, 2017], $g^{-1}(V)$ is gs_α**-closed in Y. Since f is strongly gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is closed in X and hence $g \circ f$ is strongly gs_α**-irresolute.

(ii) → (viii). Similar to the proof of (i).

Theorem: 4.5. If a function $f: (X,\tau) \rightarrow (Y,\sigma)$ is strongly gs_α**-irresolute where X is locally gs_α**-indiscrete then f is contra-continuous.

Proof. Let V be an open set in Y. By theorem 5.2[Santhini and Lakshmi Priya, 2017], V is gs_α**-open in Y.Since f is strongly gs_α**-irresolute,$f^{-1}(V)$ is open in X and hence $f^{-1}(V)$ is gs_α**-open in X. Since X is locally gs_α**-indiscrete, $f^{-1}(V)$ is closed in X and hence f is contra-continuous.

Theorem: 4.6. If a function $f: (X,\tau) \rightarrow (Y,\sigma)$ is strongly gs_α**-irresolute where Y is locally indiscrete then f is contra gs_α**-continuous.

Proof. Let V be an open set in Y. Since Y is locally indiscrete, V is closed in Y. By theorem 3.2 [Santhini and Lakshmi Priya, 2017], V is gs_α**-closed in X. Since f is strongly gs_α**-irresolute, $f^{-1}(V)$ is closed in X and hence f is contra gs_α**-continuous.

Theorem: 4.7. If a function $f: X \rightarrow Y$ is gs_α**-irresolute where X is a ${}_\alpha T_{s**}$-space then f is strongly gs_α**-irresolute.

Proof. Let U be a gs_α**-closed set in Y. Since f is gs_α**-irresolute, $f^{-1}(U)$ is gs_α**-closed in X. But X is a ${}_\alpha T_{s**}$-space, $f^{-1}(U)$ is closed in X and so f is strongly gs_α**-irresolute.

Theorem: 4.8. Let X and Z be any topological spaces and Y be a ${}^\alpha T_{s**}$-space then $g \circ f : (X,\tau) \rightarrow (Z,\mu)$ is strongly gs_α**-irresolute if g is gs-irresolute and f is strongly gs_α**-irresolute.

Proof. Let U be any gs_α**-closed set in Z. Since g is gs-irresolute, $g^{-1}(U)$ is gs-closed in Y. But Y is a ${}^\alpha T_{s**}$-space implies $g^{-1}(U)$ is gs_α**-closed in Y. Since f is strongly gs_α**-irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is closed in X and hence $g \circ f$ is strongly gs_α**-irresolute.

Completely gs_α**-Irresolute functions

In this section, the concepts of completely gs_α**-irresolute functions are introduced and studied.

Definition: 5.1. A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called a completely gs_α**-irresolute if $f^{-1}(V)$ is regular open in (X,τ) for every gs_α**-open set V in (Y,σ).

Example: 5.2. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c},{a, b, c}, {a, b, d}} and σ = {φ, Y, {a, b}}. Then $f: (X,\tau) \rightarrow (Y,\sigma)$ defined by f(a) = f(d)= c, f(c) = a, f(b) = b is completely gs_α**-irresolute.

Theorem: 5.3.

The following are equivalent for a function $f: (X,\tau) \rightarrow (Y,\sigma)$.

(1) f is completely gs_α** -irresolute.

(2) For each $x \in X$ and each gs_α**-open set V and Y containing f(x) there exists a regular open set U in X containing x such that $f(U) \subseteq V$.

(3) For every gs_α**-closed set V of Y, $f^{-1}(V)$ is regular closed in X.

Proof.

(1) ⇒ (2). Let $x \in X$ and V be a gs_α**-open set in Y containing f(x). Since f is completely gs_α**-irresolute, $f^{-1}(V)$ is regular open in X containing x. Take $U = f^{-1}(V)$. Then U is regular open in V containing x such that $f(U) \subseteq V$.

(2) ⇒ (1). Let V be a gs_α**-open set in Y such that $x \in f^{-1}(V)$. Then V is an gs_α**-open set containing f(x).By the assumption, there exists a regular open set U_x in X containing x such that $f(U) \subseteq V$ which implies $U \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V) = \cup \{U_x : x \in f^{-1}(V)\}$. Consequently $f^{-1}(V)$ regular open in X.

(1) ⇒ (3). Let V be an gs_α**-open set in Y. Then Y–V is gs_α**-open set in Y. By (1), $f^{-1}(Y - V) = X - f^{-1}(V)$ is regular open in X which implies $f^{-1}(V)$ is regular closed set in X.

(3) $\Rightarrow$ **(1).** Similar to the proof of **(1)** $\Rightarrow$ **(3).**
(2) $\Rightarrow$ **(3).** Similar to the proof of **(2)** $\Rightarrow$ **(1).**
(3) $\Rightarrow$ **(2).** Similar to the proof of **(1)** $\Rightarrow$ **(2).**

Theorem: 5.4. Every completely gs_α**-irresolute function is gs_α**-irresolute but not conversely.

Proof. Let V be a gs_α**-closed set in Y. Since f is completely gs_α**-irresolute, $f^{-1}(V)$ is regular closed in X. By theorem 3.2[13], $f^{-1}(V)$ is gs_α**-closed in X and so f is gs_α**-irresolute.

Example: 5.5. Let X = Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}} and σ = {φ, Y, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = c is gs_α**-irresolute but not completely gs_α**-irresolute.

Theorem: 5.6. Every completely gs_α**-irresolute function is strongly gs_α**-irresolute but not conversely.

Proof. Let V be gs_α**-closed set in y. Since f is completely gs_α**-irresolute, $f^{-1}(V)$ is regular closed in X and so $f^{-1}(V)$ is closed in X and hence f is strongly gs_α**-irresolute.

Example: 5.7. Let X = Y = {a,b,c} , τ = {φ,X, {a}, {b}, {a, b}, {b, c}} and σ ={φ, Y, {a}, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) = b, f(b) = a, f(c) = c is strongly gs_α**-irresolute but not completely gs_α**-irresolute.

Theorem: 5.8. Every completely gs_α**-irresolute function is completely continuous but not conversely.

Proof. Let V be an open set in Y. By theorem 5.2[13], V is gs_α**-open set in Y. Since f is completely gs_α**-irresolute, $f^{-1}(V)$ is regular open in X and so f is completely continuous.

Example: 5.9. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {d}, {a, d}, {b.d},{a, b}, {a, b, d}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X,τ) → (Y,σ) defined by f(a) =b, f(b) = f(c) = c, f(d) = a is completely-continuous but not completely gs_α**-irresolute.

Theorem: 5.10. Every completely gs_α**-irresolute function is gs_α**-continuous but not conversely.

Proof. Similar to the proof of theorem 5.8.

Example: 5.11. Let X = Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}} and σ = {φ, Y, {a, b}}. Then f : (X, τ) → (Y,σ) defined by f(a) = c, f(b) = a, f(c) = b is gs_α**-continuous but not completely gs_α**-irresolute.

Theorem: 5.12. Every completely gs_α**-irresolute function is a R-map but not conversely.

Proof. Let V be a regular open set in Y. By theorem 5.2 [Santhini and Lakshmi Priya, 2017], V is gs_α**-open in Y. Since f is completely gs_α**-irresolute, $f^{-1}(V)$ is regular open in X and so f is a R-map.

Example: 5.13. Let X = {a,b,c,d}, Y = {a,b,c}, τ = {φ,X, {a}, {b}, {d}, {a, d}, {b, d},{a, b}, {a, b, d}} and σ = {φ, Y, {a}, {b}, {a, b}, {b, c}}. Then f: (X,τ) → (Y,σ) defined by f(a) = f(c) = c, f(b) = a, f(d) = b is a R-map but not completely gs_α**-irresolute.

Theorem: 5.14. Every completely gs_α**-irresolute function is an irresolute but not conversely.

Proof. Let V be a semi-closed set in Y. By theorem 3.2[Santhini and Lakshmi Priya, 2017], V is gs_α**-closed in Y and so $f^{-1}(V)$ is regular closed in X. Consequently $f^{-1}(V)$ is semi-closed in X and so f is an irresolute function.

Example: 5.15. Let X = Y = {a,b,c}, τ = {φ,X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, Y, {a}, {a, b}}.Then f: (X,τ) → (Y,σ) defined by f(a) = c, f(b) = a, f(c) = b is irresolute but not completely gs_α**-irresolute.

Characterizations of completely gs_α-irresolute Functions**

Theorem: 6.1. If A is gs_α**-closed in (X,τ) and if f: (X,τ) → (Y,σ) is semi*α- irresolute and semi-closed then f(A) is gs_α**-closed in (Y,σ).

Proof. Let F be any semi*α-open set in (Y,σ) such that f(A) $\subseteq$ F. Then A $\subseteq$ $f^{-1}(F)$. Since f is semi*α -irresolute, $f^{-1}(F)$ is semi*α-open in (X,τ). Now A is gs_α**-closed implies scl(A) $\subseteq$ $f^{-1}(F)$. Then f(scl(A)) $\subseteq$ F. Since f(scl(A)) is a semi-closed set in (Y,σ). By theorem 3.2[Santhini and Lakshmi Priya, 2017], f(scl(A)) is gs_α**-closed in (Y,σ) and hence scl(f(scl(A))) $\subseteq$ F. Now, A $\subseteq$ scl(A) implies f(A) $\subseteq$ f(scl(A)). Hence scl(f(A)) $\subseteq$ scl(f(scl(A))) $\subseteq$ F and so f(A) is gs_α**-closed in (Y,σ).

Theorem: 6.2. Let F: X → Y be a completely gs_α**-irresolute function where X is a locally indiscrete space then f is contra gs_α**-continuous.

Proof. Let V be an open set in Y. By theorem 5.2 [Santhini and Lakshmi Priya, 2017], V is gs_α**-open set in Y. By hypothesis, f^{-1} (V) is regular open in X. Since X is locally indiscrete, f^{-1} (V) is closed in X. By theorem 3.2[Santhini and Lakshmi Priya, 2017], f^{-1} (V) is gs_α**-closed set in X and so f is contra gs_α**-continuous.

Theorem: 6.3. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute map where Y is a locally gs_α**-indiscrete space and g: (Y,σ) → (Z,μ) is strongly gs_α**-irresolute function then g f is contra-continuous.

Proof. Let V be any closed set in Z. By theorem 3.2[Santhini and Lakshmi Priya, 2017], V is gs_α**-closed in Y. Since g is strongly gs_α**-irresolute, g^{-1} (V) is closed in Y. But Y is locally gs_α**-indiscrete, g^{-1} (V) is open in Y. By theorem 5.2[Santhini and Lakshmi Priya, 2017], g^{-1} (V) is a gs_α**-open in Y. Since f is completely gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g\ f)^{-1}(V)$ is regular open in X and hence g f is contra-continuous.

Theorem: 6.4. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute map where Y is a locally gs_α**-indiscrete space and g: (Y,σ) → (Z,μ) is strongly gs_α**- irresolute function then g f is contra gs_α**-continuous.

Proof. By theorem 6.3 and by theorem 5.6 in [Santhini and Lakshmi Priya, 2017].

Lemma: 6.5. *[6]* Let S be an open subset of a space(X,τ). Then the following hold:

(i) If U is regular open in X, then so is U ∩ S in the subspace (S, τ_s).
(ii) If B ⊂ S is regular open in (S, τ_s), then there exists a regular open set U in (X,τ) such that B = U ∩ S.

Theorem: 6.6. If f : (X,τ) → (Y,σ) is a completely gs_α**-irresolute function and A is any open subset of X, then the restriction f/A: A → Y is completely gs_α**-irresolute.

Proof. Let F be a gs_α**-open set of Y. Since f is completely gs_α**-irresolute, f^{-1}(F) is regular open in X. Since A is open in X. By Lemma 4.3, $(f/A)^{-1}(F) = f^{-1}(F) \cap A$ is regular open in A and hence f/A is completely gs_α**-irresolute.

Theorem: 6.7. If a function f: X → Y be a function and g: X → X × Y be the graph of f defined by g(x) = (x,f(x)) for every x ∈ X. If g is completely gs_α**-irresolute, Then f is completely gs_α**-irresolute.

Proof. Let U be an gs_α**-open set in Y, then X × U is an gs_α**-open set in X × Y. Since g is completely gs_α**-irresolute , $f^{-1}(U) = g^{-1}(X \times U)$ is regular open in X. Thus f is completely gs_α**-irresolute.

Theorem: 6.8. Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any functions. Then (i) g f: (X,τ) → (Z,μ) is completely continuous if g is gs_α**-continuous and f is completely gs_α**-irresolute.

(ii) g f: (X,τ) → (Z,μ) is completely gs_α**-irresolute if g is gs_α**-irresolute and f is completely gs_α**-irresolute.
(iii) g f: (X,τ) → (Z,μ) is gs_α**-continuous if g is gs_α**-irresolute and f is completely gs_α**-irresolute.
(iv) g f: (X,τ) → (Z,μ) is completely gs_α**-irresolute if g is completely gs_α**-irresolute and f is completely gs_α**-irresolute.
(v) g f: (X,τ) → (Z,μ) is completely gs_α** -irresolute if g is completely gs_α**-irresolute and f is a R-map.
(vi) g f: (X,τ) → (Z,μ) is contra gs_α**-continuous if g is completely gs_α**-irresolute and f is contra gs_α**-continuous.
(vii) g f: (X,τ) → (Z,μ) is strongly gs_α**-irresolute and g is completely gs_α**-irresolute and f is continuous.
(viii) g f: (X,τ) → (Z,μ) is strongly gs_α**-irresolute and g is completely gs_α**-irresolute and f is R-map.
(ix) g f: (X,τ) → (Z,μ) is gs_α**-continuous and g is completely gs_α**-irresolute and f is strongly gs_α**-irresolute.

Proof.

(i) Let V be an open set in Z. Since g is gs_α**-continuous, g^{-1} (V) is gs_α**-open in Y. Since f is completely gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g\ f)^{-1}(V)$ is regular open in X. Hence g f is completely continuous.
(ii) → (ix). Similar to the proof of (i).

Theorem: 6.9.

Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,μ) be any functions. Then

(i) g f: (X,τ) → (Z,μ) is ω-irresolute if g is completely gs_α**-irresolute and f is ω-irresolute.
(ii) g f: (X,τ) → (Z,μ) is semi*-irresolute if g is completely gs_α**-irresolute and f is semi*-irresolute.
(iii) g f: (X,τ) → (Z,μ) is irresolute if g is completely gs_α**-irresolute and f is irresolute.
(iv) g f: (X,τ) → (Z,μ) is g*s-irresolute if g is completely gs_α** -irresolute and f is g*s-irresolute.
(v) g f: (X,τ) → (Z,μ) is gs_α**-irresolute if g is completely gs_α**-irresolute and f is gs_α**-irresolute.
(vi) g f: (X,τ) → (Z,μ) is gs_α**-irresolute if g is completely gs_α**-irresolute and f is irresolute.
(vii) g f : (X,τ) → (Z,μ) is gs_α**-irresolute if g is completely gs_α**-irresolute and f is g*s-irresolute.

Proof.

(i) Let V be a ω-closed set in Z. By theorem 3.2[Robert and Pious Missier, 2014], V is gs_α**-closed in Y. Since g is completely gs_α**-irresolute, g^{1} (V) is regular closed in Y. Since f is ω-irresolute, $f^{1}(g^{1}(V)) = (g \quad f)^{1}(V)$ is ω-closed in X. Hence g f is ω-irresolute.

(ii) → (vii). Similar to the proof of (i).

Theorem: 6.10. Let X and Z be any topological spaces and Y be a ${}^{\alpha}T_{s**}$-space then g f : (X,τ) → (Z,μ) is gs_α**-continuous if g is gs-irresolute and f is completely gs_α**-irresolute.

Proof. Let U be any closed set in Z. Then U is gs-closed in Z. Since g is gs-irresolute, $g^{1}(U)$ is gs-closed in Y. Now Y is a ${}^{\alpha}T_{s**}$-space implies $g^{1}(U)$ is gs_α**-closed in Y. Since f is completely gs_α**-irresolute, $f^{1}(g^{1}(U)) = (g \quad f)^{1}(U)$ is regular closed in X . By theorem 3.2[Santhini and Lakshmi Priya, 2017], $f^{1}(U)$ is gs_α**-closed in X. Hence g f is gs_α**-continuous.

Theorem: 6.11. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute where X is locally gs_α**-indiscrete then f is contra-continuous.

Proof. By theorem 4.5 and by theorem 5.6 in [Santhini and Lakshmi Priya, 2017].

Theorem: 6.12. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute and the space X is locally gs_α**-indiscrete then f is contra gs_α**-continuous.

Proof. By theorem 4.5 and by theorem 5.6 in [Santhini and Lakshmi Priya, 2017].

Theorem: 6.13. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute where Y is a locally gs_α**-indiscrete space and g: (Y,σ) → (Z,μ) is a gs_α**-continuous function then g f is contra-continuous.

Proof. Let V be any closed set in Z. Since g is gs_α**-continuous, $g^{1}(V)$ is gs_α**-closed. But Y is locally gs_α**-indiscrete implies $g^{1}(V)$ is open in Y. By theorem 5.2[Santhini and Lakshmi Priya, 2017], $g^{1}(V)$ is gs_α**-open in Y. Since f is completely gs_α**-irresolute, $f^{1}(g^{1}(V)) = (g \quad f)^{1}(V)$ is regular open in X and hence g f is contra-continuous.

Theorem: 6.14. If a function f: (X,τ) → (Y,σ) is completely gs_α**-irresolute where Y is a locally gs_α**-indiscrete space and g: (Y,σ) → (Z,μ) is gs_α**-continuous function then g f is contra gs_α**-continuous.

Proof. By theorem 6.13 and by theorem 5.6 in [Santhini and Lakshmi Priya, 2017].

Theorem: 6.15. If f: (X,τ) → (Y,σ) is completely gs_α**-irresolute injective function and Y is gs_α**-T_1 then X is r-T_1.

Proof. Let x,y be any distinct points of X. Since f is injective, then f(x) ≠ f(y). Since Y is gs_α**-T_1, there exists gs_α**-open sets V and W in Y such that $f(x) \in V$, $f(y) \in W$, $f(x) \notin W$, $f(y) \notin V$. Since f is completely gs_α**-irresolute, $f^{1}(V)$ and $f^{1}(W)$ are regular open sets in X such that $x \in f^{1}(V)$, $y \in f^{1}(W)$, $x \notin f^{1}(W)$, $y \notin f^{1}(V)$. It follows that X is r-T_1.

Theorem: 6.16. Let f: (X,τ) → (Y,σ) be a completely gs_α**-irresolute injective function if Y is gs_α**-T_2 then X is r-T_2.

Proof. Let x, y be any distinct points of X. Since f is injective, then f(x) ≠ f(y). Since Y is gs_α**-T_2, there exists gs_α**-open sets V and W in Y such that $f(x) \in V$, $f(y) \in W$ and $V \cap W = \varphi$. Since f is completely gs_α**-irresolute, $f^{1}(V)$ and $f^{1}(W)$ are regular open sets in X such that $x \in f^{1}(V)$, $y \in f^{1}(W)$ and $x \in f^{1}(V) \cap f^{1}(W) = \varphi$. This shows that X is r-T_2.

Remark: 6.17. The following table shows the relationships between gs_α**-irresolute maps, completely gs_α**-irresolute maps and strongly gs_α**-irresolute maps. The symbol ”1” in a cell means that a map implies the other maps and the symbol ”0” means that a map does not imply the other maps.

Irresolute map	gs_α**	completely gs_α**	strongly gs_α**
gs_α**	1	0	0
completely gs_α**	1	1	1
strongly gs_α**	1	0	1

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