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RESEARCH ARTICLE

NUMERICAL SOLUTION OF BURGERS' EQUATION USING FOURIER EXPANSION BASED ON DIFFERENTIAL QUADRATURE METHOD

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ABSTRACT

The Fourier expansion-based differential quadrature (FDQ) method was applied in this work to solve one-dimensional Burgers' equation with appropriate initial and boundary conditions. In the first step for the given problem we have discretized the interval and replaced the differential equation by the Fourier expansion basis based on differential quadrature (FDQ) to obtain a system of ordinary differential equation (ODE). The obtained ordinary differential equation was solved by fourth order classical Range-Kutta method. Finally the validation of the present scheme was demonstrated by numerical example and compared with some existing numerical methods in literature. The method is analyzed for stability and convergence. It is found that the proposed numerical scheme produce accurate results and quite easy to implement.

INTRODUCTION

Numerical analysis is the branch of mathematics that plays a prominent role in pure, and applied mathematics, as well as in sciences. The design and computation of a numerical algorithm is one of the mathematical challenges that we are facing these days. Despite the rapid development of computational methods, problems involving non-linearity, discontinuity, multiple scale, singularity and irregularity continue to pose challenges in the field of computational science and engineering (Shu, 2000). Many scientists in the field of computational mathematics are trying to develop algorithm for numerical methods by using modern computers. One of this is a Differential Quadrature Method. Of the various numerical solutions, differential quadrature (DQ) methods have distinguished themselves because of their high accuracy, straightforward implementation and generality in a variety of problems (Shu and Chew, 1999).

Burger's Equation

The one dimensional Burgers' equation is given by

$$\frac{\partial}{\partial t}u(x,t)+u(x,t)\frac{\partial}{\partial x}u(x,t)=v\frac{\partial^2}{\partial x^2}u(x,t)$$

is a nonlinear PDE and has a wide application in various areas of applied mathematics, such as fluid mechanics, nonlinear acoustics, gas dynamics, traffic flow (Cole,1951). The study of the solution of Burgers' equation has been carried out for last half Century and still it is an active area of research to develop some better numerical scheme to approximate its solution.

Fourier Expansion Basis

The polynomial approximation is suitable for most of the engineering problems, but for some problems, especially for those with periodic behaviors, Fourier series expansion could be a better choice for the true solution instead of polynomial approximation

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(Shu and Richard, 1992) and (Shu *et al.,* 1995). For a continuous function $f(x)$ on the interval [0, 2π], the Fourier series expansion can be given by

$$f(x) = c_0 + \sum \left(c_k \cos kx + d_k \sin kx \right)$$

Where the coefficient c_0, c_k and d_k are expressed as

$$c_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx$$

$$c_k = \frac{1}{\pi} \int_0^{2\pi} f(x)\cos kxdx$$

$$d_k = \frac{1}{\pi} \int_0^{2\pi} f(x)\sin kxdx$$

In practice, the truncated Fourier series expansion is usually used. Thus,

$$f(x) \approx F_{N+1}(x) = a_0 + \sum_{k=1}^{N} a_k \cos kx + \sum_{k=1}^{N} b_k \sin kx$$

The convergence of the above expansion to $f(x)$ as N tends to infinity is guaranteed by Weierstrass's second theorem.

***Theorem* 1**: Let $f(x)$ be a continuous function on the interval [0, 2π]. Then for any ε >0, there exists an integer N and a trigonometric sum S_N such that the inequality

$$\max_{x\in[0,2\pi]} \left| f(x) - S_N(x) \right| < \varepsilon$$

is satisfied, where

$$S_N(x) = a_0 + \sum_{k=1}^{N} a_k \cos kx + \sum_{k=1}^{N} b_k \sin kx \,.$$

The proof of this theorem can be found in the book of (Achieser, 1992) and (Weierstrass, 1886) [1 and 25]. It is shown that the approximation in Eq. (7) satisfies the operations of vector addition and scalar multiplication. So, $F_{N+1}(x)$ consists of a linear vector space V_{N+1} in which there exists a linearly independent set of base vectors

$$1, \cos x, \sin x, ..., \cos Nx, \sin Nx$$

It is of great importance to determine the function values at discrete nodes for the numerical solution of a partial differential equation. Therefore, the Fourier expansion should be expressed in discrete form. Supposing that $x_i, i = 0,1,2,...,N$ are the coordinates of $(N+1)$ nodes on the interval [0, 2π], and $f(x_i)$ is the function values at the $i-th$ point, the Fourier expansion $F_{N+1}(x)$ can be expressed as follows:

$$F_{N+1}(x) = \sum_{i=0}^{N} f(x_i).g_i(x)$$

Where

$$g_i(x) = \frac{G(x)}{G'(x_i)\sin(x - x_i)}$$

$$G(x)=\prod_{i=0}^{N}\sin(x-x_i)$$

$$G'(x)=\prod_{k=0,k\neq i}^{N}\sin(x_k-x_i)$$

It is clear that the coefficients $x_i, i=0,1,...,N$, form a set of linearly independent vectors in V_{N+1}. Thus it is also a basis of V_{N+1}.

Statement of the Problem

Consider the Burgers' equation:

$$\frac{\partial u}{\partial t}-v\frac{\partial^2 u}{\partial x^2}+u\frac{\partial u}{\partial x}=0,\ (x,t)\in\Omega\times(0,T],\ \text{where}\ \ \Omega=(0,1)$$

With initial condition

$$u(x,0)=f(x),\quad 0\le x\le 1$$

and the boundary condition

$$(0,t)=0, u(1,t)=0,\quad 0\le t\le T$$

Where $v>0$ is the coefficient of kinematic viscosity and the prescribed function $f(x)$ is sufficiently smooth, by using Fourier expansion basis based on weighted average differential quadrature method, the Burgers' equation given by (10) is nonlinear and parabolic and one expects to find its solution numerically by using some approximation method. In this regard Kutluary *et al.* solve (10) by the finite difference approximation based on the standard explicit method (Kutluary, 1999) [7] and using least-squares quadratic B-spline finite element method (Kutluary *et al.,* 2004) same scholars applied a spectral approximation method for one dimensional Burgers' equation (Mittal and Singhal, 1996); R.C Mittal, Ram Jiwari and K.K Sharma used quasilinearization to tackle the nonlinearity and followed by semi discretization for spatial direction using DQM (Mittal *et al.,* 2013); Ram Jiwari, R.C Mittal and K.K Sharmausing weighted average DQM (Jiwari *et al.,* 2013). In this study, the researcher look for a solution of one dimensional Burgers' equation using the Fourier expansion basis based on differential quadrature method. As a result, this study attempted to answer the following questions:

1. How do we describe the Fourier expansion basis based on differential quadrature method for one dimensional Burgers' equation?
2. To what extent the method approximate the existing solutions?
3. To what extent the present method converges?

MATERIALS AND METHODS

This chapter consists of the following methods and materials that used to carry out the study. These are; study design, study site, and period, source of information, study procedure, and ethical considerations.

Study Site and Period

The study was conducted at Jimma University, which is Ethiopia's first innovative community oriented education institution of higher learning, department of Mathematics from September 2015 to 2016.

Study Design

This research employed mixed design;

- Documentary review design
- Experimental design

Study Area

Conceptually this study focus on Fourier Expansion based on Differential Quadrature method for Burgers' Equation to approximate two point boundary condition.

Source of Information

This study mostly depends on documentary materials and the data which have been obtained by the help of MATLAB software. So, the sources of information for the study are books, journals and different related studies from internet services and numerical data obtained by MATLAB. In addition to this, workshop on MATLAB software was designed and conducted.

Study Procedure

The study is an experimental as it involves entirely laboratory work with the help of computer and MATLAB software. Farther, important materials for the study were collected by the researcher using documentary analysis. The required numerical data was collected by coding and running using MATLAB software to get the numerical results and the graphs of some examples that have exact solution, to show the validity and efficiency the method. In order to achieve the above mentioned objectives, the study is follow steps:

- Problem preparation/formulation
- Discretizing the given interval
- Replacing the differential equation by the Fourier Expansion basis based on Differential Quadrature method to obtain a system of ODE.
- The Obtained system of Ordinary Differential Equations can be solved by fourth order classical Range - Kutta method.
- Writing MATLAB code for the tri-diagonal system obtained.
- Validation of the present scheme by implementing it on numerical examples.

RESULTS AND DISCUSSION

Differential Quadrature Method

For simplicity, the one dimensional problem is chosen to demonstrate the differential quadrature method. Following the idea of an integral quadrature that uses a linear weighted summation of all the functional values to approximate an integral over a closed domain, the DQ method approximate the derivative of a smooth function at a grid point by a linear weighted summation of all the functional value in the whole computational domain. For example the first and second order derivatives of $u(x)$ at a point x_i are approximated by

$$u_x(x_i) = \sum a_{ij}\, u(x_i),\ for\ i=1,2,\ldots\ldots,\ N \qquad (1)$$

$$u_{xx}(x_i) = \sum b_{ij} u(x_i),\ \ for\ i=1,2,\ldots,N \qquad (2)$$

Where N is the number of grid points, and a_{ij}, b_{ij} are the weighting coefficients. It is noted that equations (1) and(2) are similar except that they use different weighting coefficients. Obviously, the key procedure in DQ is to determine the weighting coefficients a_{ij} and b_{ij} .

Fourier Expansion Based on Differential Quadrature Method (FDQ)

For this case, the solution of a differential equation is approximated by a Fourier series expansion of the form

$$u(x) = c_0 + \sum_{k=1}^{\infty}(c_k \cos k\pi x + d_k \sin k\pi x) \qquad (3)$$

It is easy to show that $u(x)$ in equations (3) constitutes an $(N+1)$ dimensional linear vector space with respect to the operation of addition and multiplication. Here, if $r_k(x), k=0,1,\ldots,N,$ are the base functions, any function in the space can be expressed as a linear combination of $r_k(x), k=0,1,\ldots,N.$ It is obviously observed from equations (3) that one set of base functions $1, \sin\pi x, \cos\pi x, \sin 2\pi x, \cos 2\pi x, \ldots, \sin N\pi x, \cos N\pi x$. For generality two sets of base functions are used in FDQ. And the other, the Lagrange interpolated trigonometric polynomials are taken as one set of base functions

$$r_k(x) = \frac{\sin\pi(x-x_0)\sin\pi(x-x_1)\ldots\sin\pi(x-x_{k-1})\sin\pi(x-x_{k+1})\ldots\sin\pi(x-x_N)}{\sin\pi(x_k-x_0)\sin\pi(x_k-x_1)\ldots\sin\pi(x_k-x_{k-1})\sin\pi(x_k-x_{k+1})\ldots\sin\pi(x_k-x_N)}, \qquad (4)$$

$$for\ k=1,2,\ldots,N$$

Setting

$$M(x) = \prod_{k=0}^{N} \sin \pi(x - x_k) = N(x, x_k).\sin \pi(x - x_k) \quad \text{……… (5)}$$

Where

$$N(x_i, x_k) = \prod_{k=0, k\neq i}^{N} \sin \pi(x_i - x_k) = p(x_i) \quad \text{……… (6)}$$

$N(x_i, x_j) = N(x_i, x_j).\delta_{ij}$,Where δ_{ij} is the kronecker delta operator.

Eq. (4) can then be reduced to

$$r_k(x) = \frac{N(x, x_k)}{p(x_k)} \quad \text{……… (7)}$$

We let all the base functions given by Eq. (7) satisfy two linear constrained relation (1) and (2). This results in the following two formulations

$$a_{ij} = \frac{N'(x_i, x_j)}{p(x_j)} \quad \text{……… (8)}$$

$$b_{ij} = \frac{N''(x_i, x_j)}{p(x_j)} \quad \text{……… (9)}$$

It is observed from Eq. (8) and (9) that the computation of a_{ij} and b_{ij} is equivalent to evaluation of $N'\left(x_i, x_j\right)$ and $N''\left(x_i, x_j\right)$ since $p\left(x_j\right)$ can easily be calculated by Eq. (6). To evaluate $N'(x_i, x_j)$ and $N''\left(x_i, x_j\right)$ we successively differentiate Eq. (5) and then obtain

$$M'(x) = N'(x, x_k).\sin \pi(x - x_k) + \pi N(x, x_k).\cos \pi(x - x_k) \quad \text{……… (10)}$$

$$M''(x) = N''(x, x_k) sin\pi(x - x_k) + 2\pi N'(x, x_k) \cos \pi(x - x_k) - \pi^2 N(x, x_k) \sin \pi(x - x_k) \quad \text{……… (11)}$$

$$\begin{aligned} M'''(x) = {} & N'''(x, x_k) \sin \pi(x - x_k) + 3\pi N''(x, x_k) \cos \pi(x - x_k) \\ & - 3\pi^2 N'(x, x_k) \sin \pi(x - x_k) - \pi^3 N(x, x_k) \cos \pi(x - x_k) \end{aligned} \quad \text{……… (12)}$$

From the above equations, we can obtain the following results

$$N'(x_i, x_j) = \frac{\pi.p(x_i)}{\sin \pi(x_i - x_j)}, \quad when\ j \neq i \quad \text{……… (13)}$$

$$N'(x_i, x_i) = \frac{M''(x_i)}{\pi} \quad \text{……… (14)}$$

$$N''(x_i, x_j) = \frac{M''(x_i) - \pi N'(x_i, x_j) \cos \pi(x_i - x_j)}{\sin \pi(x_i - x_j)}, \quad when\ j \neq i \quad \text{……… (15)}$$

$$N''(x_i, x_i) = \frac{2}{3\pi}\left[M'''(x_i) + \left(\frac{\pi}{8}\right)^3 N(x_i, x_i) \right] \quad \text{……… (16)}$$

Substituting Equations (13), (14) into Eq. (8) we obtain

$$a_{ij} = \pi.\frac{p(x_i)}{\sin\pi(x_i - x_j), p(x_j)}, \ when\ j \neq i \qquad (17)$$

$$a_{ii} = \frac{M''(x_i)}{\pi.p(x_i)} \qquad (18)$$

Similarly, substituting Eq. (15), (16) into Eq. (9) and using equations (17), (18), we obtain

$$b_{ij} = a_{ii}\left[2a_{ii} - \pi cot\pi(x_i - x_j)\right], \ when\ j \neq i \qquad (19)$$

$$b_{ii} = \frac{2}{3\pi}\left[\frac{M'''(x_i)}{p(x_i)} + \frac{\pi^3}{8}\right] \qquad (20)$$

From equations (17) and (19), a_{ij} and $b_{ij}(i \neq j)$ can be obtained. However, the calculation of $a_{ii}(Eq.(18))$ and $b_{ii}(Eq.(20))$ involve the computation of $M''(x_i)$ and $M'''(x_i)$ which are not easy to compute. By applying the second set of base functions $1, \sin\pi x, \cos\pi x, \sin 2\pi x, \cos 2\pi x, ..., \sin(N\pi x), \cos(N\pi x)$ into equation (1) and (2) and hence

$$\sum_{j=1}^{N} a_{ij} = 0 \qquad (21)$$

$$\sum_{j=1}^{N} b_{ij} = 0 \qquad (22)$$

From Eq. (21) and (22), a_{ii} and b_{ii} can easily calculated from $a_{ij}(i \neq j)$ and $b_{ij}(i \neq j)$.

Using equations (17), (19), (21) and (22), the weighting coefficients of the first and second order derivatives in FDQ can be calculated. It should be indicated that these equations can be applied to the periodic problems and the non-periodic problems. For the non-periodic problems, the x range in the computational domain is $0 \le x \le \pi$, while for the periodic problems, the x range in the computational domain is $0 \le x < 2\pi$.

Burgers' Equation and Numerical Discretization

The one-dimensional Burgers' equation

$$\frac{\partial u}{\partial t} - v\frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} = 0 \qquad (23)$$

Can be discretized in the cartesian coordinate system as

$$\sum_{k=1}^{N} a_{ik}u_{kj} - v\sum_{k=1}^{N} b_{ik}u_{kj} + u_{kj}\sum_{k=1}^{N} a_{ik}u_{kj} = f_{ij} \qquad (24)$$

Where N is the number of grid points, a_{ik} and b_{ik} are the weighting coefficients in the x direction. When FDQ method is used a_{ik} and b_{ik} are computed by equations (17),(19),(21) and (22).

Error Estimate and Convergence Analyses

To demonstrate the efficiency and accuracy of FDQ method the sample problem which have exact solutions are chosen for the study. The computational domain is with length L_x. In this study FDQ method use the Chebyshev-Gauss-Lobatto point distribution.

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right]L_x, \ i = 1, 2, ..., N \qquad (25)$$

Since the sample problems has exact solution, the performance FDQ method measured by $\Delta u_{\max}$ which is defined as

$$\Delta u_{\max} = \max\left|u_{ij} - u_{x_i}\right| \quad \text{(26)}$$

Where u_{ij} is the numerical solution at the mesh point x_i , u_{x_i} is the exact solution at the same mesh point. The numerical rate of convergence (ROC) is calculated using the following formula

$$ROC \approx \frac{\log\left(E(N_2)/E(N_1)\right)}{\log(N_1/N_2)} \quad \text{(27)}$$

Where $E\left(N_i\right)$ is the maximum error norm L_∞ when using N_j grid points (Jiwari *et al.,* 2013)

Main Results and Discussion

To illustrate the efficiency of the proposed numerical scheme, we solve three test examples and throughout the numerical experiment we consider step length in time space Δt .

Example 1: We consider Burgers' Eq. (23) with initial and boundary conditions in the following form

$$u(x,0) = \sin\pi x,\ 0 \le x \le 1 \quad \text{(28)}$$

$$u(0,t) = u(1,t) = 0,\ t > 0 \quad \text{(29)}$$

The numerical results for the Example 1 are presented for $\nu =$ 1.0, 0.1, 0.01 in Tables 1 and 2. From Table 1, it is concluded that the present scheme gives better results than the results in (Jiwari *et al.,* 2013). Table 2 shows the maximum absolute errors and rate of convergence for different value of ν .

Table 1. Comparison between exact and numerical solutions of Example1 for $\nu =$ 0.1, 0. 01 at different time and x

X	$\nu = 0.1$				$\nu = 0.01$		
	T	(Jiwari *et al.,* 2013) $\Delta t = 0.0001$; Δx= 0.04	Present scheme	Exact solution	(Jiwari *et al.,* 2013) Δt=0.0001; Δx=0.04	Present scheme	Exact solution
0.25	0.4	0.30880	0.30884	0.30889	0.34191	0.34191	0.34191
	0.8	0.19565	0.19565	0.19568	0.22151	0.22149	0.22148
	1.0	0.16251	0.16255	0.16256	0.18814	0.18819	0.18819
	3.0	0.02729	0.02722	0.02720	0.07537	0.07512	0.07511
0.50	0.4	0.56953	0.56958	0.56963	0.66070	0.66071	0.66071
	0.8	0.35922	0.35923	0.35924	0.43913	0.43914	0.43914
	1.0	0.29190	0.29191	0.29192	0.37434	0.37441	0.37442
	3.0	0.04020	0.04022	0.04021	0.15008	0.15009	0.15018
0.75	0.4	0.62554	0.62543	0.62544	0.91927	0.91025	0.91026
	0.8	0.37409	0.37400	0.37392	0.64739	0.64740	0.64740
	1.0	0.28746	0.28748	0.28747	0.55599	0.55604	0.55605
	3.0	0.02977	0.02977	0.02977	0.22481	0.22481	0.22481

Table 2. Max. Error norms and rate of convergence of Example 1 for various numbers of grids at T = 2.0 and with Δx= 0.04, Δt = 0.0001

N	ν = 1.0		ν = 0.1		ν = 0.01	
	L_∞	ROC	L_∞	ROC	L_∞	ROC
8	3.81903E-15	_	4.66231E-7	_	3.21680E-4	_
16	8.41682E-17	7.126	2.21680E-6	4.830	2.31861E-3	4.010
32	6.41823E-18	3.563	3.28281E-8	3.001	3.41281E-4	2.892

Example 2: Consider the Burgers' Eq. (23) with the initial and boundary conditions

$$u(x,0) = 4x(1-x),\ 0 \le x \le 1 \quad \text{(30)}$$

$$u(0,t) = u(1,t) = 0,\ t > 0 \quad \text{(31)}$$

The numerical results for the Example 2 are presented for $\nu =$ 1.0; 0.1; 0.01 in Tables 3 and 4. Table 3 makes a comparison of present results with exact and numerical solutions available in literature and it is found that the present results are better than the results in (Jiwari *et al.,* 2013). Table 4 shows the maximum absolute errors and rate of convergence for different value of ν .

Table 3. Comparison between exact and numerical solutions of Example 2 for $\nu = 0.1, 0.01$ at different time and x

X	ν = 0.1				ν = 0.01		
	T	(Shu, 2000) Δt=0.0001; Δx=0.04	Present scheme	Exact solution	(Shu, 2000) Δt=0.0001; Δx=0.04	Present scheme	Exact solution
0.25	0.4	0.31744	0.31751	0.31752	0.36213	0.36222	0.36226
	0.8	0.19952	0.19954	0.19956	0.23066	0.23044	0.23045
	1.0	0.16557	0.16563	0.16560	0.19468	0.19467	0.19469
	3.0	0.02775	0.02775	0.02775	0.07613	0.07613	0.07613
0.50	0.4	0.58443	0.58450	0.58454	0.68357	0.68368	0.68368
	0.8	0.36733	0.36740	0.36740	0.45412	0.45401	0.45371
	1.0	0.29830	0.29834	0.29834	0.38563	0.38566	0.38568
	3.0	0.04106	0.04106	0.04106	0.15217	0.15218	0.15218
0.75	0.4	0.64556	0.64560	0.64562	0.92064	0.92049	0.92050
	0.8	0.38526	0.38533	0.38534	0.66303	0.66270	0.66272
	1.0	0.29582	0.29587	0.29586	0.56929	0.56931	0.56932
	3.0	0.03043	0.03044	0.03044	0.22774	0.22774	0.22774

Table 4. Max. Error norms and rate of convergence of Example 2 for various numbers of grids at T = 2.0 and with Δx = 0.04; Δt = 0.0001

N	ν = 1.0		ν = 0.1		ν = 0.01	
	L_∞	ROC	L_∞	ROC	L_∞	ROC
8	2.16823E-16	_	6.22861E-6	_	2.81281E-3	_
16	6.42816E-17	6.816	2.32812E-7	3.128	4.41291E-6	3.287
32	3.31268E-18	5.828	3.32816E-8	2.561	3.21618E-5	2.718

Example 3. Consider the Burgers' Eq. (23a) with boundary conditions

$$u(0,t) = u(1,t) = 0,\ t > 0 \qquad (32)$$

and with exact solution

$$u(x,t) = \frac{2\nu\pi e^{-\pi^2 \nu t}\sin(\pi x)}{\sigma + e^{-\pi^2 \nu t}\cos(\pi x)},\ 0 < x < 1 \qquad (33)$$

Where $\sigma > 1$ is a parameter.

For this Example, the numerical results are presented in Tables 5 and 6. Tables 5 and 6 show the L_2 and L_∞ errors at different values of T, ν and the results are compared with (Jiwari *et al.,* 2013). It is found that the present results are better than the results presented in (Jiwari *et al.,* 2013).

Table 5. Comparison of L_∞ and L_2 errors with existing numerical methods of Problem 3 for v = 0.01, $\sigma = 100$, Δt = 0.01, at T = 1.0

N	(Jiwari *et al.,* 2013)		Present scheme		
	L_∞	L_2	L_∞	L_2	ROC
10	6.001E-11	6.503E-12	3.207E-12	4.283E-12	_
20	1.010E-11	9.344E-12	2.1680E14	3.418E-13	4.816
40	1.227E-10	2.208E-11	6.128E-12	3.813E-12	2.883
80	8.147E-09	1.017E-11	7.612E-10	5.312E-12	2.582

Table 6. Comparison of L_∞ and L_2 errors with existing numerical methods of Problem 3 for v = 0.005, $\sigma = 100$, Δt = 0.01, at T = 1.0

N	(Jiwari *et al.,* 2013)		Present scheme		
	L_∞	L_2	L_∞	L_2	ROC
10	4.708E-08	6.465E-08	8.261E-09	6.561E-08	_
20	1.091E-08	4.465E-09	1.830E-08	6.418E-10	4.871
40	1.980E-09	2.786E-10	2.617E-10	3.341E-11	2.331
80	7.182E-09	2.665E-10	6.822E-10	8.810E-11	2.563

This method shows a good result as compared with the existing literature (Jiwari *et al.,* 2013).

Conclusion

This paper demonstrates the application of FDQ method to solve one-dimensional Burgers' equation. Through test examples which have exact solution, it was found that the FDQ approach can generally obtain more accurate numerical results than the existing approach in the interval $[0, 0.75]$. The accuracy of FDQ results is suddenly improved when number of mesh points is increased to more than $2N+1$, where N equals v (the wave number), the FDQ results are quite erratic. The proposed numerical scheme gives better solution for harmonic (periodic) partial differential equations than the existing scheme.

Future Scope: In the future by applying Fourier expansion based Differential Quadrature (FDQ) method I will try to solve two-/ three- dimensional Burgers' equation and other partial differential equations.

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