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RESEARCH ARTICLE

ASPECTS OF LOITSIANSKY'S TYPE OF INVARIANT

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ABSTRACT

Turbulence is seen as one of the last outstanding unsolved problems in classical physics. In the last century, great minds viz, Heisenberg, von Weizs"acker, Kolmogorov, Prandtl and G.I. Taylor had worked on it. Einstein put his last postdoc Bob Kraichnan on the subject of Turbulence. Despite the fact that isotropic turbulence constitutes the simplest type of turbulent flow, it is still not possible to render the problem analytically traceable without introducing the two point double and triple longitudinal velocity correlations to admit self-similarity solution with respect to a single length-scale, which has served as a useful hypothesis since its inception by von Karman and Howarth (1938). Rapid development of experimental and numerical techniques in this area and the growth of computing power created a lot of activities on turbulence research. Here authors have elaborated a debated concept Loitsiansky's type of invariant associated with turbulent study from analytical point.

INTRODUCTION

Homogeneous isotropic turbulence is a kind of idealization for real turbulent motion, under the assumption that the motion is governed by a statistical law invariant for arbitrary translation (homogeneity), rotation or reflection (isotropy) of the coordinate system. This idealization was first introduced by Taylor (1935) and used to reduce the formidable complexity of statistical expression of turbulence and thus made the subject feasible for theoretical treatment. Up to now, a large amount of theoretical work has been devoted to this rather restricted kind of turbulence. However, turbulence observed either in nature or in laboratory has much more complicated structure. Although remarkable progress has been achieved so far in discovering various characteristics of turbulence, our understanding of the fundamental mechanism of turbulence is still partial and unsatisfactory. The assumption of similarity and self-preservation, which permits an analytical determination of the energy decay in isotropic turbulence, has played an important role in the development of turbulence theory for more than half a century. In the traditional approach to search for similarity solutions for turbulence, the existence of a single length and velocity scale has been assumed, and then the conditions for the appearance of such solutions have been examined. Excellent contributions had been made to this direction by von Karman and Howarth (1938), who firstly deduced the basic equation and presented a particular set of its solutions for the final decaying turbulence. Later on, two Russian scholars, Loitsiansky (1939) and Millionshtchikov (1941), separately obtained the solutions for the Karman-Howarth equation after the term related to the effect of the triple velocity correlation has been neglected (Ghosh and Ghosh, 1982; Ghosh, 2001). Their work was an extension of the "small Reynolds number" solution first given by von Karman and Howarth. Dryden gave a comprehensive review on this subject (Dryden, 1943). Detailed research on the solutions of the Karman-Howarth equation was conducted by Sedov, who showed that one could use the separability constraint to obtain the analytical solution of the Karman-Howarth equation (Sedov, 1944). Sedov's solution could be expressed in terms of the confluent hypergeometric function. Batchelor (1948) readdressed this problem under the assumption that the Loitsiansky integral is a dynamic invariant, which was a widely accepted assumption, but was later found to be invalid. Batchelor concluded that the only complete self-preserving solution which was intrinsically consistent existed at low turbulence Reynolds number, for which the turbulent kinetic energy is accordant with the final period of turbulent decay. Batchelor also found a self-

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preserving solution to the Karman-Howarth equation in the limit of infinite Reynolds number, for which the Loitsiansky integral is an invariant. Objections were later raised against using the Loitsiansky integral as a dynamic invariant. In fact, at high Reynolds number this integral can be proved to be a weak function of time Proudman and Reid (1954) and Batchelor and Proudman (1956). Saffman proposed an alternative dynamic invariant which yielded another power-law decay in the limit of infinite Reynolds number (Hinze, 1975). While the results of Batchelor and Saffman formally constitute complete self-preserving solutions to the inviscid Karman-Howarth equation, it must be kept in mind that they only exhibit partial self-preservation with respect to the full viscous equation. Later on, George (1992) revived this issue concerning the existence of complete self-preserving solutions in isotropic turbulence. In an interesting paper he claimed to find a complete self-preserving solution, valid for all Reynolds numbers. George's analysis was based on the dynamic equation for the energy spectrum rather than on the Karman-Howarth equation. Strictly speaking, the solution presented by George was an alternative self-preserving solution to the equations of Karman-Howarth and Batchelor since George relaxed the constraint that the triple longitudinal velocity correlation is self-similar in the classical sense. Speziale and Bernard (1992) reexamined this issue from a basic theoretical and computational standpoint. Several interesting conclusions have been drawn from their analysis. From the development of turbulence theory, it is known that the research on decaying homogeneous isotropic turbulence is one of the most important and extensively explored topics. Despite all the efforts, a general theory describing the decay of turbulence based on the first principles has not yet been developed (Skrbek and Stalp, 2000). It seems that the theory of self-preservation in homogeneous turbulence has lots of interesting features which have not yet been fully understood and are worth of further study (Speziale and Bernard, 1992). This paper offers a short, but interesting feature of Loitsiansky invariant unified investigation of isotropic turbulence, based on the exact solutions of the Karman-Howarth equation. The statistical procedures of random fluctuation in a turbulent flow has been investigated by Taylor (1935), Robertson (Robertson, 1940) and many others. Karman and Howarth (1938) proposed the equation of translation for a second order velocity correlation of a homogeneous isotropic flow field. The equation constituting the relation of second order velocity correlation with the third order velocity correlation in collaboration with viscous damping denoted by Laplacian of second order velocity correlations.

The tensorial equation of Navier Stocks equation along with the equation of continuity is given by

$$\frac{\partial}{\partial t}\left(\overline{u^2}R_{ij}\right)-\frac{\partial}{\partial \xi_k}\left\{\left(\overline{u^2}\right)^{\frac{3}{2}}\left(T_{ik,j}+T_{jk,i}\right)\right\}=2\nu\nabla^2{}_{\xi}\overline{u}\ R_{ij} \qquad \text{..........(1)}$$

where, $\overline{u^2}R_{ij}=\overline{u_i u'}$ and $\left(\overline{u^2}\right)^{\frac{3}{2}}T_{ik,j}=\overline{u_i u_k u'_j}$ etc. also the terms like $\overline{p'u_i}, \overline{pu'_j}$ taking to be zero as they are considered to be the fluctuating terms of the turbulent flow field.

Applying the conditions isotropy and homogeneity, we have

$$\overline{u_i u'_j}=\overline{u^2}R_{ij}=\overline{u^2}\left[-\frac{1}{2r}f'\xi_i\xi_j+\left(f+\frac{r}{2}f'\right)\delta_{ij}\right]$$

and

$$\overline{u_i u'_j u_k}=\left(\overline{u^2}\right)^{\frac{3}{2}}T_{ik,j}=\left(\overline{u^2}\right)^{\frac{3}{2}}\left[\frac{rh'-h}{r^3}\xi_i\xi_j\xi_k-\frac{rh'+2h}{r}\left(\xi_i\delta_{jk}+\xi_k\delta_{ij}\right)+\frac{h}{r}\xi_j\delta_{ik}\right]$$

Now the scalar form of equation (1) is given by

$$\frac{\partial}{\partial t}\overline{u^2}f(r,t)+2\left(\overline{u^2}\right)^{\frac{3}{2}}\left[\frac{\partial h(r,t)}{\partial r}+\frac{4h(r,t)}{r}\right]=2\nu\overline{u^2}\left[\frac{\partial^2 f(r,t)}{\partial r^2}+\frac{4}{r}\frac{\partial f(r,t)}{\partial r}\right] \qquad \text{............(2)}$$

The equation (2) gives the scalar form of equation (1) in terms of two unknowns f(r,t) and h(r,t).

Now assuming Ω(r,t) be the reactant concentration field variable ,the equation for decay can be written as

$$\frac{\partial \Omega}{\partial t}+\frac{\partial}{\partial x_1}\left(u_l\Omega\right)=D\frac{\partial^2\Omega}{\partial x_k\partial x_k}-K\Omega \qquad \text{..............(3)}$$

where D be diffusivity constant and K be the chemical reaction rate constant and u_l be the lth component of velocity at the point $P(x_1,x_2,x_3)$ at a time t. If it is assumed that the equation (3) is true for fluctuating field then we can write the same equation for the point $A\left(x'_1,x'_2,x'_3\right)$ as

$$\frac{\partial \omega'}{\partial t}+\frac{\partial}{\partial x'_1}\left(u'_l\omega'\right)=D\nabla^2{}_{x'}\omega'-k\omega' \qquad \text{...........(4)}$$

where ω' be the fluctuating part of the reactant concentration field variable at the point A at the time t and u'_1 be the lth component of velocity at A at the same time t.

Now taking the gradient of equation (4) for getting the vector field equation at the point A, we have

$$\frac{\partial}{\partial t}\left(\frac{\partial \omega'}{\partial x'_i}\right)+\frac{\partial^2}{\partial x'_i\partial x'_1}\left(u'_l\omega'\right)=D\nabla^2{}_{x'}\frac{\partial \omega'}{\partial x'_i}-K\frac{\partial \omega'}{\partial x'_i} \qquad (i=1,2,3) \qquad \text{.............(5.1)}$$

Similarly for another point B, we can write the same equation as

$$\frac{\partial}{\partial t}\left(\frac{\partial \omega''}{\partial x''_j}\right)+\frac{\partial^2}{\partial x''_j\partial x''_1}\left(u''_l\omega''\right)=D\nabla^2{}_{x''}\frac{\partial \omega''}{\partial x''_j}-K\frac{\partial \omega''}{\partial x''_j} \qquad (j=1,2,3) \qquad \text{..............(5.2)}$$

Tensorial equation related to the derived field and its simplification under the assumption of homogeneity and isotropy:

Multiplying equation (5.1) by $\frac{\partial \omega''}{\partial x''_j}$ and equation (5.2) by $\frac{\partial \omega''}{\partial x'_i}$ and adding, we get

$$\frac{\partial}{\partial t}\left(\frac{\partial \omega'}{\partial x'_i}\cdot\frac{\partial \omega''}{\partial x''_j}\right)+\frac{\partial^3\left(u'_l\omega'\omega''\right)}{\partial x''_j\partial x'_i\partial x'_1}+\frac{\partial^3\left(u''_l\omega'\omega''\right)}{\partial x'_i\partial x''_j\partial x''_1}=D\left(\nabla^2{}_{x'}+\nabla^2{}_{x''}\right)\left(\frac{\partial \omega'}{\partial x'_i}\cdot\frac{\partial \omega''}{\partial x''_j}\right)-2K\left(\frac{\partial \omega'}{\partial x'_i}\cdot\frac{\partial \omega''}{\partial x''_j}\right) \text{....(6)}$$

Now, taking $x''_i-x'_i=\xi_i$, we get $\frac{\partial}{\partial x'_i}=-\frac{\partial}{\partial \xi_i}$ and $\frac{\partial}{\partial x''_j}=\frac{\partial}{\partial \xi_j}$

Taking the averages of respective terms the equation (6) can be written as

$$\frac{\partial}{\partial t}\left(\overline{\frac{\partial \omega'}{\partial x_i'}\cdot\frac{\partial \omega''}{\partial x_j''}}\right)-\frac{\partial}{\partial \xi_i}\left[\overline{\frac{\partial}{\partial x_i'}(u_l'\omega')\frac{\partial \omega''}{\partial x_j''}-\frac{\partial}{\partial x_j''}(u_l''\omega'')\frac{\partial \omega'}{\partial x_i'}}\right]$$
$$=2D\nabla^2{}_\xi\left(\overline{\frac{\partial \omega'}{\partial x_i'}\cdot\frac{\partial \omega''}{\partial x_j''}}\right)-2K\left(\overline{\frac{\partial \omega'}{\partial x_i'}\cdot\frac{\partial \omega''}{\partial x_j''}}\right) \qquad ....(7)$$

This equation can be written in tensorial form as

$$\left(\frac{\partial}{\partial t}+2K\right)\left(\overline{grad\,\omega}\right)^2 R_{ij}-\frac{\partial}{\partial \xi_i}\left\{2\left(\overline{u^2}\right)^{\frac{1}{2}}\left(\overline{grad\,\omega}\right)^2 T_{ijl}\right\}=2D\nabla^2{}_\xi\left(\overline{grad\,\omega}\right)^2 R_{ij} \qquad ....(8)$$

Assuming the condition of homogeneity and isotropy and simplifying the equation (8), we have

$$\left(\frac{\partial}{\partial t}+2K\right)\left[\left(\overline{grad\,\omega}\right)^2\left(\frac{1}{r}\frac{\partial R}{\partial r}\xi_i\xi_j+K\delta_{ij}\right)\right]+2\left(\overline{u^2}\right)^{\frac{1}{2}}\left(\overline{grad\,\omega}\right)^2$$
$$\left[\left(\frac{1}{r}\frac{\partial^3 M}{\partial r^3}+\frac{4}{r^2}\frac{\partial^2 M}{\partial r^2}-\frac{4}{r^3}\frac{\partial M}{\partial r}\right)\xi_i\xi_j+\left(\frac{\partial^2 M}{\partial r^2}+\frac{4}{r}\frac{\partial M}{\partial r}\right)\delta_{ij}\right]$$
$$=2D\left(\overline{grad\,\omega}\right)^2\left[\left(\frac{1}{r}\frac{\partial^3 R}{\partial r^3}+\frac{4}{r^2}\frac{\partial^2 R}{\partial r^2}-\frac{4}{r^3}\frac{\partial R}{\partial r}\right)\xi_i\xi_j+\left(\frac{\partial^2 R}{\partial r^2}+\frac{4}{r}\frac{\partial R}{\partial r}\right)\delta_{ij}\right] \qquad ....(9)$$

Taking $\overline{AB}$ in the perpendicular direction and putting i = j = 2 or i = j = 3 and also using $\xi_1 = r, \xi_2 = \xi_3 = 0$, the new form of equation (9) may be written as

$$\left(\frac{\partial}{\partial t}+2K\right)\left[\left(\overline{grad\,\omega}\right)^2 R\right]+2\left(\overline{u^2}\right)^{\frac{1}{2}}\left(\overline{grad\,\omega}\right)^2\left[\frac{\partial^2 M}{\partial r^2}+\frac{4}{r}\frac{\partial M}{\partial r}\right]$$
$$=2D\left(\overline{grad\,\omega}\right)^2\left[\frac{\partial^2 R}{\partial r^2}+\frac{4}{r}\frac{\partial R}{\partial r}\right] \qquad ....(10)$$

where R and M be scalars dependent on $\left(\vec{\xi}.\vec{\xi}\right)=r^2$

For isotropic turbulence, the Karman-Howarth equation, which stems from the Navier-Stokes equations, fully describes the dynamics of the two-point velocity correlation. It does not, however, provide a very clear picture of the processes involved in the energy cascade. Some further insights can be gained by examining the Navier-Stokes equations in the wave-numbers space. It is essential to examine the energy spectrum of isotropic turbulence based on the exact solutions. Also it is worthwhile to note that using the four different kinds of the two-point correlation functions, one can obtain the asymptotic behavior of energy spectrum, depending on the different distributions of turbulence parameters.

Loitsiansky's type of invariant as obtainable from equation (10)

Now we are going to have an integral from the equation (10) which is equivalent to the scalar form of Karman-Howarth equation. A Loitsiansky type invariant can be obtained from the above equation as follows.

Multiplying equation (10) by ω^m and integrating it with respect to t, we get

$$\frac{\partial}{\partial t}\left[\left(\overline{grad\,\omega}\right)^2\int_0^\infty \omega^m R(r,t)dr+\left\{\left(\overline{u^2}\right)^{\frac{3}{2}}\left(\overline{grad\,\omega}\right)^2\omega^m M'(r,t)+\left(\overline{grad\,\omega}\right)^2\omega^m R'\right\}_0^\infty\right]=0 \qquad ....(11)$$

Assuming the case of isotropic turbulent flow field along with isotropic $grad\,\omega$ field, we have

$$\underset{\omega\to\infty}{Lt}\ r^4 R'(\omega)=0,\quad \underset{\omega\to\infty}{Lt}\ r^4 M'(\omega)=0$$

where
$R'=\dfrac{\partial R(r,t)}{\partial \omega}$, $M'=\dfrac{\partial M(r,t)}{\partial \omega}$

From equation (11) we can get

$$\frac{d}{dt}\int_0^\infty \omega^m\left(\overline{grad\,\omega}\right)^2 R(r,t)dr \quad \begin{aligned} &> 0 \quad for\ m>4 \\ &= 0 \quad for\ m=4 \\ &< 0 \quad for\ m<4 \end{aligned} \qquad ....(13)$$

Considering the second possibility of equation (13) we can get the integral of Loitsiansky type neglecting the rate constant of chemical reaction of the couple field.

$$\int_0^\infty r^4\left(\overline{grad\,\omega}\right)^2 R(r,t)dr=\Lambda\, grad\,\omega \qquad ....(14)$$

where $\Lambda\, grad\,\omega$ is termed as Loitsiansky invariant of the $grad\,\omega$ field and it will remain constant in the absence of any internal chemical rate constant i.e. K=0.

Numerical calculations and graphical representations for analysis

Now we can write

$$R(r,t)=\frac{f-g}{r^2}\xi_i\xi_j+g\delta_{ij} \qquad ....(15)$$

where $f(r,t)=\dfrac{\overline{u_\alpha u_\alpha'}}{\overline{u^2}}$ and $g(r,t)=\dfrac{\overline{u_\beta u_\beta'}}{\overline{u^2}}$. Here u_α *and* u_α' be the velocities acting horizontally on the points A and B respectively separated by a distance r. Similarly u_β *and* $u_{\beta'}$ be the respective velocities acting on the same points vertically.

u_α →A________r________B→ u_α' u_β

↑A________r_________B↑ u_β'

Now, taking i = j, we have $\xi_i\xi_j = r^2$ and $\delta_{ij} = 1$

Using these equations (15) can be written as

$$R(r,t)=f(r,t) \qquad ....(16)$$

Now, using equation (16), the L.H.S. of the equation (14) can be written as

$$I=\left(\overline{grad\omega}\right)^2\frac{\overline{u_\alpha u_\alpha'}}{\overline{u^2}}\int_0^\infty r^4 dr \quad (17)$$

$$I = \left(\overline{grad\ \omega}\right)^2 \frac{\overline{u_\alpha u'_\alpha}}{\overline{u^2}} \frac{B^5}{5} \qquad \text{.........................(18)}$$

where B is a very large quantity and I is the Loitsiansky type Invariant of $grad\,\omega$ field.

Now, we can plot I against u_α and u'_α respectively.

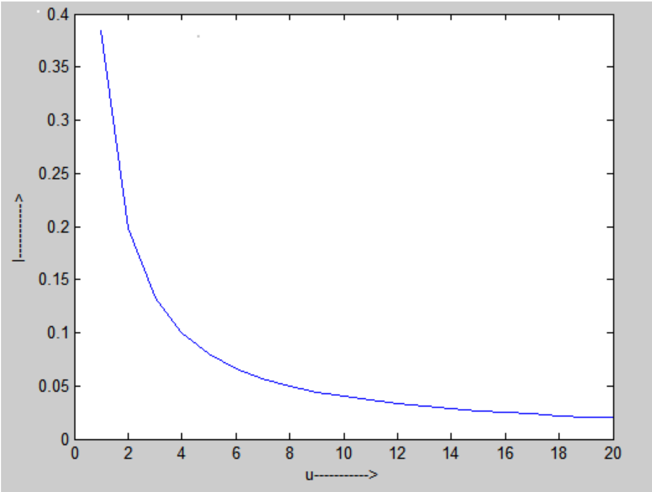


Fig. 2. Loitsiansky type Invariant vs horizontal velocity component at A

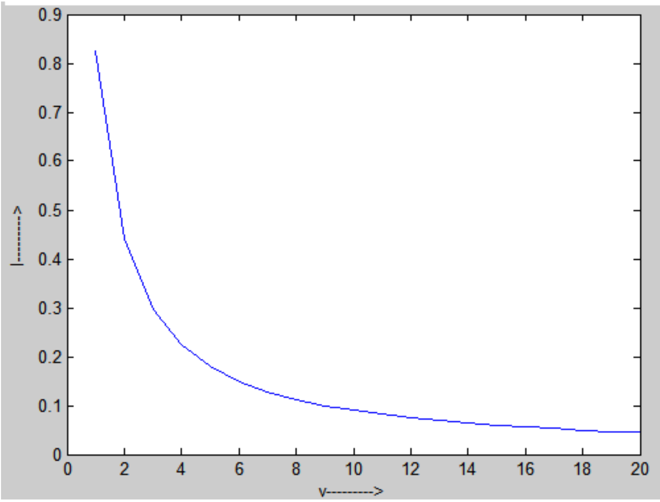
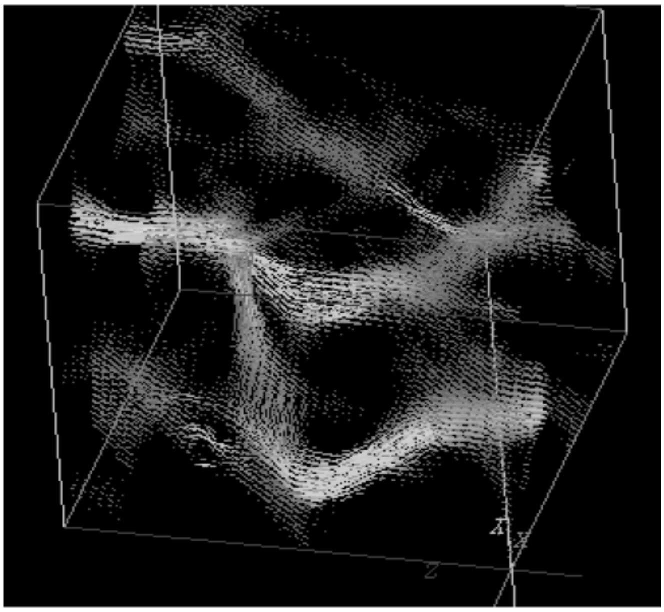


Fig. 3. Loitsiansky type Invariant vs. horizontal velocity component at B

As evident from the Figures 2 and 3 for horizontal velocities at A and B denoted by u and v respectively, the Loitsiansky type Invariant remains constant except for the very small horizontal velocities.

DISCUSSION AND CONCLUSION

The investigation which has been presented in this paper indicates that the exact statistical theory for isotropic turbulence tractable based on the new exact solution of Karman-Howarth equation. The results obtained confirm the qualitatively consistency of the approximation and suggest a satisfactory quantitative agreement with experiment in the range of which is treated. The computation establishes the fact that Loitsiansky type Invariant remains constant during the couple field decay in absence of any internal chemical reaction except for very small horizontal velocities. Analytical study of present theory may be useful in understanding the nature of isotropic turbulence. A difficulty in comparing the present results with experiment is the dependence of the results on the essentially arbitrary choice of turbulent parameters and the initial conditions. Analytical study of present theory may be useful in understanding the nature of isotropic turbulence. A difficulty in comparing the present results with experiment is the dependence of the results on the essentially arbitrary choice of turbulent parameters and the initial conditions.



To the extent that comparison is possible, the final times in the present calculation probably should be compared with early period of decay of grid turbulence. Subject to the reservations stated, the values of shown in this paper appears to be consistent with experiment values and other turbulence theories, such as DIA, EDQNM, qualitatively. Self-preservation is an old topic, but the mathematical aside is still open. It is essential to lock of the exact analysis on the invariance of Karman-Howarth equation. Tennekes and Lumley's remarks on invariance will help to understand this issue: Associated with, but distinct form, asymptotic invariance is the concept of "self-preservation" or local invariance. In simple flow geometries, the characteristics of the turbulent motion at the some point in time and space appear to be controlled mainly by immediate environment. The time and length scales of the flow may vary slowly downstream, but if the turbulence time scales are small enough to permit adjustment to the gradually changing environment, it is often possible to assume that the turbulence is dynamically similar everywhere if nondimensionalized with local length and time scales. One can seems to say that: in present paper, new appropriate length and kinetic energy scales may have been found. From mathematical aspects, the group invariant theory will lead a rich analysis on the meaning of what have done. The invariants of Karman-Howarth equation will be become new direction. At last, we note that : We have revisited the old problem firstly presented by Sedov (1982), and found richer mathematical structure in this paper compared to Sedov's work. The results help us to offer a unified investigation of isotropic turbulence. Deep insights into internal structure of isotropic turbulence have been gained based on a new complete set of the exact solutions of the Karman-Howarth equation. Simple comparison shows that the special solution found by Sedov (1982) belongs to one kind of our new set of solutions. Here, the author would like to emphasize the idea presented by Sedov at the end this paper, that is: At the first glance, not more than one function can be found from a single equation. Nevertheless, a careful consideration of the mathematical structure of this equation makes it possible to carry out an analysis of all possible cases and to find, to the accuracy of one basic constant α, all admissible solutions of the problem in question. This aspect and the appropriate mathematical analysis of the problem escaped the 3D LGA's used for chemical attention of a number of scientists who developed the theory of reactions, other complex fluids turbulent motions in fluids and processed the experimental data (Sedov, 1982; Zheng Ran, 2006). This outcome is of great importance for many investigations e.g. atmospheric science, oceanic flow studies, industrial problems. Plume flow, flow in turbines or pipe flows particularly in chemical factories, power plants (thermal, nuclear) are of important areas of investigations where Loitsiansky's invariant has notable constant to consider.

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