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RESEARCH ARTICLE

SPECIAL PAIRS OF PYTHAGOREAN TRIANGLES AND HARSHAD NUMBERS

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ABSTRACT

We present pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is six times the Harshad number. Also we present the number of pairs of primitive and Non-primitive Pythagorean triangles.

INTRODUCTION

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer (Sierpinski, 2003; Gopalan and Janaki, 2008; Gopalan and Vijayashankar, 2010; Gopalan and Leelavathi, 2008 & 2007; Gopalan and Gnanam, 2007; Gopalan and Devibala, 2006; Gopalan and Sivakami, 2013 & 2012; Meena *et al.,* 2014; Gopalan and Janaki, 2008; Gopalan and Sangeetha, 2010; Gopalan *et al.,* 2010; Gopalan and V.Geetha, 2013). Apart from the other patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhurva numbers. These numbers have been presented in (Kapur, 1997; Bert Miller, 1980; Charles Bown, 1981; Sastry, 2001). In (Gopalan *et al.,* 2013; Gopalan and Janaki, 2008; Gopalan and Janaki, 2008), special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. In (Mita Darbari, 2014), special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited.

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In (Janaki and Radha, 2016), special Pythagorean triangles connected with Harshad numbers. In (Janaki and Saranya, 2016), special pairs of Pythagorean triangles and Dhurva numbers are presented. Recently in, special pairs of Pythagorean triangles and Jarasandha numbers are presented. In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is six times the Harshad number.

Basic Definitions

Pythagorean Equation

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equations are also referred to as Pythagorean triangle and denote it by T(x, y, z). Also, in Pythagorean triangle T(x, y, z): $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Primitive

Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$ where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and gcd (m,n) =1.

Harshad Number: It is an integer that is divisible by the sum of its digits.

Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q (m > q > 0), and p, q (p > q > 0) respectively, such that m+ p + q = the three digit Harshad number 171. Let P_1, P_2 be the perimeters of PT_1, PT_2 such that $P_1 - P_2 = 6$ times the 3-digit Harshad number 171.

The above relation leads to the equation

We have presented below in table 1 the values of m, p, q, P_1 and P_2.

S.No.	m	q	p	P_1	P_2	$\frac{P_1 - P_2}{6}$
1.	60	54	57	13680	12654	171
2.	61	52	58	13786	12760	171
3.	62	50	59	13888	12862	171
4.	63	48	60	13986	12960	171
5.	64	46	61	14080	13054	171
6.	65	44	62	14170	13144	171
7.	66	42	63	14256	13230	171
8.	67	40	64	14338	13312	171
9.	68	38	65	14416	13390	171
10	69	36	66	14490	13464	171
11	70	34	67	14560	13534	171
12	71	32	68	14626	13600	171
13	72	30	69	14688	13662	171
14	73	28	70	14746	13720	171
15	74	26	71	14800	13774	171
16	75	24	72	14850	13824	171
17	76	22	73	14896	13870	171
18	77	20	74	14938	13912	171
19	78	18	75	14976	13950	171
20	79	16	76	15010	13984	171
21	80	14	77	15040	14014	171
22	81	12	78	15066	14040	171
23	82	10	79	15088	14062	171
24	83	8	80	15106	14080	171
25	84	6	81	15120	14094	171
26	85	4	82	15130	14104	171
27	86	2	83	15136	14110	171

Thus, it is seen that there are 27 pairs of Pythagorean triangles such that for each pair the difference in the perimeters is six times the 3-digit Harshad number 171. Out of these 27 pairs of Pythagorean triangles 10-pairs are non-primitive and in each of the remaining 17 pairs, one of the triangle is primitive and the other is non-primitive triangle.

A similar observation, regarding 4-digit ,5-digit and 6-digit Harshad numbers are exhibited in the table 2 below:

Harshad Number	Pairs of Pythagorean triangles	Pairs of non-primitive Pythagorean triangles	Pairs of primitive and non-primitive Pythagorean triangles
3675	611	217	394
11025	1836	1163	673
155655	25940	10381	15559

Conclusion

One may search for the connections between the pairs of Pythagorean triangles and other Harshad numbers and other number patterns.

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