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# RESEARCH ARTICLE

## VERTEX-MAGIC TOTAL LABELING OF FAMILY OF GRAPHS

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## ARTICLE INFO

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## ABSTRACT

A vertex- magic total labelling of a (p,q)-graph G with p-vertices and q-edges is defined as one-to-one correspondence by taking vertices and edges onto the integers $\{1, 2, - - - |V|+|E|\}$ with the property that the sum of labels on a vertex and the edges incident on it is a constant, independent for all the vertices of graph. In this paper, the properties of these labelings are studies. It explains how to construct labelings for various families of graphs which includes paths, cycles, complete graph of odd order, complete bipartite graphs, products of cycles, generalized Petersen graph P (n, m) for all $n \geq 3, 1 \leq m \leq \lfloor (n-1)/2 \rfloor$.



## INTRODUCTION

Throughout this paper, we let the graph G considered as finite, undirected with vertex set V(G) and edge set E(G) and let $p = |V(G)|$ & $q = |E(G)|$ be the number of vertices and number of edges of G respectively. We will deal only with connected graphs, although the concepts apply equally to graphs with more than one connected components. There is a wide variety of graph labelings. Some of most studied are graceful and harmonious labelings, which have a number of applications. When the idea of magic squares is generalized for graphs, we obtain magic type of labelings. The first author who applied the idea of magic labelling of graphs in early 60's was Sedlàček. He defined a graph to be magic if it has an edge-labeling, with real number such that the sum of labels around any vertex is equal to the same constant. Kotzig and Rosa defined magic labelling to be a total labelling if the labels are consecutive integer starting from 1, labelled to both vertices as well as edges.

### Vertex-magic total labelling

### Definition 1

In [6] this paper we introduce the notion of a vertex –magic total labelling. A one-to-one map λ from $V(G) \cup E(G)$ onto the integers $\{1, 2, - - - p+ q\}$ is a vertex-magic total labeling if there is a constant **k** so that for every vertex x

$$\lambda(x) + \sum \lambda(x,y) = k \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where the sum is over all vertices 'x' adjacent to vertex 'y'. The constant 'k' is called the magic constant for λ.

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**Definition 2**

**Cycles and paths**

A walk of length k from node $v_0$ to node $v_k$ is a non-empty graph P = (V, E) of the form V = {$v_0$,$v_1$,...,$v_k$} E = {($v_0$,$v_1$),...,($v_{k-1}$,$v_k$)} where edge j connects nodes j   1 and j (i.e. |V| = |E|+1). A trail is a walk with all different edges. A path is a walk with all different nodes (and hence edges). A walk or trail is closed when $v_0 = v_k$. A cycle is a walk with different nodes except for $v_0= v_k$.

Define map $\lambda^{'}$ on $E \cup V$ by $\lambda^{'}(x_i) = M + 1 - \lambda(x_i)$ where M=p+q for any vrtex $x_i$ & $\lambda^{'}(x_i x_{i+1}) = M + 1 - \lambda(x_i x_{i+1})$ for any edge $x_i x_{i+1}$. We will call $\lambda^{'}$ the dual λ.

**Vertex –magic total labelling of cycles and path:**

**Theorem 1:** [1]The n-cycle $C_n$ has a labelling for any $n \geq 3$.

**Proof:** consider edge-magic total labelling $\lambda^{'}$ for $C_n$ for every $n \geq 3$.

There is a constant $k$ so that $\lambda^{'}(x_i) + \lambda^{'}(x_i x_{i+1}) + \lambda^{'}(x_{i+1}) = k$ for all vertices $x_i$ of $C_n$. If we define a new mapping λ by $\lambda(x_i) = \lambda^{'}(x_i x_{i+1})$ & $\lambda(x_i x_{i+1}) = \lambda^{'}(x_{i+1})$, then we clearly have $k$ as the weight at each vertex and so λ is vertex-magic total labelling of G.

**Result:** Vertex-magic total labeling of cycles of odd length.

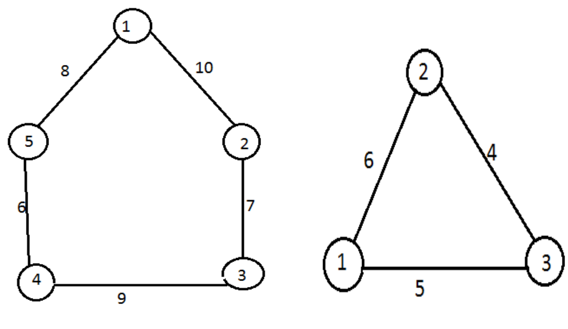
Let $C_n$ have vertices $x_i$ and $x_i x_{i+1}$ for $i = 0,1,2 - - - n-1$ .subscripts will be taken modulo n. For n odd, labels to vertices and edges are assigned as,

$$\lambda(x_i) = n - i,\ \lambda(x_i x_{i+1}) = \begin{cases} \frac{i}{2}+1, i=even \\ b+\frac{n+i}{2}+1, i=odd \end{cases}$$

Then clearly we have $k$ as the weight at each vertex and so λ is a vertex-magic total labelling of G.

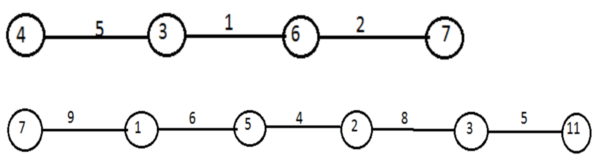
**Example 2.1** shows example for labelling of $C_5$ & $C_3$

k=19 & k=12




Corollary 1: $P_n$ , the path with n-vertices has a labeling for any $n \geq 3$.

Figure 2.2 shows example for labeling of $n \geq 3$ $P_4$ & $P_6$.



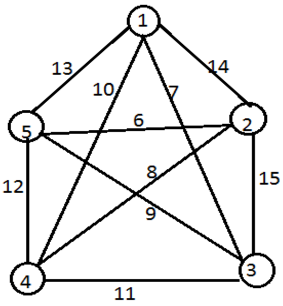
**Definition 3:**

**Complete graph:** An undirected graph is called complete or fully connected if each node is connected to every other node by an edge, that is when E = V×V. A complete graph of n vertices is denoted Kn

**Vertex magic total labelling for complete graph with odd number of vertices**

**Theorem 2:** [2]There exist a vertex magic total labelling for Kn, where n is odd. The magic constant 'k' for the labelling of complete graph is $y_1, y_2, ---, x_i x_{i+m} 1 \le i \le n$ is the maximum possible.

Example 3.1 shows example for labelling of $K_5$.




**Definition 4:**

**Complete bipartite graph**

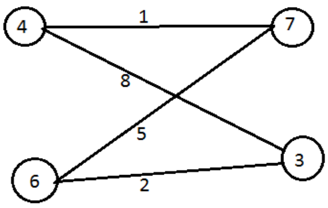
A graph is called bipartite if V can be partitioned into two subsets V1 ⊂V and V2 ⊂V, whereV1∩V2 =   andV1∪V2 = V, such that E⊆V1×V2. If |V|= m and |V|= n, and E = V1×V2, then G is called a complete bipartite graph and is denoted by Km, n.

**Vertex magic total labelling for Complete Bipartite graph with odd number of vertices.**

**Theorem 3:** [1] There is a labelling for $K_{m,m}$ for every m>1.

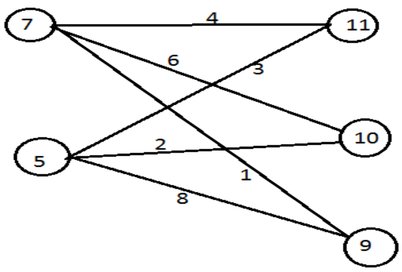
The magic constant us given by $k = \frac{1}{2}(m+1)\left(m^2 + 2m + 1\right) - (m+1)$

Example 3.1 shows example for labelling of $K_{2,2}$




**Theorem 4:** [1] There is a labelling for $K_{m,m+1}$ for all m.

Example 3.2 shows example for labelling of $K_{2,3}$




**Definition 5:**

**Generalized Petersen graph**

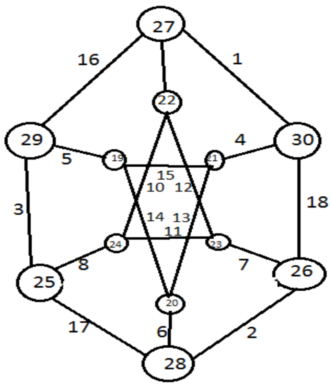
Watkins [7] defined generalized Petersen graph as, P(n,m),

$n \geq 3$ and $1 \leq m \leq \left\lfloor \frac{n-1}{2} \right\rfloor$ consists of an outer n-cycles $y_1, y_2, ---, y_n$ as a set of n spokes $y_i x_i$, $1 \leq i \leq n$ & n inner edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The standard Petersen graph is the instance P (5, 2)

**Vertex magic total labelling for Generalized Petersen graph with odd number of vertices.**

**Theorem 5:**[3] For $n \geq 3, 1 \leq m \leq \left\lfloor \frac{n-1}{2} \right\rfloor$, every generalized Petersen graph has vertex magic total labelling with the magic constant $k = 9n + 2$

Example 5.1 shows example for labelling of P (6, 2)



**Definition 6:**

**Product of cycles:** The Cartesian product G□H of graphs G & H is a graph such that, The vertex set of G□H is the Cartesian product V(G)XV(H); and Any two vertices $u' = v'$ & are adjacent in G□H if and only if either

- u=v & $u'$ s adjacent with $v'$ in H or
- $u' = v'$ & u is adjacent with v in G.

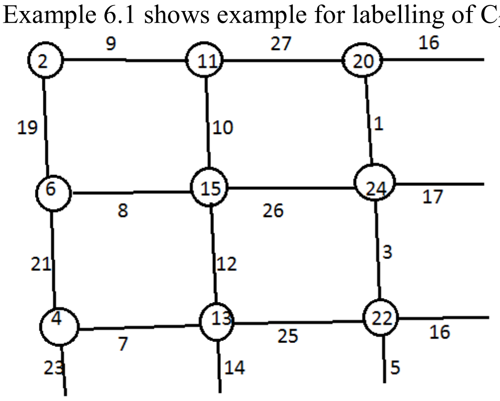
**Vertex magic total labelling for product of cycles:**

**Theorem 6:** [4] There exit a vertex magic total labelling of $C_m x C_n$ for each m,n ≥3 and n odd with magic constants

$$k = \frac{1}{2} m(15n+1) + 2, \lambda\left(v_{i,j}, v_{i,j+1}\right) = 3m\left(\frac{n+1}{2} + 1\right) - i, j =, 2i+1, i = 0, = 3mj + 2m - 2(i-1), i = 1, 2, ---, m-1$$

$$v_{i,j} v_{i+,j} i = 0, 1, 2, ---, m-1 \;\&\; j = 0, 1, 2, ---, n-1 m, n \geq 3$$

Example 6.1 shows example for labelling of $C_3 x C_3$ with magic constant.



Note: Let

$C_m x C_n$ have vertices $v_{i,j}$, $v_{i+,j}$

$i = 0, 1, 2, ---, m-1$ & $j = 0, 1, 2, ---, n-1$ for $m, n \geq 3$ and n is odd.

Consider the following labelling, where i,j all taken as modulo m & n respectively.

$$\lambda\left(v_{i,j}\right) = 3mj + 2 \qquad \text{For } i = 0$$
$$= 3mj + 2m - 2(i-1) \quad \text{For } i = 1, 2, ---, m-1$$

$$\lambda\left(v_{i,j}, v_{i+1,j}\right) = 3m[n-(j+1)] + 2i + 1$$
$$\lambda\left(v_{i,j}, v_{i,j+1}\right) = 3m\left(\frac{1}{2}+1\right) - i \quad \text{For } j = \text{even}$$
$$= 3m\left(\frac{n+1}{2}+1\right) - i \quad \text{For } j = \text{odd}$$

**Conclusion**

- This labelling can be easily altered to obtain some other values for the magic constant k.
- It is not easy to extent this method for vertex magic labelling of $C_m x C_n$ with n even (for even cycles).

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