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RESEARCH ARTICLE

NEW TYPE OF CONNECTEDNESS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper we generalize the concept of I_π-connectedness in ideal topological spaces. Further the relationship of other related classes of connected spaces are investigated.

INTRODUCTION

Ideal topology is a topological space endowed with an additional structure namely the ideal. Kuratowski (Kuratowski, 1996) introduced the concept of local functions in ideal topological spaces. The notion of Kuratowski operator plays a vital role in defining ideal topological space which has its application in localization theory in set topology by Vaidyanathaswamy (Vaidyanathaswamy, 1945). Ideals have been frequently used in the fields closely related to topology such as real analysis measure theory and lattice theory. In 1990, Jankovic and Hamlett (Jankovic and Hamlett, 1990; Jankovic and Hamlett, 1992) developed new topologies from old via ideals and introduced I-open sets with respect to an ideal I in 1992. The properties like continuity, separation axioms, connectedness, compactness and resolvability have been generalized using the concept of ideals in topological spaces. Erdal Ekici (Ekici and Noiri, 2008) *et al* introduced the notion of connectedness in ideal topological spaces. The purpose of this paper is to study the notion of I_π-connectedness in ideal topological spaces and discuss their properties.

Preliminaries

Throughout this paper (X, τ) is a topological space on which no separation axioms are assumed unless explicitly stated. The notation (X, τ,*I*) will denote the topological space (X, τ) and an ideal I on X with no separation properties assumed. For A ⊆ (X, τ), Cl(A) and Int(A) respectively denote the closure and interior of A with respect to τ.

Definition: 2.1(13)

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following properties: (1) A ∈ I and B ⊆ A implies B ∈ I.
(2) A ∈ I and B ∈ I implies A∪ B ∈ I.

An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I).

Definition: 2.2(13)

For a subset A of X, A (I) = {*x* ∈ X: U ∩ A I for each neighbourhood U of *x*} is called the local function of A with respect to I and τ. We simply write A instead of A (I).

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Definition: 2.3(13)

It is well known that $Cl^{*}(A) = A \cup A$ defines a Kuratowski closure operator for τ (I) which finar than τ.

Definition: 2.4(13)

A basis β(I, τ) for τ (I) can be described as follows: β(I, τ) = {U E: U ∈ τ and E ∈ I}.

Definition: 2.5

A subset A of an ideal topological space (X, τ, *I*) is

(1) -perfect (9), if A =A
(2) - closed (10), if A ⊆ A
(3) *-dense (5), if $Cl^{*}(A) = X$
(4) τ^{*}-closed set (10), if $A = Cl^{*}(A)$

Definition: 2.6(16)

A subset A of a space (X, τ) is said to be regular open set, if A = int(cl(A)).

Definition: 2.7(18)

Finite union of regular open sets in (X, τ) is π-open in (X, τ).The complement of π-open in (X, τ) is π-closed in (X, τ).

Definition: 2.8(1)

Given a space (X, τ,*I*), a set operator $()^{*\pi}$: P(X) → P(X) is called the π-local function of *I* with respect to τ is defined as follows: for A ⊆ X, $(A)^{*\pi}(I,\tau)$ = {*x*∈ X/ $U_x \cap A$ *I*, for every $U_x \in \pi N(x)$}, where πN(*x*)} = {U ∈ πO(*x*) | *x* ∈U}. We write π-local function as $A^{*\pi}(I)$ or $A^{*\pi}$ or $A^{*\pi}(I, \tau)$.

Definition: 2.9(7)

An ideal space (X, τ, I) is said to be -hyperconnected, if A is -dense for every open subset A ≠ϕ of X.

Definition: 2.10(8)

An ideal space (X, τ, I) is said to be -connected, if X cannot be written as the disjoint union of a nonempty open set and a nonempty -open set.

Definition: 2.11(8)

An ideal space (X, τ, I) is said to be $_s$-connected, if X cannot be written A is not the union of two - separated sets in (X, τ, I).

Definition: 2.12(8)

Let X be an ideal space and *x* ∈ X. The union of all $_s$-connected subsets of X containing *x* is called the $_s$-component of X containing *x*.

3. I_π- Hyperconnected space

Definition: 3.1

A subset A of an ideal space (X, τ, I) is said to be I_π- dense, if $cl^{*\pi}(A) = X$.

Definition: 3.2

An ideal topological space (X, τ, I) is said to be I_π-hyperconnected, if every pair of non-empty I_π-open set intersects.

Definition: 3.3

An ideal space (X, τ, I) is said to be I_π-hyperconnected, if A is I_π-dense for every non-empty open subset of X.

Example: 3.4

Let (X, τ) be an indiscrete space with ideal I = { A ⊆ X | p A} on X. Then

Then $A^{*\pi}$ = X if p∈ A
= ϕ if p A

Therefore $Cl^{*\pi}(A)$ = X if p∈ A
= A if p A

Thus $\tau^{*\pi}$ = {A ⊆ X: p A}∪ {ϕ}.
The only nonempty I_π- open set is X and $Cl^{*\pi}(X)$ = X.
Hence (X, τ, I) is I_π- hyperconnected.

Example: 3.5

X = {a, b, c}
τ = {X, ϕ, {a}, {c}, {a, c}}
I = {ϕ, {b}, {c}, {b, c}}
$\tau^{*\pi}$ = {X, ϕ, {a}, {c}, {a, b}, {a, c}}
Now A= {a} is a nonempty I_π-open set in X and $Cl^{*\pi}(A)$ = {a, b} ≠ X.
Hence (X, τ, I) is not I_π- hyperconnected.

Theorem: 3.6

Every -hyperconnected space is I_π-hyperconnected.

Proof

Given X is a -hyperconnected space. Since X is -hyperconnected, A is -dense and so $Cl^{*}(A)$ = X. Then every -open set in X intersects. Since τ^{*} is finer than $\tau^{*\pi}$, every I_π-open set intersects. Hence it is I_π-hyperconnected.
The converse of the above need not be true as shown in the following example:

Example: 3.7

X = {a, b, c, d}
τ = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}}
I = {ϕ, {d}}
τ^{*} = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}}
$\tau^{*\pi}$ = {X, ϕ, {b}, {a, b, c}}
Here every nonempty I_π-open set in X intersects but every nonempty -open set in X does not intersect. Therefore (X, τ, I) is I_π-hyperconnected space but not -hyperconnected.

Definition: 3.8

Let f: (X, τ, I) (Y, σ) be a function from an ideal topological space to a topological space, then f is said to be I_π- continuous, if f^{-1}(A) is I_π-open in X for every open set A in Y.

Theorem: 3.9

Let (X, τ, I) be an ideal topological space then the following are equivalent:
1. X is I_π-hyperconnected space
2. Every I_π- continuous function of X into a Hausdorff space is constant
3. Every I_π- continuous function f: X {a,b} with discrete topology is constant.

Proof

(1) (2)

Suppose there exists a Hausdorff space Y and a I_π- continuous function, such that f is not constant. Then there exists two distinct points x and y such that f(x) ≠ f(y). Since Y is Hausdorff space, there exists two disjoint open set G and H such that f(x) ∈ G, f(y)

∈ H and G ∩ H = ϕ. Since f is I_π- continuous f^{-1}(G) and f^{-1}(H) are I_π-open sets in X, then f^{-1}(G) ∩ f^{-1}(H) = f^{-1}(G ∩ H) = ϕ which is a contradiction. Hence f is a constant function.

(2) (3) : Straight forward
(3) (1)

By hypothesis it is obvious that f^{-1}({a}) = X or f^{-1}({b}) = X. In both cases X is I_π-hyperconnected space.

Theorem: 3.10

If X is a I_π-hyperconnected space, f : X Y is continuous and G(f) is closed in X × Y then f is constant.

Proof

Suppose that f is not constant, then there exists two points *x* and *y* such that f(*x*) ≠ f(*y*). Then we have (*x*, f(*y*)) ∈ (X × Y) G(f). Since G(f) is closed, there exists open neighbourhood U and V of *x* and f(*y*) such that (U × V) ∩ G(f) = ϕ which contradicts the hypothesis. Hence f is contant.

Definition: 3.11

A function f: (X, τ, I) (Y, σ, J) said to be I_π- irresolute, if f^{-1}(A) is I_π-open in X for every I_π-open set A in Y.

Theorem: 3.12

If X is a I_π-hyperconnected space and f: (X, τ, I) (Y, σ, J) is I_π- irresolute surjection then Y I_π-hyperconnected space.

Proof

Suppose Y is not I_π-hyperconnected space, then there exists non-empty disjoint I_π-open sets G and H of Y. Since f is I_π-irresolute, f^{-1}(G) and f^{-1}(H) exists, f^{-1}(G) ∩ f^{-1}(H) = f^{-1}(G ∩ H) = ϕ which is a contradiction to the fact that X is I_π-hyperconnected space.

Definition: 3.13

Let f: (X, τ, I) (Y, σ, J) be a function from an ideal topological space to an ideal topological space such that f(I) ⊆ J, then f is said to be $M^{*\pi}$- open if f(A) is I_π-open in Y for every I_π-open set A in X.

Theorem: 3.14

If Y is a I_π-hyperconnected space and f: (X, τ, I) (Y, σ, J) is an injection $M^{*\pi}$-open map, then X is I_π-hyperconnected space.

Proof

Let U and V be any nonempty I_π- open sets of X. Since f is $M^{*\pi}$-open, f(U) and f(V) exists such that f(U) and f(V) are nonempty and f(U) ∩ f(V) = ϕ which implies that U∩ V ≠ ϕ. Thus X is I_π-hyperconnected space.

Definition: 3.15

An ideal topological space (X, τ, I) is said to be I_π- ultraconnected space, if every pair of I_π- closed sets intersects.

Remark: 3.16

I_π- ultraconnectedness and I_π- hyperconnectedness are independent of each other.

Example: 3.17

X = {a, b, c}
τ = {X, ϕ, {a}}
I = {ϕ, {b}, {c}, {b, c}}
$\tau^{*\pi}$ = {X, ϕ, {a}, {a, b}, {a, c}}

Here every non empty I_π- open sets intersect. Therefore (X, τ, I) is I_π- hyperconnected space. But the complement of I_π- open sets do not intersect. Hence it is not I_π- ultraconnected space.

Example: 3.18

X = {a, b, c}
τ = {X, ϕ, {a}, {b}, {a, b}}
I = {ϕ, {c}}
$\tau^{*\pi}$ = {X, ϕ, {a}, {b}, {a, b}}
Here (X, τ, I) is not I_π- hyperconnected space, but it is I_π- ultraconnected space.

4. I_π- connected space

Definition: 4.1

An ideal space (X, τ, I) is called I_π- separation, if every pair of proper sets with A ∈ τ and B ∈ $\tau^{*\pi}$ such that A ∩ B = ϕ and X = A ∪ B.

Definition: 4.2

An ideal space (X, τ, I) is called I_π- connected if and only if there is no I_π- separation on X. If (X, τ, I) has I_π- separation, then (X, τ, I) is said to be I_π- disconnected.

Theorem: 4.4

Every I_π- hyperconnected space is I_π- connected.

Proof

Suppose that (X, τ, I) is I_π- disconnected. Then there exists two nonempty proper sets A and B with A ∈ τ and B ∈ $\tau^{*\pi}$ such that A ∩ B = ϕ and X = A ∪ B which is contradiction. Hence (X, τ, I) is I_π- connected.

The converse of the above is not true as shown in the following example:

Example: 4.5

X = {a, b, c, d}
τ = {X, ϕ, {a},{a, c}, {a, b, c}}
I = {ϕ, {a}, {d}, {a, d}}
Let A = {a} and B = {b, c} then A∪ B ≠ X. Therefore (X, τ, I) is I_π- connected but not I_π- hyperconnected.

Theorem: 4.6

Let (X, τ, I) be the ideal topological space, then every -connected space is I_π- connected.

Proof

Suppose X is - connected space. Then X cannot be written as the disjoint union of a nonempty open set and a nonempty - open set. Since τ^* is finer than $\tau^{*\pi}$, X cannot be as the disjoint union a nonempty open set and a nonempty I_π- open set. Hence it is I_π- connected.

The converse of the above is not true as shown in the following example:

Example: 4.7

X = {a, b, c, d}
τ = {X, ϕ, {a}, {a, b}, {a, b, c}}
I = {ϕ, {a}}
Let A = {a, b} and B = {c} then A∪ B ≠ X. Therefore (X, τ, I) is I_π-connected but not *-connected.

Theorem: 4.8

Let f : (X, τ, I) (Y, σ, J) be a surjective continuous function. If (X, τ, I) is I_π-connected then (Y, σ, J) is I_π-connected.

Proof

Suppose Y is I_π-disconnected. Then there exists a I_π- separation A, B of Y with A ∈ σ and B ∈ $\sigma^{*\pi}$ such that A ∩ B = ϕ and Y = A ∪ B. Thus we have f^{-1}(A ∩ B) = f^{-1}(A) ∩ f^{-1}(B) = ϕ and f^{-1}(A ∪ B) = f^{-1}(A) ∪ f^{-1}(B) = f^{-1}(Y) = X. Thus f^{-1}(A) and f^{-1}(B) are I_π- separations of X which is contradiction to I_π- connectedness of X. Hence (Y, σ, J) is I_π-connected.

Theorem: 4.9

An ideal topological space X is I_π- connected if and only if X and ϕ are the only subsets of X which are both I_π- open and I_π-closed.

Theorem: 4.10

Let f be a I_π- irresolute function from a I_π-connected space X into an ideal topological space Y. Then f(X) is I_π-connected.

Proof

Suppose f(X) is not I_π-connected then f(X) = G ∪ H = ϕ, where G and H are I_π-open sets in f(X), since f is I_π-irresolute f^{-1}(G) ∪ f^{-1}(H) = X and f^{-1}(G) ∩ f^{-1}(H) = ϕ. But this is not possible when X is I_π-connected. Hence I_π-irresolute image of a I_π-connected space is I_π-connected.

Theorem: 4.11

If I_π-connected B ⊆ A, where A is non I_π-connected then B lies completely in any one of the component.

Proof

Given B ⊆ A and A is I_π-disconnected that is A = G ∪ H, where G and H are disjoint non empty open set and non empty I_π-open set. Given B can be written as disjoint union of disjoint non empty open set and non empty I_π-open set which leads to the contradiction. Thus B ⊆ G or B ⊆ H. Hence the proof.

Definition: 4.12

Non- empty subsets A, B of an ideal space (X, τ, I) are called I_π-separated, if $cl^{*\pi}$(A) ∩ B = A ∩ $cl^{*\pi}$(B) = ϕ.

Theorem: 4.13

Let (X, τ, I) be an ideal space. If A and B are I_π-separated sets of X and A ∪ B ∈ τ then A and B are open and I_π-open respectively.

Proof

Let A and B are I_π-separated. Then $cl^{*\pi}$(A) ∩ B = A ∩ $cl^{*\pi}$(B) = ϕ. Therefore B ⊆ X \ $cl^{*\pi}$(A). This implies that B = (A ∪ B) ∩ (X \ $cl^{*\pi}$(A)). Since A ∪ B is open, A ∪ B is I_π-open and X \ $cl^{*\pi}$(A) is I_π-open. Hence B is I_π-open. Similarly we can obtain that A is open.

Theorem: 4.14

Let (X, τ, I) be an ideal space and A, B ⊆ Y ⊆ X. The following are equivalent:
(1) A, B are I_π-separated in Y
(2) A, B are I_π-separated in X

Proof

Suppose A, B are I_π-separated in Y.

$cl_Y^{*\pi}$(A) ∩ B = A ∩ $cl_Y^{*\pi}$(B) = ϕ
$cl^{*\pi}$(A) ∩ Y∩ B = A ∩ $cl^{*\pi}$(B) ∩Y = ϕ
$cl^{*\pi}$(A) ∩ B = A ∩ $cl^{*\pi}$(B) = ϕ
A, B are I_π-separated in X.

The converse is similar.

Definition: 4.15

A subset A of an ideal space (X, τ, I) is called I_π-separated connected (briefly $I_{\pi s}$- connected), if A is not the union of two I_π-separated sets in (X, τ, I).

Theorem: 4.16

Let Y be an open subset of an ideal space (X, τ, I). Then the following are equivalent:

(1) Y is $I_{\pi s}$- connected in (X, τ, I).
(2) Y is I_π- connected in (X, τ, I).

Proof

(1) (2)

Suppose that Y is not I_π- connected. Then there exists non-empty disjoint open and I_π-open sets A, B in Y such that Y = A ∪ B. Then A, B are open and I_π-open sets in X respectively, since Y is open in X. Since A and B are disjoint then $cl^{*\pi}(A) \cap B = \phi = A \cap cl^{*\pi}(B)$. This implies that A, B are I_π-separated sets in X. Thus Y is not $I_{\pi s}$- connected in X which is a contradiction. Hence Y is I_π- connected in (X, τ, I).

(2) (1)

Suppose that Y is not $I_{\pi s}$- connected in X. Then there exists I_π-separated sets A, B such that Y = A ∪ B implies A, B are open and I_π-open sets in X respectively. This implies that A, B are open and I_π-open sets in Y respectively. Since A and B areI_π -separated sets in X, then A and B are non-empty disjoint. Thus Y is not I_π- connected which is a contradiction. Hence Y is $I_{\pi s}$- connected in (X, τ, I).

Theorem: 4.17

Let (X, τ, I) be an ideal space. If A is a $I_{\pi s}$- connected set of X and H, G are I_π- separated sets of X with A ⊆ H ∪ G then either A ⊆ H or A ⊆ G.

Proof

Let A ⊆ H ∪ G. Since A = (A∩H) ∪ (A∩G), then $(A\cap G) \cap cl^{*\pi}(A\cap H) \subseteq G \cap cl^{*\pi}(H) = \phi$. Similarly we have $(A\cap H) \cap cl^{*\pi}(A\cap H) = \phi$. Suppose that A∩H and A∩G are non-empty. Then A is not $I_{\pi s}$- connected which is a contradiction. Thus either A∩H = ϕ or A∩G = ϕ implies A ⊆ H or A ⊆ G.

Theorem: 4.18

If A is a $I_{\pi s}$- connected set of an ideal space (X, τ, I) and $A \subseteq B \subseteq cl^{*\pi}(A)$ then B is $I_{\pi s}$- connected.

Proof

Suppose that B is not $I_{\pi s}$- connected. Then there exist $I_{\pi s}$- separated sets H and G such that B = H∪G. This implies that H and G are non-empty and $G \cap cl^{*\pi}(H) = \phi = H \cap cl^{*\pi}(G)$. Hence either
A ⊆ H or A ⊆ G.

Case (i)

Suppose that A ⊆ H. Then $cl^{*\pi}(A) \subseteq cl^{*\pi}(H)$ and $G \cap cl^{*\pi}(A) = \phi$. This shows that $G \subseteq B \subseteq cl^{*\pi}(A)$ and $G = cl^{*\pi}(A) \cap G = \phi$. Thus G is an empty set which is a contradiction. Hence B is $I_{\pi s}$-connected.

Case (ii)

Suppose that A ⊆ G. Then cl(A) ⊆ cl(G) and H ∩ cl(A) = ϕ. This implies that $H \subseteq B \subseteq cl^{*\pi}(A) = cl(A)$ and H = H ∩ cl(A) = ϕ. Therefore H is an empty set which is a contradiction. Hence B is
$I_{\pi s}$- connected.

Corollary: 4.19

If A is a $I_{\pi s}$-connected set in an ideal space (X, τ, I) then $cl^{*\pi}(A)$ is $I_{\pi s}$- connected.

Theorem: 4.20

If {A_i : i ∈ I} is a non-empty family of $I_{\pi s}$-connected sets of an ideal space (X, τ, I) with $_{i\in I} A_i$ ≠ ϕ then $_{i\in I} A_i$ is $I_{\pi s}$-connected.

Proof

Suppose that $_{i\in I} A_i$ is not $I_{\pi s}$-connected. Then we have $_{i\in I} A_i$ = H ∪ G, where H and G are I_{π}- separated sets in X. Since $_{i\in I} A_i$ ≠ ϕ, we have a point x in $_{i\in I} A_i$. Since x ∈ $_{i\in I} A_i$ either x ∈ H or x ∈ G.

Case (i)

Suppose that x ∈ H. Since $x \in A_i$ for each i ∈ I, then A_i and H intersect for each i ∈ I. Then A_i ⊆ G. Since H and G are disjoint, A_i ⊆ H for all i ∈ I. Hence $_{i\in I} A_i$ ⊆ H. This implies that G is empty which is a contradiction. Hence $_{i\in I} A_i$ is $I_{\pi s}$-connected.

Case (ii)

Suppose that x ∈ G. Since $x \in A_i$ for each i ∈ I, then A_i and G intersect for each i ∈ I. Thus A_i ⊆ G. Since G and H are disjoint, A_i ⊆ G for all i ∈ I. Thus $_{i\in I} A_i$ ⊆ G. This shows that H is empty which is a contradiction. Hence $_{i\in I} A_i$ is $I_{\pi s}$-connected.

Definition: 4.21

Let X be an ideal space and x ∈ X. The union of all $I_{\pi s}$-connected subsets of X containing x is called the I_{π}-component of X containing x.

Theorem: 4.22

Each I_{π}-component of an ideal space (X, τ, I) is a maximal $I_{\pi s}$-connected set of X.

Proof

Let A be the I_{π}-component of X containing x for every x ∈ X. To prove A is maximal $I_{\pi s}$-connected set of X. Suppose that A is not maximal $I_{\pi s}$-connected. Then there exists another $I_{\pi s}$-connected set B containing A. Therefore B is $I_{\pi s}$-connected set containing x. By the definition of I_{π}-component B is contained in A. Thus A = B. Hence the proof.

Theorem: 4.23

The set of all distinct I_{π}-components of an ideal space (X, τ, I) forms a partition of X.

Proof

Let A and B be two distinct I_{π}-components of X. Suppose that A and B intersect. Then A ∪ B is $I_{\pi s}$- connected in X. Since A ⊆ A∪B, then A is not maximal. Thus A and B are disjoint. Every x ∈ X belongs to the I_{π}-component of X containing x. Therefore X is the disjoint union of all distinct I_{π}-components.

Theorem: 4. 24

Each I_{π}-component of an ideal space (X, τ, I) is I_{π}- closed in X.

Proof

Let A be a I_{π}-component of X. By corollary: 4.19, $cl^{*\pi}$(A) is I_{π}- connected and A = $cl^{*\pi}$(A). Hence A is I_{π}- closed in X.

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