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REVIEW ARTICLE

STRUCTURE OF PO-K-TERNARY IDEALS IN PO-TERNARY SEMIRING

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ABSTRACT

In this paper we introduce the notion of PO-*k*-ternary ideals, full PO-*k*-ternary ideal and characterize PO-*k*-ternary ideals. We will prove some results about these PO-*k*-ternary ideals and full PO-*k*-ternary ideal.

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INTRODUCTION

The notion of semiring was introduced by Vandiver, (1934) in 1934. In fact semiring is a generalization of ring. In 1971 Lister, (1971) characterized those additive subgroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. MadusudhanaRao, Siva Prasad and Srinivasa Rao, (2015), studied and investigated some results on partially ordered ternary semiring.

Preliminaries

Definition 2.1[6] : A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a *ternary semiring* if T is an additive commutative semigroup satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,
ii) $[(a + b)cd] = [acd] + [bcd]$,
iii) $[a(b + c)d] = [abd] + [acd]$,
iv) $[ab(c + d)] = [abc] + [abd]$ for all *a; b; c; d; e* ∈T.

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Note 2.2[6]: For the convenience we write $x_1x_2x_3$ instead of $[x_1x_2x_3]$

Note 2.3[6]: Let Tbe a ternary semiring. If A, B and C are three subsets of T, we shall denote the set ABC= $\{\Sigma abc : a \in A, b \in B, c \in C\}$.

Note 2.4[6]: Let T be a ternary semiring. If A, B are two subsets of T, we shall denote the set A + B = $\{a + b : a \in A, b \in B\}$ and 2A = $\{ a + a : a \in A\}$.

Note 2.5[6]: Any semiring can be reduced to a ternary semiring.

Definition 2.6 [6]: A ternary semiring T is said to be a *partially ordered ternary semiring* or simply *PO Ternary Semiring*or*Ordered Ternary Semiring*provided T is partially ordered set such that $a \le b$ then

(1) $a + c \le b + c$ and $c + a \le c + b$,
(2) $acd \le bcd$, $cad \le cbd$ and $cda \le cdb$ for all *a, b, c, d*∈ T.

Throughout Twill denote as PO-ternary semiring unless otherwise stated.

Theorem 2.7[6]: Let Tbe a po-ternary semiring and A ⊆T, B ⊆T and C ⊆ T. Then (i) A ⊆(A], (ii) ((A]] = (A], (iii)

(A](B](C] ⊆(ABC] and (iv) A ⊆B ⇒A ⊆(B] and (v) A ⊆B ⇒(A] ⊆(B], (vi) (A ∩ B] = (A] ∩ (B], (vii) (A ∪ B] = (A] ∪ (B].

Definition 2.8 [6]: A nonempty subset A of a PO-ternary semiring T is a *PO-ternary ideal* of T provided A is additive subsemi group of T, ATT ⊆ A, TTA ⊆ A, TAT ⊆ A and (A] ⊆ A.

Theorem 2.9[8] : Let T be a PO-ternary semiring and A, B be two PO-ternary ideals of T, then the sum of A, B denoted by A + B is a PO-ternary ideal of T where A + B = {*x* = *a* + *b* / *a*∈ A, *b*∈ B}.

PO-*k*-Ternary Ideals

In this section we will study a more restricted class of PO-ternary ideals in a PO-ternary semi ring, which is called PO-*k*-ternary ideals or subtractive, and we introduce some related results and examples.

Definition3.1: A PO-ternary ideal A of a PO-ternary semi ring T is said to be *PO-k-ternary ideal* or *subtractive* provided for any two elements *a*∈ A and *x*∈ T such that *a* + *x*∈ A ⇒*x*∈ A.

Example 3.2: In any PO-ternary semi ring of the set of real numbers R, every ideal A is PO-*k*-ternary ideal, since for any *a*∈ A, ∈ T such that *a*+ *x*∈ A then *a*+ *x*+ (-*a*) ∈ A, so *x*∈ A.

Example 3.3: In the semi ring Z^+ under the operations max and min, the set I_n= {1, 2, 3,…., n} is a PO-ternary k-ideal of Z^+. Since for any element *a*∈ I_n and *x*∈ Z^+ such that *a*+ *x*= max {*a*, *x*} ∈ I_n, implies *x*∈ I_n.

Example3.4: Consider the PO-ternary semi ring Z_0^- under the usual addition, ternary multiplication and natural ordering ≤, let A = {-3k / *k*∈ N ∪ {0}}. Then A is a PO-k-ternary ideal of Z_0^-.

Definition3.5: Let *n*, *i* being integers such that 2 ≤ *n*, 0 ≤ *i*<*n*, and B (*n*, *i*) = { 0, 1, 2, 3, ……., *n* – 1}. We define addition and ternary multiplication in B (*n*, *i*) by the following equations.

$$x + y = \begin{cases} x+y & \text{if } x+y \le n-1 \\ l & \text{if } x+y \ge n \end{cases}$$
$$\text{where } l \equiv (x+y) \bmod m,\ m = n-i,\ i \le l \le n-1.$$

$$[xyz] = \begin{cases} xyz & \text{if } xyz \le n-1 \\ l & \text{if } xyz \ge n \end{cases}$$
$$\text{where } l \equiv (x+y) \bmod m,\ m = n-i,\ i \le l \le n-1.$$

Note3.6: The set B (*n, i*) is a commutative PO-ternary semi ring under addition, ternary multiplication [ ] as defined in definition 5.1.5, and natural ordering.

Example3.6: In note 5.1.6, *n* = 10, *i* = 7, then we have B (10, 7) = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and natural ordering, the operations defined as follows:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	7
2	2	3	4	5	6	7	8	9	7	8
3	3	4	5	6	7	8	9	7	8	9
4	4	5	6	7	8	9	7	8	9	7
5	5	6	7	8	9	7	8	9	7	8
6	6	7	8	9	7	8	9	7	8	9
7	7	8	9	7	8	9	7	8	9	7
8	8	9	7	8	9	7	8	9	7	8
9	9	7	8	9	7	8	9	7	8	9

[ ]	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	7	9	8	7	9
3	0	3	6	9	9	9	9	9	9	9
4	0	4	8	9	7	8	9	7	8	9
5	0	5	7	9	8	7	9	8	8	9
6	0	6	9	9	9	9	9	9	9	9
7	0	7	8	9	7	8	9	7	8	9
8	0	8	7	9	8	8	9	8	7	9
9	0	9	9	9	9	9	9	9	9	9

Example 3.7: The B (5, 2) = { 0, 1, 2, 3, 4} is a commutative PO-ternary semi ring such that 0 ≤ 1 ≤ 2 ≤ ≤ 3 ≤ 4 and the operations defined as follows:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	2
2	2	3	4	2	3
3	3	4	2	3	4
4	4	2	3	4	2

[ ]	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	3	2
3	0	3	3	3	3
4	0	4	2	3	4

Then I_1 = {0, 3} is a PO-k-ternary ideal of B (5, 2). But I_2 = {0, 2, 3, 4} is a PO-ternary ideal, but I_2 is not PO-*k*-ternary ideal. Since 2 ∈ I_2, 2 + 1 ∈ I_2, but 1 ∉ I_2.

Theorem 3.8: In a PO-ternary semi ring T, the set of zeroed Z (T) is a PO-ternary ideal of T.

Proof: Let *a*, *b*∈ Z (T), then there exist *x*∈ T such that *a* + *x* = *x* + *a* = *x* and *b* + *x* = *x* + *b* = *x*. Now (*a* + *b*) + *x* = *a* + (*b* + *x*) = *a* + *x* = *x*⇒*a* + *b*∈ Z (T). Now let *s*, *t*∈ T. Then *stx* = *st*(*a* + *x*) = *sta* + *stx*⇒*sta*∈ Z(T), *sxt* = *s* (*a* + *x*) *t* = *sat* + *sxt*⇒*sat*∈ Z (T) and *xst* = (*a* + *x*)*st* = *ast* + *xst*⇒*ast*∈ Z (T).

Therefore Z(T) is a ternary ideal of T.

Suppose that *a*∈ Z (T), *x*∈ T such that *x* ≤ *a*. *x*≤ *a*⇒*x* + *a* ≤ *a* + *a*⇒*x* + *a* ≤ *a*⇒*x* + *a* = *a*. Therefore *x*∈ Z (T). Hence Z (T) is a PO-ternary ideal of T.

Theorem3.9: In a PO-ternary semiring T, the set of zeroed Z (T) is PO-*k*-ternary ideal of T.

Proof: By theorem 3.8, Z (T) is a PO-ternary ideal of T. To show that Z (T) is a PO-k-ternary ideal of T, let *t*∈ T and *a*∈ Z

(T) such that $a + t \in Z(T)$, therefore there exist $x \in T$ such that $a + t + x = x$. But $a + y = y$ for some $y \in T$. Then we have $x + y = a + t + x + a + y = t + (a + y + a + x) = t + (y + a + x) = t + (y + x) = t + (x + y)$. Therefore $t \in Z(T)$ and hence Z (T) of T is PO-k-ternary ideal of T.

Theorem 3.10: Let T be a PO-ternary semiring and I be a left PO-ternary ideal of T and A, B be a non-empty subsets of T, Then (I : A, B) = { $r \in T : rab \in I$, for all $a \in A$, $b \in B$} is a left PO-ternary ideal of T.

***Proof*:** Let $x, y \in$ (I : A, B). Then $xab, yab \in I$ for all $a \in A$ and $b \in B$. Then $xab = s$, $yab = t$ for some $s, t \in I$. Then $s + t = xab + yab = (x + y)ab \in I \Rightarrow (x + y) \in$ (I : A, B). Let $p, q \in T$ and $x \in$ (I : A, B). $x \in$ (I : A, B) $\Rightarrow xab \in I$. Since I is a left PO-ternary ideal of T. Hence $pq(xab) \in I \Rightarrow (pqx)ab \in I \Rightarrow pqx \in$ (I : A, B). Now, suppose that $p \in T$ and $x \in$ (I : A, B) such that $p \leq x$. $x \in$ (I : A, B) $\Rightarrow xab \in I$. $p \leq x \Rightarrow pab \leq xab$. $pab \leq xab$, I is a left PO-ternary ideal of T and hence $pab \in I \Rightarrow p \in$ (I : A, B). Therefore $p \in T$ and $x \in$ (I : A, B) such that $p \leq x \Rightarrow p \in$ (I : A, B). Hence (I : A, B) is a left PO-ternary ideal of T.

Theorem 3.11: Let T be a PO-ternary semiring and I be a lateral PO-ternary ideal of T and A, B be a non-empty subset of T, Then (I: A, B) = {$r \in T$: $arb \in I$, for all $a \in A$, $b \in B$} is a lateral PO-ternary ideal of T.

***Proof*:** Let $x, y \in$ (I : A, B). Then $axb, ayb \in I$ for all $a \in A$ and $b \in B$. Then $axb = s$, $ayb = t$ for some $s, t \in I$. Then $s + t = axb + ayb = a(x + y)b \in I \Rightarrow (x + y) \in$ (I : A, B). Let $p, q \in T$ and $x \in$ (I : A, B). $x \in$ (I : A, B) $\Rightarrow axb \in I$. Since I is a lateral PO-ternary ideal of T. Hence $p(axb)q \in I \Rightarrow paxbq = apxqb \in I \Rightarrow pxq \in$ (I : A, B). Now, suppose that $p \in T$ and $x \in$ (I : A, B) such that $p \leq x$. $x \in$ (I : A, B) $\Rightarrow axb \in I$. $p \leq x \Rightarrow apb \leq axb$. $apb \leq axb$, I is a lateral PO-ternary ideal of T and hence $apb \in I \Rightarrow p \in$ (I : A, B). Therefore $p \in T$ and $x \in$ (I : A, B) such that $p \leq x \Rightarrow p \in$ (I : A, B). Hence (I : A, B) is a lateral PO-ternary ideal of T.

Theorem 3.12: Let T be a PO-ternary semiring and I be a right PO-ternary ideal of T and A, B be a non-empty subset of T, Then (I : A, B) = {$r \in T : abr \in I$, for all $a \in A$, $b \in B$} is a right PO-ternary ideal of T.

***Proof*:** Let $x, y \in$ (I : A, B). Then $abr, aby \in I$ for all $a \in A$ and $b \in B$. Then $abx = s$, $aby = t$ for some $s, t \in I$. Then $s + t = abx + aby = ab(x + y) \in I \Rightarrow (x + y) \in$ (I : A, B). Let $p, q \in T$ and $x \in$ (I : A, B). $x \in$ (I : A, B) $\Rightarrow abx \in I$. Since I is a right PO-ternary ideal of T. Hence $(xab)pq = ab(xpq) \in I \Rightarrow xpq \in$ (I : A, B). Now, suppose that $p \in T$ and $x \in$ (I : A, B) such that $p \leq x$. $x \in$ (I : A, B) $\Rightarrow abx \in I$. $p \leq x \Rightarrow abp \leq abx$. $abp \leq abx$, I is a right PO-ternary ideal of T and hence $abp \in I \Rightarrow p \in$ (I : A, B). Therefore $p \in T$ and $x \in$ (I : A, B) such that $p \leq x \Rightarrow p \in$ (I : A, B). Hence (I : A, B) is a right PO-ternary ideal of T.

TheoreM3.13: Let T be a PO-ternary semiring and I be a PO-ternary ideal of T and A, B be a non-empty subset of T, Then (I: A, B) = {$r \in T$: rab, arb, $abr \in I$, for all $a \in A$, $b \in B$} is a PO-ternary ideal of T.

***Proof*:** By theorems 3.10, 3.11, 3.12, it is easy to verify that (I: A, B) is a PO-ternary ideal of T.

Theorem3.14: Let T be PO-ternary semiring and I be a PO-*k*-ternary ideal of T and A be a non-empty subset of T, then (I: A, B) = { $r \in T$: rba, rab, $arb \in I$, for all $a \in A$, $b \in B$} is a PO-*k*-ternary ideal of T.

***Proof*:** By theorem 3.13, (I: A, B) is a PO-ternary ideal of T. Let $r \in$ (I : A, B), $y \in T$ such that $r + y \in$ (I : A, B) then rba, arb, $abr \in I$, and $(r + y)ba$, $ab(r + y)$, $a(r + y)b \in I$ for all $a \in A$, $b \in B$. Then $rba + yba = (r + y)ba \in I$ which is PO-*k*-ternary ideal. Hence $yba \in$ I.similarly, $abt \in I$ and $aby \in I$. Therefore $y \in$ (I: A, B). Hence (I : A, B) is a PO-*k*-ternary ideal of T.

Definition3.15: A PO-ternary semiring T is said to be ***E-inverse***, provided for every $a \in T$, there exist $x \in T$ such that $a + x \in E^{+}(T)$.

Note 3.16: In a PO-ternary semiring T the set of all additive idempotents $E^{+}(T)$ is not a PO-*k*-ternary ideal.

Example3.17: Let T = {0, a, b} such that $0 \leq a \leq b$ and define the addition, ternary multiplication on T as

+	0	a	b
0	0	a	b
a	a	0	b
b	b	b	b

[ ]	0	a	b
0	0	0	0
a	0	0	0
b	0	0	b

Then T is a additive inverse PO-ternary semiring under the operations. Moreover $E^{+}(T) = \{0, b\}$ is a PO-ternary ideal of T. But $a + b = b \in E^{+}(T)$ and $a \notin E^{+}(T)$ and hence $E^{+}(T)$ is not PO-*k*-ternary ideal.

Note3.18: The sum of two PO-k-ternary ideals need not be a PO-*k*-ternary ideal.

Example3.19: Consider the PO-ternary semiring of positive integers with zero Z_0^{+} under the usual addition and ternary multiplication. Then $2Z_0^{+}$ and $3Z_0^{+}$ are PO-*k*-ternary ideals of Z_0^{+}. But $2Z_0^{+} + 3Z_0^{+} = Z_0^{+} \setminus \{1\}$ is not a PO-*k*-ternary ideal. Indeed 1 + 2 = 3, where $2, 3 \in 2Z_0^{+} + 3Z_0^{+}$, but $1 \notin 2Z_0^{+} + 3Z_0^{+}$.

Theorem3.20: Let T be a PO-ternary semiring. If A is a PO-ternary ideal of T such that A = I ∪ J, where I, J are PO-*k*-ternary ideals, then A = I or A = J.

***Proof*:** Since A = I ∪J, then I ⊆Aand J ⊆A. Now suppose A ≠I, and A ≠J, then there exist $x, y \in$ Asuch that $x \in I$, $x \notin J$, $y \in J$, $y \notin I$, but $x + y \in$ A= I ∪J, so $x + y \in$ Ior $x + y \in$ J, now if $x + y \in$ I, then $y \in$ Ias Iis PO-*k*-ternary ideal, contradiction. Also if $x + y \in$ Jthen $x \in$ Jas Jis PO-*k*-ternary ideal, contradiction. Hence A = I or A = J.

Full Po-*K*-Ternary ideals

In this section, we will study more restrictions on the po-*k*-ternary ideal and the PO-ternary semiring. We study full PO-*k*-ternary ideal in additive inversive ternary semirings, so T denotes an additive inversive ternary semiring.

Definition4.1: A PO-ternary semiring T is said to be *additively regular* if for each $a \in$ T, there exists an element $a^{\#} \in$ T such that $a = a + a^{\#} + a$.

Theorem4.2: Let T be a PO-ternary semiring and if a is an additively regular element of T. Then the element $a^{\#}$ is unique.

Proof: Assume that b, c are element of T such that $a + b + a = a = a + c + a$, $b + a + b = b$ and $c + a + c = c$. Then $b = b + a + b = b + a + c + a + b = b + a + b + a + c = b + a + c = c + b + a = c + c + b + a + a = c + c + a + b + a = c + c + a = c + a + c = c$. Therefore $b = c = a^{\#}$.

Definition4.3: A PO-ternary semi ring T is said to be *additively inverse PO-ternary semi ring* if for each $a \in$ T, there exists a unique element $b \in$ T such that $a = a + b + a$ and $b = b + a + b$.

Note4.4: In an additively inverse PO-ternary semi ring the unique inverse b of an element a is usually denoted by a'.

Definition4.5: A PO-k-ternary ideal A of a PO-ternary semi ring T is said to be a *full PO-k-ternary ideal* provided the set of all additive idempotent of T, E^+(T) contained in A.

Example 4.6: In any PO-ternary ring R every ideal A is a full PO-*k*-ternary ideal. Since 0 is the only additive idempotent element in R which belongs to any PO-ternary ideal A of R. So A is full PO-k-ternary ideal.

Example4.7: In $Z \times Z^+ = \{(a, b): a, b$ are integers $b > 0\}$, define $(a, b) + (c, d) = (a + c, lcm(b, d))$, $[(a, b)(c, d)(e, f)] = (ace, gcd(b, d, f))$ and $(a, b) \leq (c, d)$ if $a \leq c$ and $b \leq d$. Then $Z \times Z^+$ is an additive inverse PO-ternary semiring, since for any $(a, b), (c, d), (e, f) \in Z \times Z^+$

Additive Commutative

$(a, b) + (c, d) = (a + c, lcm(b, d)) = (c + a, lcm(d, b)) = (c, d) + (a, b)$.

Additive Associative

$$((a, b) + (c, d)) + (e, f) = ((a + c, lcm(b, d)) + (e, f)$$
$$= (((a + c) + e, lcm(lcm(b, d), f))$$
$$= ((a + (c + e), lcm(b, lcm(d, f)))$$
$$= (a, b) + ((c + e), lcm(d, f))$$
$$= (a, b) + ((c, d) + (e, f)).$$

Multiplicative associative: Similarly as additive associative

Distributive

$$(a, b).(c, d).((e, f)) + (g,h)) = (a, b).(c, d).((e + g, lcm(f, h))$$
$$= (a, b).(c.(e + g), \gcd(d, lcm(f, h)))$$
$$= (a.c.(e + g), \gcd(b, \gcd(d, lcm(f, h))))$$
$$= (a, b).(c, d).(e, f) + (a, b).(c, d).(g, h).$$

Similarly $(a, b).((e, f)) + (g,h)).(c, d) = (a, b).(e, f).(c, d) + (a, b).(g, h).(c, d)$ and $((e, f)) + (g,h)).(a, b).(c, d) = (e, f).(a, b).(c, d) + (g,h).(a, b).(c, d)$.

Additive inverse: For any $(a, b) \in Z \times Z^+$, there exist a unique $(-a, b) \in Z \times Z^+$ such that

$(a, b) + (-a, b) + (a, b) = (a + -a + a, lcm(b, b, b)) = (a, b)$,
$(-a, b) + (a, b) + (-a, b) = (-a + a + -a, lcm(b, b, b)) = (-a, b)$.
Moreover, the set $A = \{(a, b) \in Z \times Z^+ : a = 0, b \in Z^+\}$ is a full PO-*k*-ternary ideal of $Z \times Z^+$. Since $E^+(Z \times Z^+) = \{0\} \times Z^+ \subseteq$ A, and for any $(0, b) \in$ A, $(c, d) \in Z \times Z^+$ such that $(0, b) + (c, d) = (c, lcm(b, d)) \in A$, then $c = 0$, and hence $(c, d) \in$ A.

Theorem4.8: The intersection of two full PO-*k*-ternary ideals of a PO-ternary semiring T is a full PO-*k*-ternary ideal of T.

Proof: Let A, B be two full PO-*k*-ternary ideals of T. Then by theorem 3.5.7, A ∩ B is a PO-ternary ideal of T which is full as $E^+(T) \subseteq$ A and $E^+(T) \subseteq$ B. Now, let $t \in$ T such that $a + t \in$ A ∩ B for some $a \in$ A ∩ B, then $a + t \in$ A ∩ B, $a \in$ A and $a + t \in$ A ∩ B, $a \in$ B, then $t \in$ A, $t \in$ B as A, B be PO-*k*-ternary ideals. Therefore $t \in$ A ∩ B.

Theorem4.9: Every PO-*k*-ternary ideal of a PO-ternary semiring T is an inversive PO-ternary subsemiring of T.

Proof: Obviously that every PO-ternary ideal of T is PO-ternary subsemiring of T. Let $a \in$ A, then $a \in$ T. Therefore there exist an $a' \in$ T such that $a = a + a' + a = a + (a' + a) \in$ A. But A is PO-*k*-ternary ideal and $a \in$ A, so $a + a' \in$ A. Again A is PO-*k*-ternary ideal and $a \in$ A, so $a' \in$ A. Therefore A is an inverse PO-ternary subsemiring of T.

Definition4.10: Let A be a PO-ternary ideal of an additive inversive PO-ternary semiring T. We define *k-closure* of A, denoted by $\overline{A}$ by:

$$\overline{A} = \{a \in T : a + x \in A \text{ for some } x \in A\}.$$

Theorem4.11: Let T be a PO-ternary semiring and A be a PO-ternary ideal of T, then $\overline{A}$ is a PO-k-ternary ideal of T. Moreover $A \subseteq \overline{A}$ and $(A] \subseteq (\overline{A}]$.

Proof: Let $a, b \in \overline{A}$, then $a + x, b + y \in$ A for some $x, y \in$ A. Now $(a + b) + (x + y) = (a + x) + (b + y) \in$ A. But $x + y \in$ A and hence $a + b \in \overline{A}$.

Next $s, t \in$ T, then $sta + stx = st(a + x) \in$ A. But $stx \in$ A, therefore $sta \in \overline{A}$. Similarly sat and $ast \in \overline{A}$. Hence $\overline{A}$ is a ternary ideal of T. Now let $a \in \overline{A}$ and $t \in$ T such that $t \leq a$. $a \in \overline{A} \Rightarrow a + x \in$ A for some $x \in$ A. Since A is PO-ternary ideal of T, so $t \leq a \Rightarrow t + x \leq a + x$. Since $a + x \in$ A $\Rightarrow t + x \in$ A $\Rightarrow t \in \overline{A}$

and hence $\overline{A}$ is a PO-ternary ideal of T. To show that $\overline{A}$ is a PO-*k*-ternary ideal, let *c*, $c + d \in \overline{A}$, then there exist *x* and *y* in A such that $c + x \in$ A and $c + d + y \in$ A. Now $d + (c + x + y) = (c + d + y) + x \in$ A $\Rightarrow c + d + y \in$A. Hence $d \in \overline{A}$. Therefore $\overline{A}$ is a PO-*k*-ternary ideal. Finally, since $a + a \in$ A for all $a \in$ A, it follows thatA $\subseteq \overline{A}$. By theorem 2.7, (A] $\subseteq$ ($\overline{A}$].

Lemma4.12: Let T be a PO-ternary semiring and A be a PO-ternary ideal of T. Then A = $\overline{A}$ if and only if A is a PO-*k*-ternary ideal of T.

Proof: Suppose that A = $\overline{A}$, then by theorem 4.11, $\overline{A}$ is a PO-*k*-ideal o f T, and hence A is PO-*k*-ideal of T. Conversely, suppose that A is a PO-*k*-ternary ideal of T. Again by theorem 4.11, A $\subseteq \overline{A}$. On the other hand, let $a \in \overline{A}$ then $a + x \in$ A for some $x \in$ A. But A is a PO-*k*-ternary ideal of T and $x \in$ A implies that $a \in$ A. There fore $\overline{A} \subseteq$ A. Hence A = $\overline{A}$.

Lemma4.13: Let T be a PO-ternary semiring and A, B be two PO-ternary ideals of T such that A $\subseteq$ B, then $\overline{A} \subseteq \overline{B}$.

Proof **:** Let A, B be two PO-ternary ideals of T such that A $\subseteq$ B, let $a \in \overline{A}$, then $a + x \in$ A for some $x \in$A, but A $\subseteq$ B and hence $a + x \in$ B for some $x \in$ B, therefore $a \in \overline{B}$. Hence $\overline{A} \subseteq \overline{B}$.

Lemma4.14: Let T be a PO-ternary semiring and A be a PO-ternary ideal of T. Then $\overline{A}$ is the smallest PO-*k*-ternary ideal of T containing A.

Proof: Let B be a PO-*k*-ternary ideal of T such that A $\subseteq$ B, let $x \in \overline{A}$. Then $x + a = b$ for some $a, b \in$ A. Since A $\subseteq$ B and B is a PO-*k*-ternary ideal of T, then $x \in$ B. There fore $\overline{A} \subseteq$ B.

Lemma4.15: Let T be a PO-ternary semiring and A, B be two full PO-*k*-ternary ideals of T, then $\overline{A+B}$ is a full PO-*k*-ideal of T such that A $\subseteq \overline{A+B}$ and B $\subseteq \overline{A+B}$.

Proof **:** By theorem 2.9, A + B is a PO-ternary ideal of T. then by theorem 4.11, $\overline{A+B}$ is a PO-*k*-ternary ideal of T and A + B $\subseteq \overline{A+B}$. Now $E^+(T) \subseteq$ A and $E^+(T) \subseteq$ B. so far any $e \in E^+(T)$, $e + e = e$. Therefore $E^+(T) \subseteq$ A + B $\subseteq \overline{A+B}$. This implies that $\overline{A+B}$ is a full PO-*k*-ternary ideal of T. Finally, let $a \in$ A, then $a = a + a' + a = a + (a' + a) \in$ A + B as $(a' + a) \in E^+(T) \subseteq$ B. Hence A $\subseteq \overline{A+B}$. Similarly B $\subseteq \overline{A+B}$.

Conclusion

In this paper mainly we studied about po-*k*-ternary ideals and full po-*k*-ternary ideals in PO-ternary semiring.

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