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RESEARCH ARTICLE

OBSERVATIONS ON THE HYPERBOLA $y^2 = 102x^2 + 33$

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ABSTRACT

The binary quadratic equation is considered and a few interesting properties among the solutions are presented. Employing the integer solutions of the equation under consideration, a special Pythagorean triangle is obtained.

INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values (Dickson, 2005; Mordell, 1970; Carmicheal, 1959; Gopalan and Geetha, 2010). For an extensive review of various problems, one may refer (Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2012; Gopalan *et al*., 2013; Gopalan *et al*., 2013; Gopalan *et al*., 2013; Gopalan *et al*., 2013; Gopalan *et al*., 2013; Gopalan *et al*., 2014; Gopalan *et al*., 2014; Meena *et al*.,, 2014; Gopalan *et al*., 2014; Meena *et al*.,, 2014; Santhi *et al*., 2014; Meena *et al*., 2015). In this communication, yet another interesting hyperbola given by $y^2 = 102x^2 + 33$ is considered and infinitely many integer solutions are obtained.

MATERIALS AND METHODS

Consider the binary quadratic equation

$$y^2 = 102x^2 + 33 \tag{1}$$

with the least positive integer solutions $x_0 = 2, y_0 = 21$
To obtain the other solutions of (1), consider the Pellian equation

$$y^2 = 102x^2 + 1$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is given by,

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$$\tilde{x}_n = \frac{g}{2\sqrt{102}} \quad \text{and} \quad \tilde{y}_n = \frac{f}{2}$$

In which,

$$f = \left(101+10\sqrt{102}\right)^{n+1} + \left(101-10\sqrt{102}\right)^{n+1}$$

$$g = \left(101+10\sqrt{102}\right)^{n+1} - \left(101-10\sqrt{102}\right)^{n+1}, \text{ where } n = -1,0,1,2,\ldots$$

Applying Brahmagupta lemma between the solutions of (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the general solution of (1) is found to be

$$x_{n+1} = f + \frac{21g}{2\sqrt{102}} \tag{2}$$

$$y_{n+1} = \frac{21}{2} f + \sqrt{102}g \tag{3}$$

where n = -1,0,1,2,….

Thus (2) and (3) represent non-zero distinct integral solutions of (1) which represents a hyperbola. The recurrence relations satisfied by the values of x and y are respectively

$$x_{n+3} - 202x_{n+2} + x_{n+1} = 0; \qquad y_{n+3} - 202y_{n+2} + y_{n+1} = 0$$

A few numerical examples are presented in the table below.

n	x_{n+1}	y_{n+1}
-1	2	21
0	412	4161
1	83222	840501
2	16810432	169777041

A few interesting relations among the solutions are presented below.

- x_{n+1} is always even and y_{n+1} is always odd.
- $x_{n+1} \equiv 0(\mathrm{mod}\ 2)$
- $y_{n+1} \equiv 0(\mathrm{mod}\ 8)$
- $x_{3n+1} \equiv 0(\mathrm{mod}\ 3)$
- $\frac{6}{11}(14y_{2n+2} - 136x_{2n+2}) + 12$ is a Nasty number.
- $\frac{1}{11}(14y_{2n+2} - 136x_{2n+2}) + 2$ is a quadratic number.
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- $\frac{1}{11}(14y_{3n+3} - 136x_{3n+3}) + \frac{3}{11}(14y_{n+1} - 136x_{n+1})$ is a Cubic integer.
- $(14y_{3n+3} - 136x_{3n+3} + 42y_{n+1} - 408x_{n+1})$ $-\frac{1}{12342}(1428x_{n+1} - 136y_{n+1})^2(14y_{n+1} - 136x_{n+1}) = 4(14y_{n+1} - 136x_{n+1})$
- $x_{n+2} = 10y_{n+1} + 101x_{n+1}$.
- $x_{n+3} = 2020y_{n+1} + 20401x_{n+1}$.

- $y_{n+2} = 101y_{n+1} + 1020x_{n+1}$.
- $y_{n+3} = 20401y_{n+1} + 206040x_{n+1}$.
- $\frac{1}{11}(14y_{2n+2} - 136x_{2n+2}) + 2 = \frac{1}{121}(14y_{n+1} - 136x_{n+1})^2$.
- $\frac{1}{11}(14y_{3n+3} - 136x_{3n+3}) + \frac{3}{11}(14y_{n+1} - 136x_{n+1}) = \frac{1}{1331}(4y_{n+1} - 30x_{n+1})^3$.
- $x_{n+3}y_{n+1} - x_{n+1}y_{n+3} = 66660$.
- $102x_{n+1}x_{n+3} - y_{n+1}y_{n+3} = -673233$
- $x_{n+2}y_{n+1} - x_{n+1}y_{n+2} = 330$
- $102x_{n+2}x_{n+1} - y_{n+1}y_{n+2} = -3333$

Remarkable observations

- Define $X = 14y_{n+1} - 136x_{n+1}$ and $Y = 1428x_{n+1} - 136y_{n+1}$, then the pair (X, Y) satisfies the hyperbola

$\frac{1}{121}X^2 = \frac{1}{12342}Y^2 + 4$

- Let $p = (x_{n+1} + y_{n+1}),\ q(= x_{n+1})$ be any two non-zero distinct positive integers ,note that $p > q$.

Treat p, q as the generaters of the Pythagorean triangle $T(\alpha, \beta, \gamma)$, where $\alpha = 2pq$, $\beta = p^2 - q^2$, $\gamma = p^2 + q^2$.
Let A,P are triangle. In the above A,P represent the area and perimeter of the Pythagorean triangle T, then the following relations are observed:

(i) $\alpha + 51\beta + 50\gamma = -33$

(ii) $\frac{4A}{P} - 51\gamma + 52\beta = 33$

(iii) $51\alpha - \gamma + 33 = \frac{204A}{P}$

Conclusion

In this paper, infinitely many non-zero distinct integer solutions for the hyperpola $y^2 = 102x^2 + 33$ are obtained. As binary quadratic diophantine equations are rich in variety, one may search for integer solutions and the corresponding properties for other choices of binary quadratic diophantine equations.

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