Available Online at http://www.journalajst.com

ISSN: 0976-3376

ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

Asian Journal of Science and Technology
Vol. 5, Issue 11, pp.722-726, November, 2014

RESEARCH ARTICLE

OPTIMAL DESIGN OF A FABRIC SHELL USING A COUPLED FEM-OPTIMIZATION PROCEDURE

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ARTICLE INFO

Article History:
Received 25th August, 2014
Received in revised form
10th September, 2014
Accepted 08th October, 2014
Published online 19th November, 2014

Key words:
Fabric Shell, Wind effects,
Optimization Procedure,
Prestressed cables

ABSTRACT

On this work it is shown the behavior of a prestressed spatial steel structure joined to a catenoid fabric shell. This structure was projected based on a minimalist concept, and was designed to cover large spans with a minimum weight. The bearing structure is formed by tube bars joined by prestressed cables. The cover is a catenoid shell made of polyester fiber. The study has focused on the analysis and design of this complex structure by using Finite element method (FEM). Wind effects are included together with gravitational and prestressed loads, the analytical procedure couples a dynamic structural analysis with a non linear optimization procedure. Results show the final stresses in the structure and sections design obtained according to the Standards LRFD. Computer simulation was carry on using Ansys software.

INTRODUCTION

Natural structures offer unique examples of high structural stiffness and performance while preserving the aesthetic beauty of their shapes. There exist interesting examples of man made structural forms which have been developed by mimicking natural structures. The result is a 3-D structure which, by itself, offers great stiffness while diminishing its weight. One particularly attractive approach to the above is the use of a “minimal” (term valid both from the mathematics as from the architectural standpoints) structure under the “tensegrity” concept, which describes a structure stabilized by means of tensors, which allows to associate the functional relationship of the structure as a closed circuit of structural elements. These structures possess a minimum weight with the capability of sustaining important mechanical loads under a rational scheme and efficiency (Foster, 2003; Sanchez, 2007). The present study deals with a spatial structure constituted by a special arrangement of elements that provide substantial stiffness to the structure. The original structure was proposed by Uehara H. (2007). The outstanding features of this building corresponds to a original structure that combines a catenoid membrane with a tensegrity-based structure, being the wind effect of particular interest due to its membrane nature (see Fig 1). This shell is coupled through top and bottom rings to the structure, thus creating a complex interaction of the whole system. The study was numerically performed by using the Finite element ANSYS software (2007), by coupling the structural analysis with a non-linear optimization model. The analysis of such structures, involving a highly complex geometry, involves the study of minimal surfaces (Rash, 2003). Due to the fluctuating wind loads on the flexible structures, it is customary to use the so-called Dynamic Magnification Factors, as a result of fully coupled fluid-structure interaction analysis, in addition to experimental studies in wind tunnels (Haug, 2003). Determining the shape of the membrane is the key to manufacture the fabric shell without defects, which could result detrimental by diminishing the prestress conditions.

The analysis of prestressed structures involves a non-linear model, in which the stresses are evaluated over and over again until a prestablished control value. The model we are using in this work involves, additionally, an optimization procedure which allows, simultaneously, to obtain a minimal structure in terms of the design and state variables.

MATERIALS AND METHODS

Figure 2 shows schematically the analyzed structure. Four relevant elements of the structure can be observed: pinned tubular sections; rings with flexure stiffness; supporting cables of the membrane and cables applied inside the hexagons. The final project of the structure includes five concrete columns and a bearing wall.

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Fig. 1 (a). Prestressed structure coupled to a catenoid membrane. Prototype was built in Alfalfares Park in Queretaro City

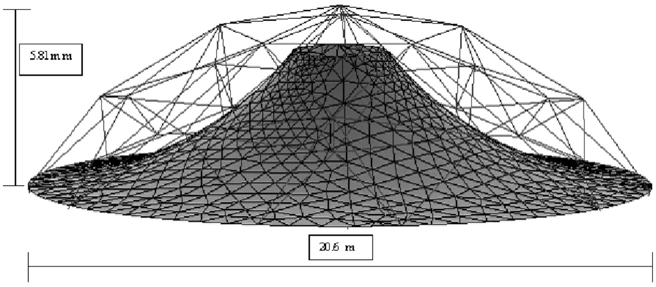

Fig. 2. Discrete model of the roof cover

Covering membrane is an orthogonal mesh of woven polyester fibers, coated with polyvinyl layers. The stress fields of the membrane (fabric shell) are of a biaxial type (Shaeffer, 1996), for this reason, they are highlighted along the directions of the overlapping of the fibers: warp and weft. The conformation of the fabric varies, in terms of number of fibers per line, protective layers and even the interwoving of the fibers. The weft direction are also prestressed, due to the tensile stress of the fibers in the warp direction. In any case, the analysis of the geometry of the fabric itself has been studied in detail through the mechanics of composites (West, 1991). The materials properties in this work are summarized in Table 1.

According to data from the supplier, the membrane has the same rupture and tear stresses in the warp and weft directions. Also the yield and rupture stresses of cables are indicated according to those that produce tensile effects on the membrane located at the top boundary, and those that give prestressed effects in the inner hexagons of the structure.

Theory/Calculations

The model applied to the structure is linear elastic, with the exception of the membrane. The finite elements of the bars correspond to link 3-D elements with tensile or compression bearing capacity. The cables are considered as linked elements too, with the capability of accepting initial tensile strains. The membrane is modeled using shell elements of 8 nodes with a minimum stiffness normal to its plane in order to stabilize its behavior under loading. Due to its thickness and geometry, the membrane tends to get unstable even with small loads (Bathe k., 1982). This leads to a non linear analysis of the membrane. Therefore, the membrane is only able to sustain tensile stresses, collapsing under compression. Similarly, the cables only sustain tensile stresses. The design of the structural elements is made introducing a subroutine integrated in the macro of the analysis; this design is made according to the current Standards LRFD (Load Resistance Factor Design) Standards AISC (2005). The corresponding matrix equation to the applied structural model is as follows:

$$\left(\int [B]^T [D][B] dv + k\int_A [N]^T [N] dA\right)\{U\} = \int_A [N]^T \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} dA + \int_V [N]^T \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix} dv +$$
$$+ \int [B]^T [D]\{\varepsilon_t\} dv + \int [B]^T [D]\{\varepsilon_i\} dv + \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix} \quad (1)$$

Where:

{U}: Displacement vector
{p}: External pressure vector
{P }: Punctual forces vector
{b}: Body forces vector
$\{\varepsilon_t\}$: Temperature strains vector
$\{\varepsilon_i\}$: Initial Strains vector
[B] : Matrix of derivatives of shape functions
[D]: Elastic constants matrix
[K]: Stiffness matrix
[N]: Shape functions matrix

This equation involves the whole stiffness of the system, the supporting effect on the membrane, as well the external pressures on the membrane, the weight of structural elements, temperature strains and the punctual forces applied in the nodes. Von-Misses membrane stresses involve the main stresses σ_i (Pascoe, 1981):

Table 1. Materials properties

Parameter	Cables	Fabric Shell	Structural Tube (Schedule 40)
Elasticity modulus E	1.68 GPa (1.68x10^6 Kg/cm^2)	0.753 GPa (7530 Kg/cm^2)	210 GPa (2.1x10^6 Kg/cm^2)
Yield Stress F_y	776.9 MPa (7769 Kg/cm^2)	-------------------	253.0 MPa (2530 Kg/cm^2)
Specific Weigth W	6.12 T/m^3	1.22 T/m^3	7.8 T/m^3
Poisson Modulus	0.25	0.1	0.2
Rupture Stress (Warp,Weft)	-------------------	51.2 MPa (512 Kg/cm^2)	-------------
Tear stress (Warp,Weft)	-------------------	48.8 MPa (488 Kg/cm^2)	-------------

$$\sigma_i = \left(\frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)\right)^{\frac{1}{2}} \quad (2)$$

The stresses on bars and cables are of the axial type. Top and bottom rings develop flexural stresses. Matrix $[K_l]$ is commonly used to take into account the prestressed condition. This is the so-called Stress stiffening Matrix. On the other hand, according ASCE recommendations (Shaeffer R.E., 1996):

Membrane maximum operating stress (overstressed) < 2.5 MPa + 30%
Membrane maximum operating stress (Relaxed) < 2.5 MPa
Operating tensile stress of a cable of ½ inch < 2.4 Ton

The propose coupled procedure takes into account the initial prestress strains in two groups of cables: Those supporting the membrane named: DEFM, and those that carry tensile stress to the inner hexagons: DEFC. Then, it is possible to perform the analysis of the initial strains including the prestress values mentioned earlier. These values change in every cycle until an equilibrium is reached in which the stresses in the different structural members, as well as the tensile stresses in cables and membrane are of the desired level (Table 2).

Table 2. Min-max constrain values for parameters a and b

Optimization State Equation	a	b
Displacement on Structure	0.0	L/240 + 0.5 (cm)
Von-Misses Stress on Fabric shell	1.5 MPa (150T/m^2)	1.8 MPa (180T/m^2)
Tensile on Shell Cables	3.5 kN (0.35 Ton)	10.0 kN (1.0 Ton)
Tensile on primary cables	10.0 kN (1.0 Ton)	30.0 kN (3.0 Ton)
Tensile on secondary cables	10.0 kN (1.0 Ton)	30.0 kN (3.0 Ton)

Wind Loads

Careful monitoring by Japanese researchers (Kohichi, 1986) in similar structures, show that stresses in the membrane increases significantly under wind pressure. Figure 3 shows the coefficients recorded (Kazuo, 1986) for a catenoid membrane similar to the one used in this study. Wind generates pressure zones on the leeward and suction zones on the winward.

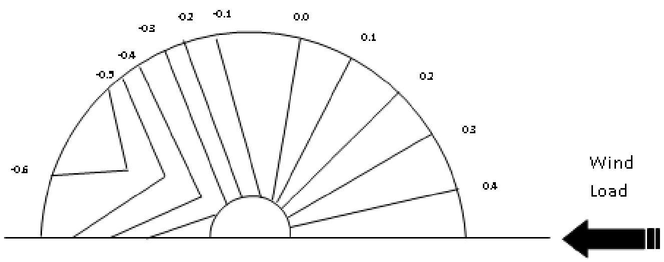

Fig. 3. Wind effect on the catenoid membrane. Pressure and suction coefficients are shown around circle

The present study was carried out in the city of Queretaro, in Central Mexico. According to the Mexican Standards Code (NTCRDF, 2004) was estimated the wind pressure by utilizing the following equation:

$$P = 0.0048\ C\ V_D^2 \qquad (3)$$

Where:
V_D = Design velocity in Km/h
C = the pressure coefficient

The coefficient C corresponds to the values shown in Figure 4. To take into account dynamic effects, a dynamic magnification factor F_D was evaluated; this factor will multiply the pressure P (equation 3).

A modal analysis was carried on to determine the first resonant period, resulting equal to 0.18 seg. corresponding to a rotational mode on the top zone of the structure. Value of F_D was obtained close to unity.

Optimization Model

An optimization model is required to establish an optimum equilibrium state in the system, due to the initial prestress tension at the cables that develop stresses on the entire system. These change when the loads are applied, thus generating new pre-stress tensile stresses and a new equilibrium state. If pre-stress losses are involved in the membrane and the cables, the final evaluation of such structures becomes complex. Solution proposed herein is to evaluate the initial and final pre-stress tensile stresses in the cables before and after the stress distribution of the entire system, so these values comply with the range of pre-established values. Also, the maximum stress of the membrane has to lie within the pre-established range. All this needs to be achieved, under the restriction of minimum weight of the structure.

Representing the structure weight by the objective function Z (Frangopol et. al., 1997), and the design variables before mentioned DEFC and DEFM, corresponding to the initial strains of group of cables 1 and 2 respectively, optimization problem can be enunciated as follows:

Minimize the function: Z = f (DEFC, DEFM)
Under the state conditions: $a_i \leq g_i$ (DEFC, DEFM) $\leq b_i$; $i = 5$
So, design variables be in the range: $0 \leq$ (DEFC, DEFM) $\leq c$; $c = 0.02$

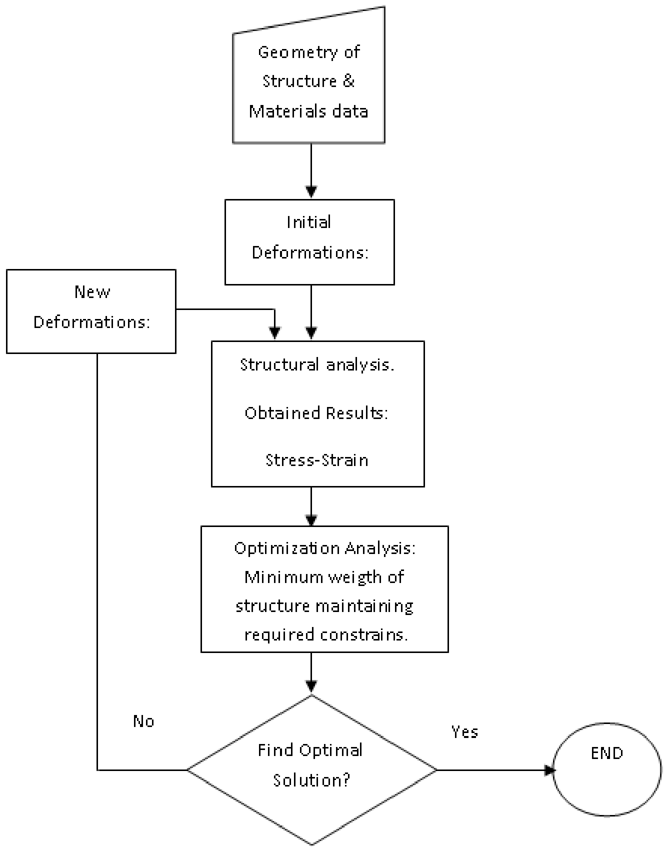

Fig. 4. Block-Scheme of coupled Structural-Optimization analysis

State equations refer to the code constraints on stresses and displacements of the structure. The final tensile constrains in the cables are due to the design conditions (LRFD Norms above). Also, the membrane stress ranges have been constrained in such a way that generates minimum stress and creates a stable geometrical shape, without weak zones where the material could wrinkle. On the other hand, a maximum stress that avoids material rupture is needed. According to the scheme shown in Figure 4, the analysis-design process is coupled with the non-linear optimization subroutine. In addition to the considered aspects aforementioned, the study couples a design phase based on limit states. Every cycle of optimum analysis-design gives a group of design solutions which could belong to the space of feasible solutions. The process repeats with new variable design values given by optimization procedure or even by the user, until a local minimum of the objective function is reached and the user considers it to be an appropriate solution. The analysis was carry out by taking the next load combinations:

i). (Dead load + Augmented presstress)(1.0)
ii). (Dead load + Live load + Presstress load (relaxed))(1.4)
iii). (Dead load + Prestress in cables (relaxed) + Temperature)(1.0)
iv). (Dead load + Wind + Presstress (service))(1.1)
v). (Dead load + Presstress in cables (relaxed) + rain)(1.1)

Prestress losses in the steel cables range from 1 to 2 % after 1000 hours, according to the type of cable, also it is recommended membrane cables be subjected to a tensile pre-stress of 4 to 10% of the cable rupture stress, if it is to be overstressed (Shaefer, 1996). On the other hand, the membrane operating tensions range from 1 to 4 kN/m for polyester membranes with vinyl coatings. This implies that, for a thickness of 0.85 mm a maximum operating stress of 50 kg/cm^2 (5.0 MPa) is applicable. Also, Nobuhiko (1986) made in situ experimental studies of a membrane, by measuring tensile relaxation stresses of about 30% of that which was initially applied. In addition to the stress relaxation on the fabric shell, its degradation due to ultraviolet rays also becomes a very important factor. Due to all these factors, the present study has taken into account the following pre-stress values:

Membrane maximum operating stress (overstressed) <2.5 MPa + 30%
Membrane maximum operating stress (Relaxed) < 2.5 MPa
Operating tension stress of a cable of ½ inch < 2.4 Ton

The coupled procedure takes into account the initial pre-stress strains in two groups of cables: those supporting the membrane (DEFM), and those that carry tensile stress to the inner hexagons (DEFC). Then, it is possible to perfom the analysis of the initial strains, including the pre-stress values mentioned earlier. These values change in every cycle until equilibrium is reached, in which the stresses in the different structural members, as well as the tensile stresses in cables and membrane are at the desired level (see Table 2).

RESULTS

The proposed procedure of analysis-design of this singular roof cover shows that the structure has a natural stiffness and stability, being the displacements under loading very small with respect to Mexican Regulations. Also, the stresses in all the structural elements have ample security factors that discard the possibility of its failure under loads. The grid structure without tensors is, by itself, an unstable structure, and due to the presence of cable tensors, becomes stable. Coupling of the fabric shell and grid steel bars makes the entire structure stiffer, creating a pre-stressed closed circuit. The membrane maximum stresses occur in the region close to the top ring. Therefore, this zone has to be properly reinforced.

Wind loading generates suction effects on windward side tending to uplift the membrane in lower zones. Also it has been observed that maximum stresses are concentrated on the top. These zones must be reinforced to overcome the acting stresses. Some representative results are shown as follow: Figures 5 shows the wind effects on the shell. Also, Figure 6 shows tensile loads in the structural tubes pinned elements.

Final sections of the structure, as well as the initial prestress strains applied to the cables are shown in Table 3. The cables considered here are of the Extra High Strenght type (776.9 Mpa).

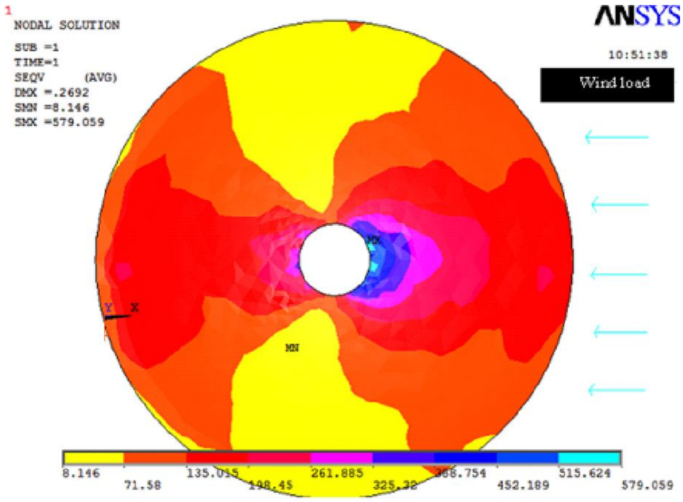

Fig. 5. Load combination 4. Membrane Von-Misses stresses due to wind load (T/m^2) (x10^{-2} MPa)

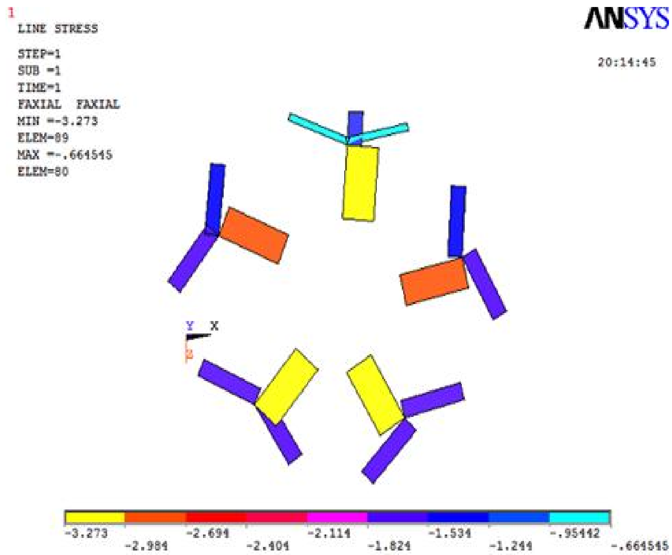

Fig. 6. Load combination 4. Tensile load in steel structural tubes (Ton)

Table 3. Obtained steel sections and Initial deformations

Structural element	Obtained section	Deformation
Bottom Steel Ring	6" Pipe Sched. 40	---
Top Steel Ring	4" Pipe Sched. 40	---
Hexagons, Central Pentagon and Inner steel elements	4" Pipe Sched. 40	---
Steel cables (Group 1)	Extra-High strength Cable ϕ 3/8"	---
Steel Cables (Group 2)	Extra-High strength Cable ϕ 1/2"	---
Membrane: Polyester with vinyl covers	Thickness:0.82mm Weight: 1.0 Kg/m^2	---
Initial Strain in Hexagon cables	---	DEFC: 0.0005
Initial strain in hexagon cables	---	DEFM: 0.023

DISCUSSION

Generally speaking, the analysis of prestressed structures with complex geometries, such as the roofing presented her, involve non-linear models, able to adjust the complicated interactions stress-strain taking place in the structure. The approach we have explained above allows to simultaneously determining stresses and strains in the prestressed condition over the closed structural system, while ensuring a minimum weight structure. This is achieved by coupling the mechanical-structural model with a subroutine for optimization. Nevertheless, the global analysis of these roofing structures must take into account other parameters, such as the wind fluctuations, which cause aerodynamical instabilities on the membrane. These can be taken into account by feed backing into our model, experimental data from wind tunnels. The coupled model presented herein allows including many other design/ performance parameters.

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