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RESEARCH ARTICLE

EFFECTS OF HALL CURRENTS AND VARIABLE FLUID PROPERTIES ON MHD FLOW PAST STRETCHING VERTICAL PLATE

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ABSTRACT

The effects of Hall currents on free-convective steady laminar flow of fluid of variable properties, along a semi-infinite vertical plate for large temperature differences, in the presence of Hall current has been investigated. The fluid density is vary exponentially and the thermal conducting linearly with temperature, while the fluid viscosity is vary as a reciprocal of a linear function of temperature. The usual Boussinesq approximation is neglected. The sysem of nonlinear equations governing the problem under consideration transformed into non-similar partial differential equations which have been solved numerically by the forth-order Runge-Kutta method. The effects of the magnetic parameter *M*, the Hall parameter m, the density / temperature parameter *n*, the thermal conductivity parameter *S*, the viscosity temperature θ_r, and the temperature ratio parameter θ_w on the velocity and temperature distribution as well as the coefficient of heat flux and shearing stress at the plate are investigated.

INTRODUCTION

The study of free convection boundary layer flow of an electrically conducting fluid along a continuously stretching semi-infinite plate is very important to understand the behavior of the fluid motion in several environmental and engineering applications, like industry, oil refinement, cooling of an infinite electrically conducting plate in a cooling path and others. Sakiadis (Sakiadis, 1961; Erickson *et al*., 1966) is the first researcher who studied the laminar flow along a continuously stretching electrically conducting plate. Cogley *et al*. (1968) improved sakiadis' to include blowing or suction at the moving plate. Vayjravelu and Hadyinicolaou (1997) studied the free convective heat transfer in mgnetohydrodynamics flow along a stretching sheet with uniform free stream. This problem with various aspects has been investigated by Grffith (1964), Ghin (1975), Gupta *et al*. (1977) and Gorla (1978). Also Chakrabarti *et al*. (1979), Rajagopal *et al*. (1987) and Chamkha (1999) studied the flow past a stretched sheet with a linear velocity and different thermal boundary conditions. Free-convective flow with mass transfer along a vertical plate subjected to a uniform magnetic field has been investigated by Elbashbeshy (1998). Beside the magnetic field, the Hall current affected the flow and heat transfer as shown by Abo-Eldahab *et al* (1996), Khaled K. Jaber (2014), Pop and Watanabe (1993) and Abo-Eldahab (2001). Also, Khaled K. Jaber (2013) studied the combined effects of Hall and ion slip currents on MHD free-convective flow past a semi-infinite vertical plate with heat generation. Joule heating effect on MHD fee-convective flow of a micropolar fluid was studied by Abd El-Hakiem *et al*. (1999). Large temperature differences between the plate and the fluid affects the physical properties of the fluid so, they cannot assumed to be constants. Also Boussinesq approximation can no longer be used. Recently, K. K. Jaber (2012) studied the combined effects of Hall currents and variable Viscosity on Non-Newtonian MHD flow past a stretching vertical plate. He showed that the variable viscosity effect the temperature and flow velocity. Hence, in the present work, I improved my previous work, density, viscosity and thermal conductivity are considered variables for high temperature differences and neglect the Boussinesq approximation. The nonlinear boundary layer equations, governing the problem, are solved numerically by the forth-order Runge-Kutta method. The two components of the velocity, temperature distributions, the coefficient of heat flux and the shearing stress at the plate are determined for different values of the parameters involved in the problem namely, Hall parameter m, the temperature ratio parameter θ_w, the thermal conductivity parameter *S*, the viscosity-temperature parameter θ_r and the magnetic field M.

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Mathematical formulation

This study considered the steady free-convective flow of an incompressible electrically conducting non gray gas past an isothermal semi-infinite vertical plate in the presence of a transverse uniform magnetic field. The x-axis is taken along the plate and the y –axis is taken as normal to it (see Fig. A). The magnetic Reynolds number is taken very small so that the induced magnetic field can be neglected.

The fluid density is assumed to vary exponentially with temperature, see [19]

$$\rho = \rho_\infty e^{-\beta(T-T_\infty)} \tag{1}$$

where

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{\mathrm{P}} \tag{2}$$

The fluid thermal conductivity is taken as

$$K = k_\infty\left[1 + b(T - T_\infty)\right] \tag{3}$$

Where b is a constant depends on the fluid. In general, b> 0 for water and air, while b <0 for fluids like lubricating oils.
Also fluid viscosity is assumed as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma(T - T_\infty)\right] \tag{4}$$

Or

$$\frac{1}{\mu} = a\left[T - T_r\right] \tag{5}$$

Where $a = \dfrac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \dfrac{1}{\gamma}$ are constants depend on the reference state and the thermal property of the fluid γ. In general a > 0 for liquids and a < 0 for gases.

The governing equations for continuity, momentum and energy are:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{6}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\rho_\infty\left(1 - e^{-\beta(T-T\infty)}\right) - \frac{\sigma_o B_0}{1+m^2}(u + m\,w) \tag{7}$$

$$\rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\sigma_o B_0}{1+m^2}(m u - w) \tag{8}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \tag{9}$$

The initial and boundary condition are given as:

u = v = 0, $T = T_\omega$ at y= 0 ; u $\longrightarrow$ 0, $T \longrightarrow T_\infty$ as y $\longrightarrow \infty$ (10)

Using equations (1), (3) and (5), equations (7), (8) and (9) become

$$e^{-\beta(T-T\infty)}\left(u\frac{\partial u}{\partial x}+\nu\frac{\partial u}{\partial y}\right)=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\frac{1}{a(T-T_r)}\frac{\partial u}{\partial y}\right)+g(1-e^{-\beta(T-T\infty)})-\frac{\sigma_o B_o u}{\rho_\infty(1+m^2)}\left(u+m\,w\right) \tag{11}$$

$$e^{-\beta(T-T\infty)}\left(u\frac{\partial w}{\partial x}+\nu\frac{\partial w}{\partial y}\right)=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\frac{1}{a(T-T_r)}\frac{\partial w}{\partial y}\right)-\frac{\sigma_o B_o u}{\rho_\infty(1+m^2)}\left(mu-w\right) \tag{12}$$

$$e^{-\beta(T-T\infty)}\left(u\frac{\partial T}{\partial x}+\nu\frac{\partial T}{\partial y}\right)=\alpha\frac{\partial}{\partial y}\left[\left\{1+b(T-T_\infty\right\}\frac{\partial T}{\partial y}\right] \tag{13}$$

Introducing the following dimensionless variables

$$\psi=4\upsilon_\infty CX^{\frac{3}{4}}f(\xi,\eta)\ ,\ \ \xi=X^{\frac{1}{2}}L^{\frac{-1}{2}}\ ,\ \eta=CX^{\frac{-1}{4}}\int_0^y\frac{\rho}{\rho_\infty} \tag{18}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\qquad ,\ C^4=\frac{g(1-e^{-n})}{4\upsilon^2{}_\infty}$$

The continuity equation is satisfied by

$$u=\frac{\rho_\infty}{\rho}\frac{\partial\psi}{\partial y}\qquad ,\ \nu=-\frac{\rho_\infty}{\rho}\frac{\partial\psi}{\partial x} \tag{14}$$

From (14) and (15) we find that

$$u=4\upsilon_\infty C^2X^{\frac{1}{2}}f^{/}\qquad ,\ \nu=-\frac{\rho_\infty}{\rho}\upsilon_\infty CX^{-\frac{1}{4}}(3f+2\xi\frac{\partial f}{\partial\xi}-\eta f^{/}) \tag{15}$$

Also, let w the component of the velocity in the z-direction has the similar form

$$w=4\upsilon_\infty C^2X^{\frac{1}{2}}g(\xi,\eta) \tag{16}$$

Using the above transformations the governing equations are transformed into:

$$\begin{aligned}&2f^{/2}-3ff^{//}+2\xi[f^{/}\frac{\partial f^{/}}{\partial\xi}-f^{//}\frac{\partial f}{\partial\xi}]=\frac{\theta_r}{(\theta_r-\theta)^2}e^{-n\theta}f^{//}\theta^{/}+\frac{\theta_r}{\theta_r-\theta}e^{-n\theta}(f^{///}-nf^{//}\theta^{/})\\&-(\frac{1-e^{n\theta}}{1-e^{-n}})-2\xi\frac{Me^{n\theta}}{G_r^{1/2}\left(1+m^2\right)}\left(f^{/}+mg\right)\end{aligned} \tag{17}$$

$$\begin{aligned}&2f^{/}g-3fg^{/}+2\xi[f^{/}\frac{\partial g}{\partial\xi}-g^{/}\frac{\partial f}{\partial\xi}]=\frac{\theta_r}{(\theta_r-\theta)^2}e^{-n\theta}g^{/}\theta^{/}+\frac{\theta_r}{\theta_r-\theta}e^{-n\theta}(g^{//}-ng^{/}\theta^{/})\\&+2\xi\frac{Me^{n\theta}}{G_r^{1/2}\left(1+m^2\right)}\left(mf^{/}-g\right)\end{aligned} \tag{18}$$

$$(1+S\theta)\frac{\partial}{\partial\eta}(e^{-n\theta}\theta^{/})+Se^{-n\theta}\theta^{/2}+3p_rf\theta^{/}+2\xi P_r[\theta^{/}\frac{\partial f}{\partial\xi}-f^{/}\frac{\partial\theta}{\partial\xi}]=0 \tag{19}$$

The boundary conditions are transformed into

$$\eta = 0 \quad : f^{\backslash} = o \quad , \theta = 1 \quad , 3f + 2\xi \frac{\partial f}{\partial \xi} = 0$$

$$\eta \longrightarrow \infty \quad : f^{\backslash} \longrightarrow o, \quad \theta \longrightarrow 0 \tag{20}$$

where n = $\beta(T_w - T_\infty)$ is the density temperature number, S = $b(T_w - T_\infty)$ is the thermal conductivity number, $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$ is the viscosity-temperature number, $Gr = \frac{g(1-e^{-n})L^3}{\upsilon_\infty{}^2}$ is the Grash of number , M=$\frac{\sigma_o B_o L^2}{\rho_\infty \upsilon_\infty}$ is the magnetic number, $\theta_w = \frac{T_w}{T_\infty}$ is temperature ratio parameter, $P_r = \frac{\upsilon_\infty}{\alpha}$ is the Prandtl number, where primes denote differentiation with respect to η only.

The shearing stress at the plate are found as:

$$\tau_{wx} = -\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = -\frac{4(2-\theta_w)\theta_r}{\theta_r - 1} \mu_\infty \upsilon_\infty C^3 X^{1/4} f^{//}(\xi, 0) \tag{21}$$

$$\tau_{wz} = -\mu \left.\frac{\partial w}{\partial y}\right|_{y=0} = -\frac{4(2-\theta_w)\theta_r}{\theta_r - 1} \mu_\infty \upsilon_\infty C^3 X^{1/4} g^{/}(\xi, 0) \tag{22}$$

and the rate of heat transfer at the plate (Nusselt number) as:

$$N_u = L(1+S)CX^{-1/4} e^{-n} \theta^{/}(\xi, 0) \tag{23}$$

RESULTS AND DISCUSSION

Equation (17), (18) and (19) with the boundary conditions (20) , the partial derivative with respect to ξ is replaced by two-point backward finite difference with step h = 0.1. The transformed system is solved numerically by using the fourth-order Runge-Kutta method with an estimation of $f^{//}(\xi, \eta)$, $g^{/}(\xi, \eta)$ and $\theta^{/}(\xi, \eta)$ by the shooting technique to obtain $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$. The value of η at infinity is fixed at 2. Solutions are obtained for the Prandtl number $\Pr = 0.7$ and the Grash off number Gr = 0.5.

In view of Equation (14) Equation (1) can be written as

$$\rho = \rho_\infty e^{-n\theta} \tag{24}$$

since θ varies from 0 , at the edge of the boundary layer , to 1 at the vertical plate surface , the density of the fluid adjacent to the plate is related to its free- stream value by the following expression:

$$\rho_w = \rho_\infty e^{-n}$$

This can be approximated by $\rho = \rho_\infty (1 - n\theta)$ in case the temperature difference between the fluid and plate is small, so the density can be treated as a variable only in the buoyancy term of the momentum equation (Boussinesq approximation). Therefore, when $n \longrightarrow 0$, Equations (6)-(9) reduce to the Boussinesq equations and for large temperature differences the condition $n \longrightarrow 0$ is disregarded.

Figure 1and 2 show that the increasing of the magnetic field parameter M decreases the dimensionless primary flow velocity $f^{/}$ and increases the dimensionless secondary flow velocity g. The increasing of the magnetic field parameter M increases the Lorentz force which acts in the opposite direction of the flow which in turn decreases the dimensionless primary flow velocity $f^{/}$. Figure 3 demonstrate that the increasing of Hall parameter m increases the secondary flow velocity g. $^{/}$. Form Figures 4, 5 and 6 it is observed that the dimensionless primary velocity $f^{/}$ and secondary velocity g increase while the dimensionless temperature θ decreases as the density-temperature parameter n increases. The increase in the density temperature parameter n increases the buoyancy forces so increases the fluid velocities and decreases the temperature so as the velocities of the fluid.

Figures 7, 8 and 9 show as expected, that the increasing of the thermal conductivity S increases the temperature of the fluid and the fluid velocities. Figures 10, 11 and 12 show that the velocities $f^{/}$, g and the dimensionless temperature θ increase due to the increase in the temperature ratio parameter θ_{ω} . Also, it is observed from Figures 13, 14 and 15 that as the viscosity -temperature parameter θ_r increased the dimensionless temperature θ decreases and accordingly the dimensionless velocities $f^{/}$, g increase. Table 1 shows that the dimensionless wall-velcioty gradient $f''(x, 0)$ increases as N, m, S, θ_r and θ_w increase and decreases as M increases, the dimensionless wall-velcioty gradient $g^{/}(x, 0)$ increases as n, M, S, θ_r and θ_w increase where as it decreases as m increases . Moreover, the dimensionless rate of heat transfer- $\dot{\theta}'(x, 0)$ increases as m, n and θ_r increase and decreases as S , M and θ_w increases .

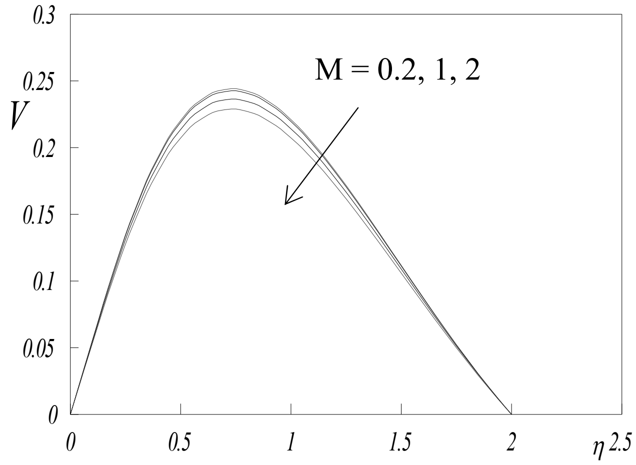

Fig.1. Effect of magnetic parameter M on the primary flow velocity V

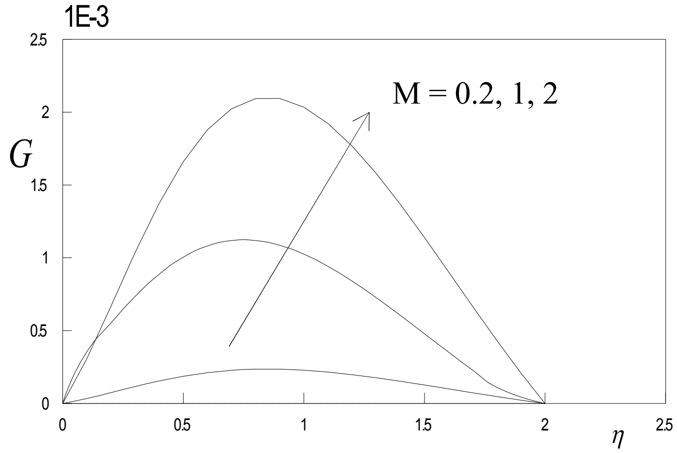

Fig.2. Effect of magnetic parameter M on the secondary flow velocity G

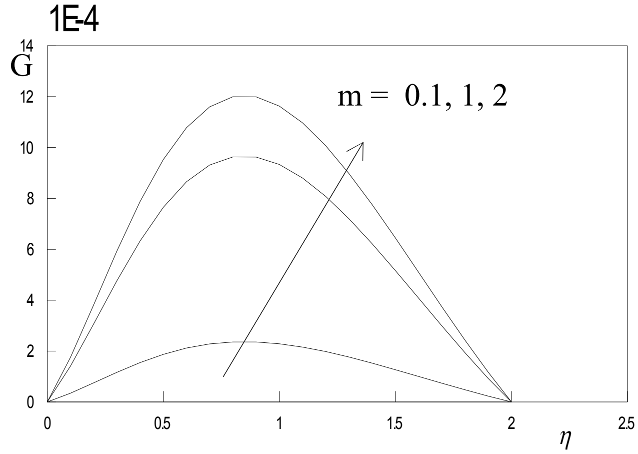

Fig.3. Effect of Hall parameter m on the primary flow velocity V

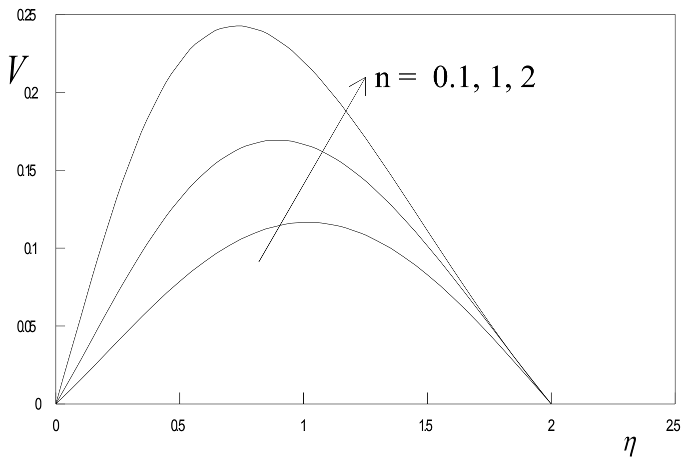

Fig.4. Effect of the parameter n on the primary flow velocity V

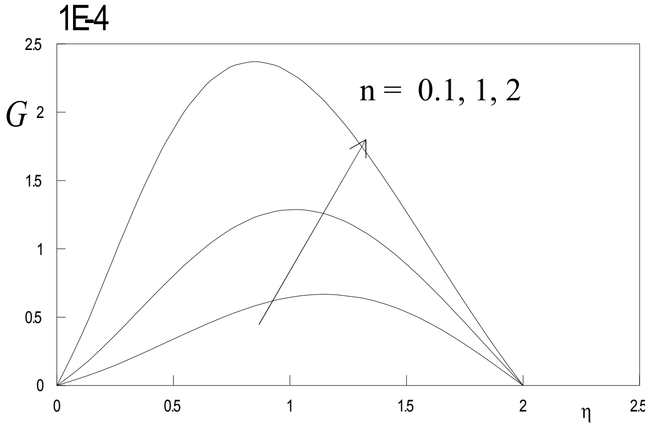

Fig.5. Effect of the parameter n on the secondary flow velocity G

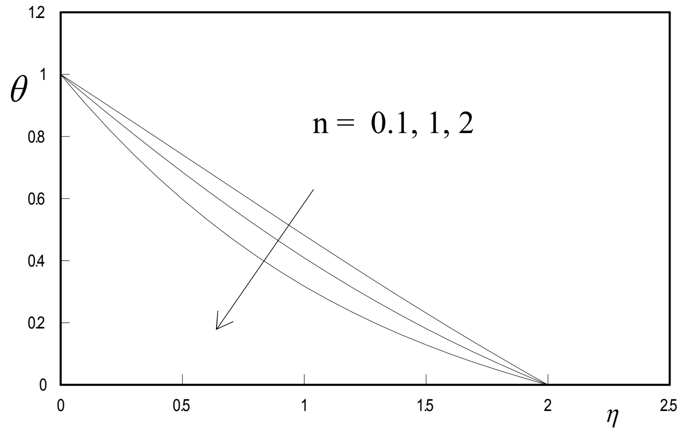

Fig.6. Effect of the parameter n on the temperature

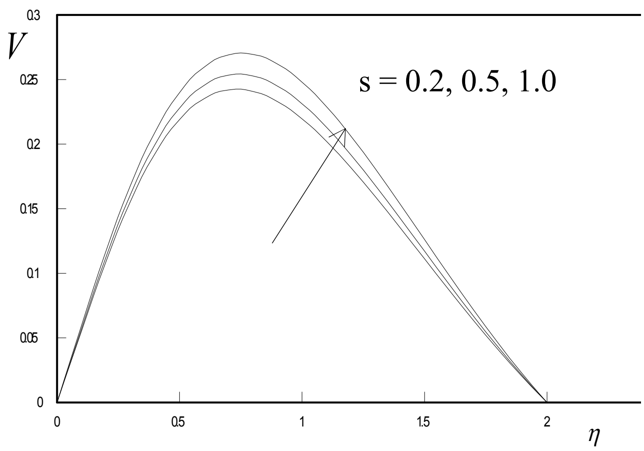

Fig. 7. Effect of the parameter s on the primary flow velocity V

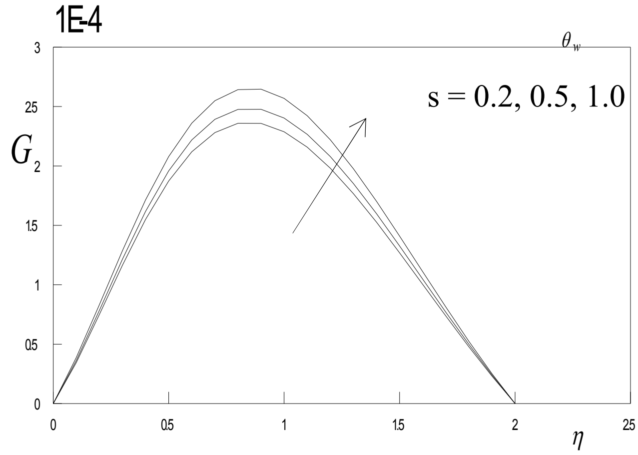

Fig. 8. Effect of the parameter s on the secondary flow velocity G

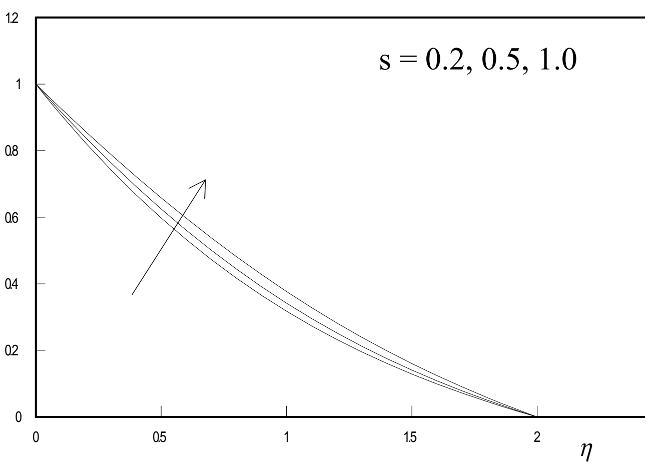

Fig. 9. Effect of the parameter s on the temperature

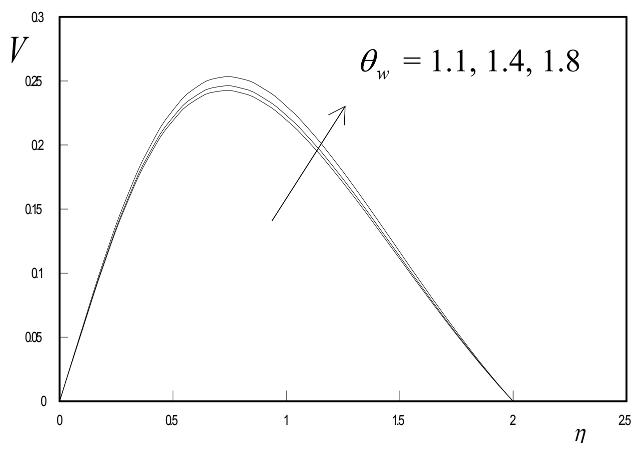

Fig. 10. Effect of the parameter θ_w on the primary flow velocity G

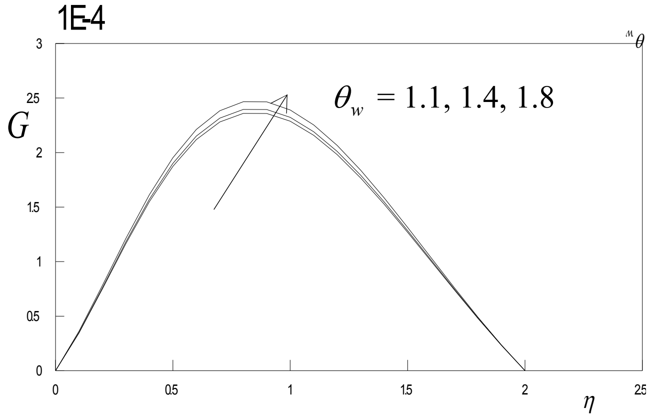

Fig. 11. Effect of the parameter θ_w on the secondary flow velocity G

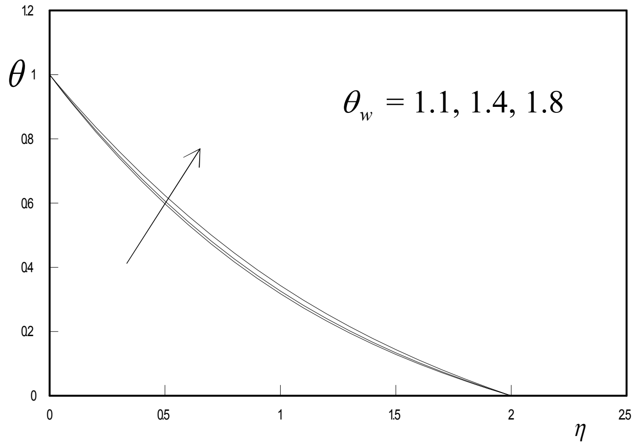

Fig. 12. Effect of the parameter θ_w on the temperature θ

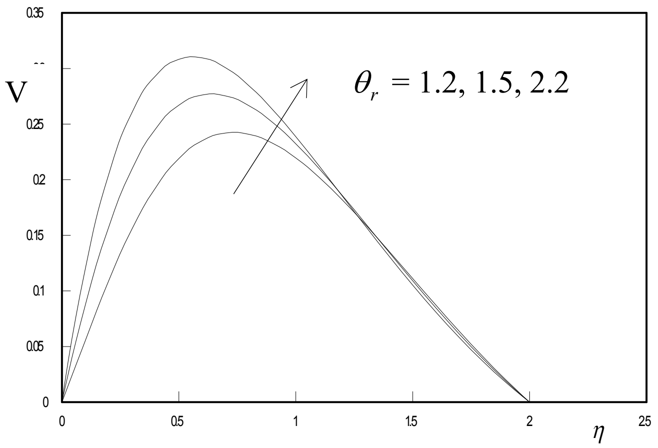

Fig. 13. Effect of the parameter θr on the primary flow velocity

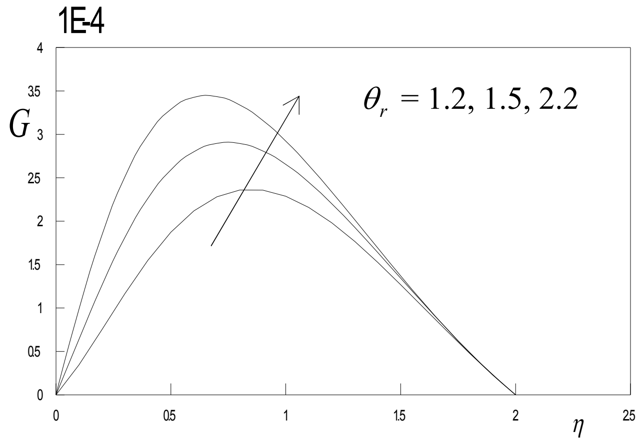

Fig. 14. Effect of the parameter θr on the secondary flow velocity

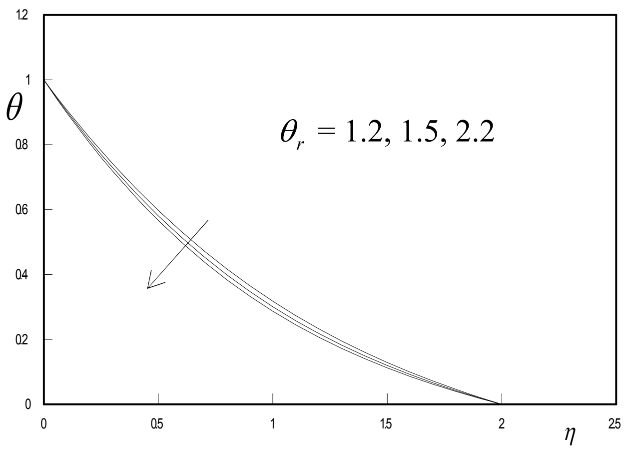

Fig. 15. Effect of the parameter θr on the temperature θ

Table 1. Variation of dimensionless wall-velocity gradient and dimensionless rate of heat transfer at the plate with the dimensionless θw, N ,M ,S, m and θr for Prandtl number=0.72 and Gr=0.5

N	M	M	S	θw	θr	f'	g	- θ
0.1	0.2	0.1	0.2	1.1	1.2	0.147592	0.147592	0.523182
0.5	0.2	0.1	0.2	1.1	1.2	0.265337	0.000111372	0.68406
1	0.2	0.1	0.2	1.1	1.2	0.523542	0.000288411	0.999698
1	1	0.1	0.2	1.1	1.2	0.515713	0.0013724	0.993265
1	2	0.1	0.2	1.1	1.2	0.506043	0.00258017	0.983145
1	0.2	1	0.2	1.1	1.2	0.52473	0.00146627	1.00076
1	0.2	2	0.2	1.1	1.2	0.535081	0.00121293	0.961244
1	0.2	0.1	0.5	1.1	1.2	0.547747	0.000308797	0.885734
1	0.2	0.1	1	1.1	1.2	0.578901	0.000336635	0.767793
1	0.2	0.1	0.2	1.4	1.2	0.523617	0.000288783	1.00016
1	0.2	0.1	0.2	1.8	1.2	0.523564	0.000288472	0.999789
1	0.2	0.1	0.2	1.1	1.5	0.941763	0.000606392	1.04983
1	0.2	0.1	0.2	1.1	2.2	1.38567	0.000995202	1.09654

Concluding remarks

In this paper, we have studied the effects of Hall currents and variable fluid properties on the MHD free convective steady lamina boundary layer flow past an isothermal semi-infinite vertical plate. The fluid density is assumed to vary exponentially and the thermal conductivity linearly with temperature, the fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature. The Boussinesq approximation is neglected due to the large temperature differences between the fluid and the plate. This paper demonstrates the fluid density has to be taken as variable in the continuity equation, energy equation and all terms of the momentum equation.

Besides, it is observed that:

1) The increasing in the Hall parameter m yields to a significant increasing in the secondary flow velocity, a slight increasing in the fluid velocities $f^{/}$ and the fluid temperature the dimensionless wall-velocity gradients and the rate of heat transfer from the plate to the fluid.
2) The increasing in the magnetic parameter M tends to increase the fluid temperature, the secondary flow velocity, the dimensionless wall-velocity gradient and the rate of heat transfer and to decrease the fluid velocities.
3) The increasing in the thermal conductivity parameter s yields to an increasing in the fluid velocities, temperature and the dimensionless wall-velocity gradients.
4) The increasing in the viscosity-temperature parameter θ_r increases the fluid velocities, the dimensionless wall-velocity gradient and decreases the fluid temperature and the rate of the heat transfer between the plate and the fluid.
5) The increasing in the density-temperature parameter n produce an increasing in the fluid velocities, and the dimensionless wall-velocity gradients and produce a decreasing in the fluid temperature and the rate of the heat transfer between the plate and the fluid.
6) The increasing in the temperature ratio parameter θ_w increases the fluid velocity components, the fluid temperature, the dimensionless wall-velocity gradient and a decreases the fluid temperature and the rate of heat transfer between the plate and the fluid.

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