



ISSN: 0976-3376

Available Online at <http://www.journalajst.com>

ASIAN JOURNAL OF
SCIENCE AND TECHNOLOGY

Asian Journal of Science and Technology
Vol. 5, Issue 7, pp. 378-383, July, 2014

RESEARCH ARTICLE

TWO STAGE MULTI-RATE ASYNCHRONOUS SUB-NYQUIST SAMPLING IN COGNITIVE RADIOS

* Sabitha R.

Department of ECE, Sri Krishna Coll. of Eng. & Technol., Coimbatore, India

ARTICLE INFO

Article History:

Received 06th April, 2014

Received in revised form

28th May, 2014

Accepted 04th June, 2014

Published online 31st July, 2014

ABSTRACT

Multi-rate asynchronous sub-Nyquist sampling (MASS) followed by a two stage sensing scheme for cognitive radios where energy detection is used in the first stage and cyclostationary detection in the second stage is proposed. The detection parameters in both the stages are used to maximize the probability of detection and to minimize the probability of false alarm. Compared to previous approaches, two-stage MASS offers lower sampling rate, robust against lack of time synchronization, more accuracy and is more robust to fading environments. Also it is an attractive approach for cognitive radio networks.

Key words:

Cognitive Radio,
Wideband Spectrum,
Two-Stage Spectrum Sensing,

Copyright © 2014 Sabitha R. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

INTRODUCTION

The economical problem with fixed spectrum assignment policy has the suboptimal use of spectrum resource leading to overutilization in some bands and underutilization in others [2-4]. This observation has led to the recent spectrum reforms by the U.S. Federal Communication Commission (FCC). The Dynamic Spectrum Access (DSA) for enhanced spectrum utilization for adaptive networks is achieved via the CR [5,6]. CR is an emerging wireless communication technology aims at using DSA to allow the unused, licensed TV frequency spectrum to be used by unlicensed users on a non-interfering basis [7,10]. Cognitive radio (CR) is one of the promising solutions for addressing this spectral under-utilization problem [1]. In [8], Y.-C. Liang, K.-C et al. provided systematic overview of CR networking at the physical and MAC layers. In [9], S. Chaudhari et al. introduced computationally efficient signal system schemes for multicarrier based primary user signal. In [12] H. Sun et al. exposed a point that CR with a broader spectral awareness could potentially explain more spectral opportunities and achieve greater capacity. In [13] Z. Tian et al. Wavelet based detection and in [14] Z. Quan S. multiband joint detection was specified for sensing wideband spectrum. The addition of [13] is that the flexibility in adapting to a dynamic wide band spectrum. The advantage is that it performs well when applied to practical conditions. In [11] E. Axell et al. defined the concept of CR to explain the underutilized spectral resources

of reusing unused spectrum in an opportunistic manner and creates awareness about the existence of PU in a given geographical area. In [16] B. Farhang et al. proposed a filter band method and in [17] D. Donoho, and in [18] E. Candes et al. proposed compressive signal reconstruction from incomplete to implement wide band SS. Also in [19-21] and in [17-24] and in [23] the authors developed a cooperative approach to wideband SS using compressive sampling mechanism. In [25] H. Sun et al. defined the approximation of the average probability of detection over a slow fading channel. All the above mentioned techniques (samples the transmitted signal at operation) use sub Nyquist sampling rates. In this paper we develop a new combined two-stage MASS applied to wideband spectrum sensing. The proposed combined MASS differs from [28] in two ways. First in [28], after spectral reconstruction of compressive sampling only the traditional energy detection technique is used. But in this proposed method detection is accomplished by both energy detection and cyclostationary detection. Second, accuracy in sensing the spectrum is measured in terms of FFT size and down sampling factor. Hence, combined MASS or two-stage MASS can be used interchangeably. The proposed two stage MASS system has superior compression capability compared with the Nyquist sampling system. Compared with existing wideband spectrum sensing approaches, the new combined two-stage MASS has lower implementation complexity, higher energy efficiency, better data compression capability, show good accuracy and is more applicable to CR networks. The rest of the paper is organized as follows. We briefly introduce the CS-based sensing scheme in Section II. We then propose the MASS system in Section III. Simulation results

*Corresponding author: Sabitha R.

Department of ECE, Sri Krishna Coll. of Eng. & Technol.,
Coimbatore, India

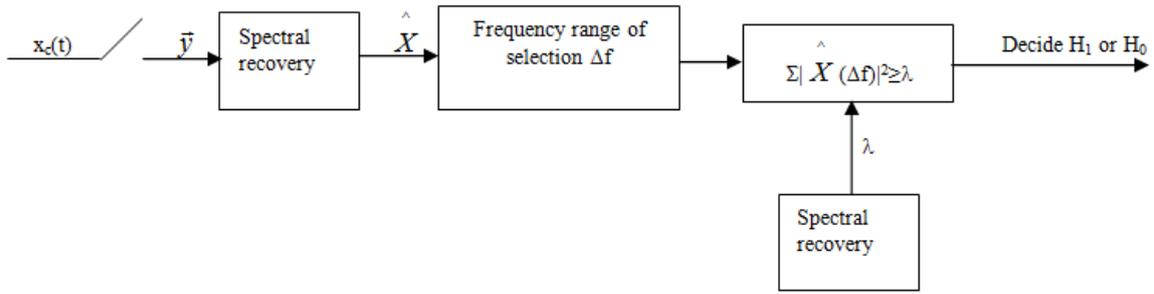


Fig.1. Diagram of CS-based spectrum sensing when using the spectral domain energy detection approach

are presented in Section IV, followed by conclusions in Section V.

Problem Statement

The assumptions used in this paper are 1)all CRs keep quiet during the spectrum sensing interval as enforced by protocols, e.g., at the medium access control (MAC) layer. Therefore, the observed signal at a CR arises only from PUs and background noise. 2) The continuous time signal $x_c(t)$ is received at a CR, and the frequency range of $x_c(t)$ is $0 \sim W(Hz)$. The signal $x_c(t)$ is sampled at the sampling rate $f_s(Hz)$ for an observation time T . The discrete Fourier transform (DFT) spectrum of it can be calculated by $\vec{X} = F\vec{x}$, where F denotes an N -by- N DFT matrix. If Shannon-Nyquist sampling theorem is followed, the sampling rate needs to be at least twice the bandwidth of the signal, i.e., $f_s \geq 2W$ which results in excessive memory requirements and prohibitive energy costs. To overcome these disadvantages the researchers have to search for technologies to reduce the sampling rate f_s while maintaining W by using CS theory. First stage of spectrum sensing approach is spectral domain energy detection. As shown in Fig. 1, this approach extracts the reconstructed spectrum \vec{X} in the frequency range of interest, e.g., Δf , and then calculates the signal energy in the spectral domain. The output energy is compared with a detection threshold (denoted by λ) to decide whether the corresponding frequency band is occupied or not, i.e., choosing between hypotheses H_1 (presence of PUs) and H_0 (absence of PUs). In the second stage of cyclostationary detection the FFT of the received signal is used to decide the presence (H1) or absence (H0) of Pus. In this paper, we will present a novel system, i.e., two-stage MASS, to sample the signal using sub-Nyquist sampling techniques, while enabling the detection to be more accurate even at low SNR with two-stage detection.

System and Signal Model

Suppose that a CR has ν sub-Nyquist sampling branches as shown in Fig. 2. The spectrum of interest is taken from the output of wideband filter with bandwidth W . At the i -th branch, the low-rate sampler samples the received signal at the sub-Nyquist rate $f_i(Hz)$. In the observation time T (second), the numbers of samples in these ν sampling branches

are M_1, M_2, \dots, M_ν , respectively where $M_i = f_i T (\forall i \in [1, \nu])$. In addition, M_1, M_2, \dots, M_ν are chosen to be different prime numbers that are of the order of \sqrt{N} , i.e., $M_i \sim O(\sqrt{N})$, by controlling the sampling rate $f_i (\forall i \in [1, \nu])$. The DFT spectra are used to reconstruct the wideband spectrum. Suppose that the received signal $x_c(t)$ is of finite support and absolutely summable. Using the sub-Nyquist rate $f_i < 2W$, we obtain the sampled signal $y_i[m] = x_c(\frac{m}{f_i}) = x_c(\frac{mT}{M_i}), m = 0, \dots, M_i - 1$. The DFT spectrum of \vec{y}_i is then calculated by $\vec{Y}_i = F_s \vec{y}_i$, where F_s denotes the M_i -by- M_i DFT matrix. The DFT spectrum of \vec{y}_i is related to the continuous-time Fourier transform of $x_c(t)$ by

$$Y_i(f) = f_i \sum_{l=-\infty}^{\infty} X_c(f + lf_i) \tag{1}$$

Where $X_c(f) = \int_{-\infty}^{\infty} x_c(t) e^{-j2\pi ft} dt$ is the Fourier transform of $x_c(t)$

Furthermore, if $x_c(t)$ is sampled at or above the Nyquist rate, i.e., $f_s = \frac{N}{T} \geq 2W$, the sampled signal can be

written as $x[n] = x_c(\frac{n}{f_s}) = x_c(\frac{nT}{N}), n = 0, 1, \dots, N - 1$. The spectrum of \vec{x} will be related to the continuous-time Fourier transform of $x_c(t)$ by $X(f) = f_s \sum_{l=-\infty}^{\infty} X_c(f + lf_s)$. As the signal $x_c(t)$ is band-limited to W and the sampling rate $f_s \geq 2W$, there will be no spectral aliasing phenomena in $X(f)$; thus, we can rewrite this relationship by $X(f) = f_s X_c(f), \forall f \in [-\frac{W}{2}, \frac{W}{2}]$. Because $X(f)$ has all information in $[-\frac{W}{2}, \frac{W}{2}]$, we assume that $X(f)$ is zero everywhere except $f \in [-\frac{W}{2}, \frac{W}{2}]$. Substituting $X(f) = f_s X_c(f)$ into, we can obtain.

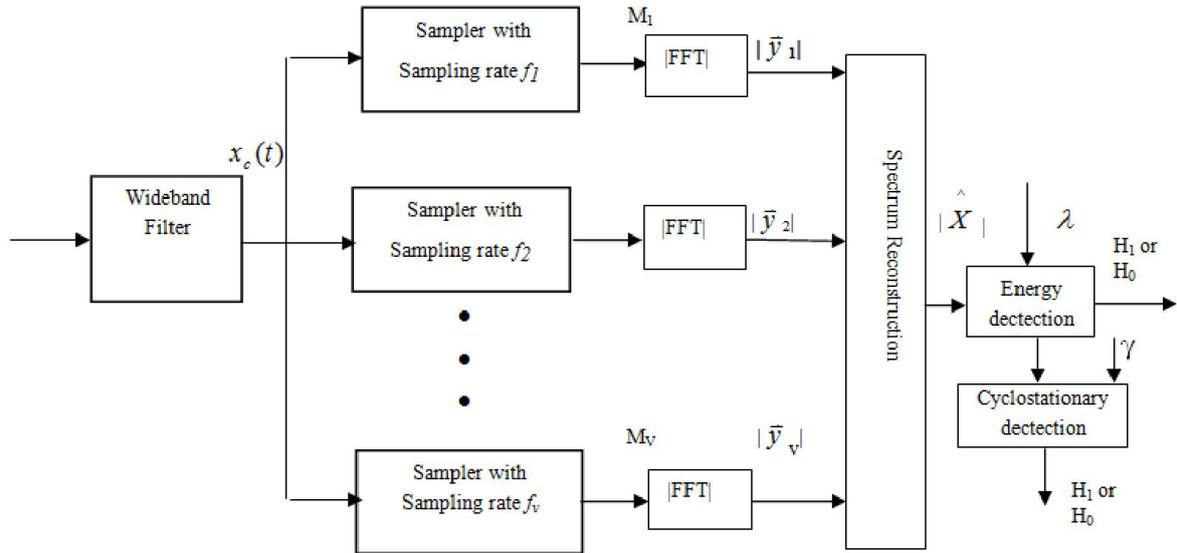


Fig.2. Schematic Illustration of the Two-stage Multirate Asynchronous sub-Nyquist sampling system in one CR node

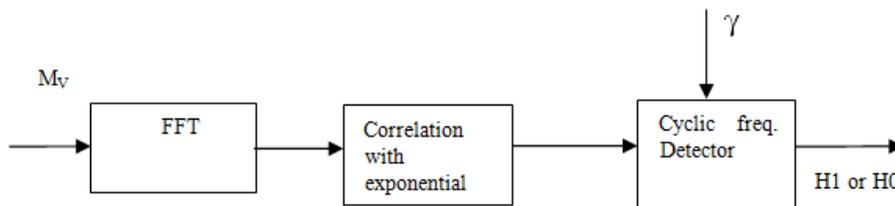


Fig.3. Diagram of CS-based spectrum sensing when using the cyclostationary detection approach

$$Y_i(f) = \frac{f_i}{f_s} \sum_{l=-\infty}^{\infty} X(f + lf_i), f + f_i \left[-\frac{W}{2}, \frac{W}{2} \right], \quad (2)$$

$$Y[m] = \frac{M_i}{N} \sum_{l=-\infty}^{\infty} X[m + lM_i], m + lM_i \in \left[-\left\lfloor \frac{N}{2} \right\rfloor, \left\lfloor \frac{N}{2} \right\rfloor \right], \quad (3)$$

$$= \frac{M_i}{N} \sum_{l=-\infty}^{\infty} X[n] \sum_{l=-\infty}^{\infty} \delta[n - (m + lM_i)],$$

$$m \in \left[-\left\lfloor \frac{M_i}{2} \right\rfloor, \left\lfloor \frac{M_i}{2} \right\rfloor \right] \quad (4)$$

Where $\lfloor a \rfloor$ is the floor function that gives the largest integer not greater than a, and $\delta[n]$ denotes the Dirac delta function that is zero everywhere except at the origin, where it is one. In matrix form, we write

$$\vec{Y}_i = \frac{M_i}{N} \Phi_i \vec{X}, \quad (5)$$

Where the elements of $\Phi_i \in R^{M_i \times N}$ ($M_i < N$) can be

$$\text{represented by } \Phi_i \left[m + \left\lfloor \frac{M_i}{2} \right\rfloor + 1, n + \left\lfloor \frac{N}{2} \right\rfloor + 1 \right] = \sum_{l=-\infty}^{\infty} \delta[n - (m + lM_i)].$$

Multi-Rate Asynchronous Sub-Nyquist Sampling

Due to the sub-Nyquist sampling in each sampling branch, we have to consider the effects of spectral aliasing. Even then, when the spectral sparsity $k \ll N$ and the sampling rate satisfy $M_i \sim O(\sqrt{N})$, the probability of signal overlap is very small [28]. In such case, consider two cases: no signal at a particular value m and one signal at m . For a signal to be non-overlapped, the following equation has to be satisfied:

$$|\vec{Y}_i| = \left| \frac{M_i}{N} \Phi_i \vec{X} \right| = \frac{M_i}{N} \Phi_i |\vec{X}|, \quad (6)$$

The elements of Φ_i are either zeros or ones, hence each frequency bin of \vec{Y}_i has no signal overlap from \vec{X} .

The proposed system doesn't require exact synchronization between sub-Nyquist samplers. The time offset between sampling branches need to be sufficiently small. Because energy detection is used in the first stage for spectrum recovery, there is an advantage of applying the proposed technique to co-operative CR networks with same spectral environments. For successful spectrum reconstruction Mutual coherence [17] μ to be minimum

$$\mu = \max_{l \neq z \in [1, N]} \left| \left\langle \hat{\phi}_l, \hat{\phi}_z \right\rangle \right| \quad (7)$$

Mutual coherence of the measurement matrix Φ at v samples is given by,

$$\mu = \max_{l \neq z} \left| \left\langle \hat{\phi}_l, \hat{\phi}_z \right\rangle \right| = \frac{1}{\nu} \tag{8}$$

When $\mu < \frac{1}{2k-1}$, the k -sparse signal can be successfully recovered. Thus, we know that if $\mu = \frac{1}{\nu} < \frac{1}{2k-1}$, i.e., $\nu > 2k-1$, the spectral magnitude $|\vec{X}|$ can be exactly reconstructed. The number of sampling branches $\nu > 2k-1$ in order to reconstruct the spectrum. Sensing error is also been calculated as the ratio of addition of difference between the estimated spectrum and the actual spectrum in both upper and lower bounds and the difference between the upper and lower bands.

Simulation Results

It is assumed that there are ν sub Nyquist sampling branches used for sampling the received signal $x_c(t)$ at various sub Nyquist rates. In the i^{th} sampling branch the received signal denoted as $x_c(t)$ is sampled at sub Nyquist rate f_i , and is deserved for time T. $x_c(t)$ is denoted as,

$$x_c(t) = \sum_{i=1}^{N_b} \sqrt{E_i} B_i * \sin(B_i(t-\Delta)) * \cos(2\pi f_i(t-\Delta)) + z(t) \tag{9}$$

Where $\sin c(x) = \frac{\sin(\pi x)}{\pi}$, Δ denotes a random time offset between sampling branches, and $z(t)$ is unit additive white Gaussian noise (AWGN). The received power E_i don't change for the entire T but vary randomly from branch to branch. We assume there are $N_b = 30$ non-overlapping sub bands, whose bandwidths $B_i = 0.5 \sim 5MHz$, with carrier frequencies $f_i = 0 \sim 20GHz$. Here fix $u=23$ sampling branches with different sub Nyquist rates $M_i \sim O(\sqrt{N})$.

After reconstruction using (7) the presence or absence of user is decided by combined energy detection and cyclostationary detection. Fig.4. depicts the Receiver Operating characteristic curve for Probability of detection at various SNR for fixed bandwidth factors. Fig.5. shows the effect of sparsity level k

and the compression ratio $\frac{M}{N} \left(M \triangleq \frac{\sum_{i=1}^{\nu} M_i}{\nu} \right)$ on the detection performance.

From the fig it is inferred that lower the sparsity level better is the detection performance. Also fig depicts that the higher the compression ratio lower is the probability of false alarm and higher is the probability of detection. Fig.6. shows the comparison of MSE reconstruction of the spectrum for the compression ratio below 0.25 the proposed combined MASS can achieve smaller MSE when compared to both CS based approach and the future rate of innovation approach. Fig.7. shows the effect of lack of synchronization between sampling branches, time effects considered here is the range $0 \sim 0.8 \mu s$

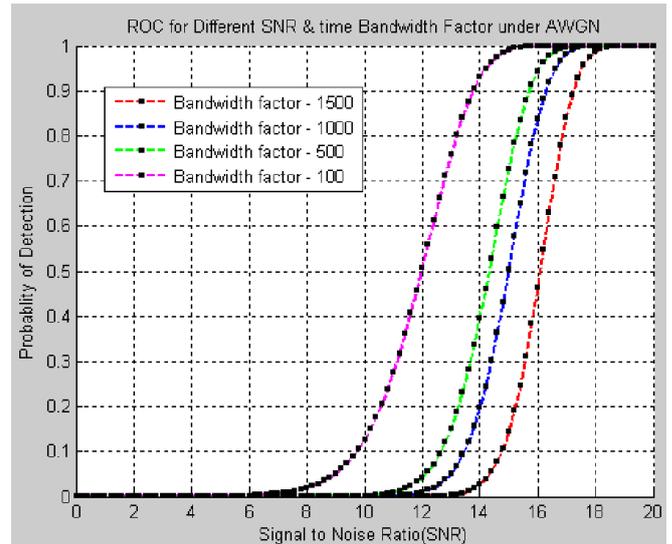


Fig.4. Receiver Operating characteristic curve for Probability of detection at various SNR for fixed bandwidth factors

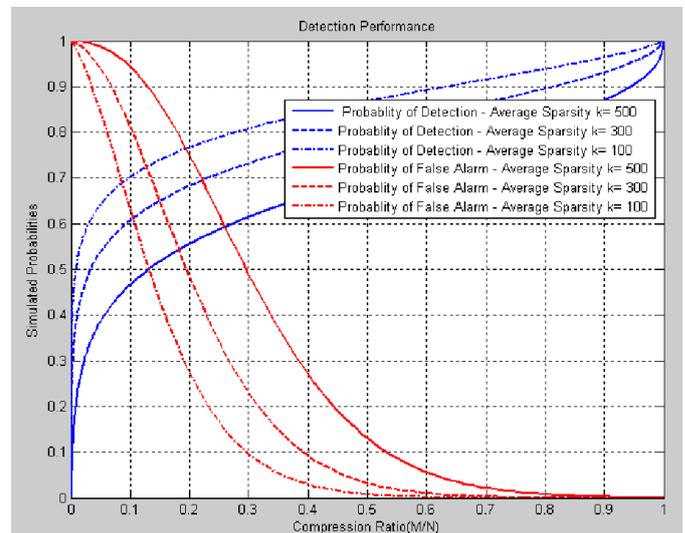


Fig 5. Shows the influence of sparsity level k and the compression ratio on the detection performance of Combined MASS with average SNR=10Db

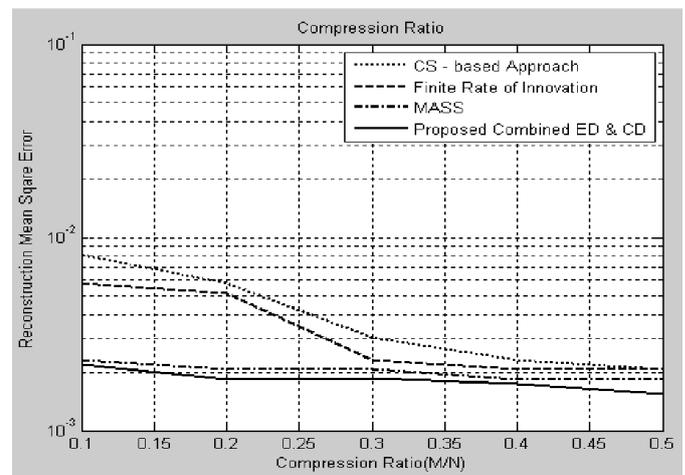


Fig.6. Comparison between the proposed system and the existing approaches in terms of MSE reconstruction of the spectrum and the compression ratio

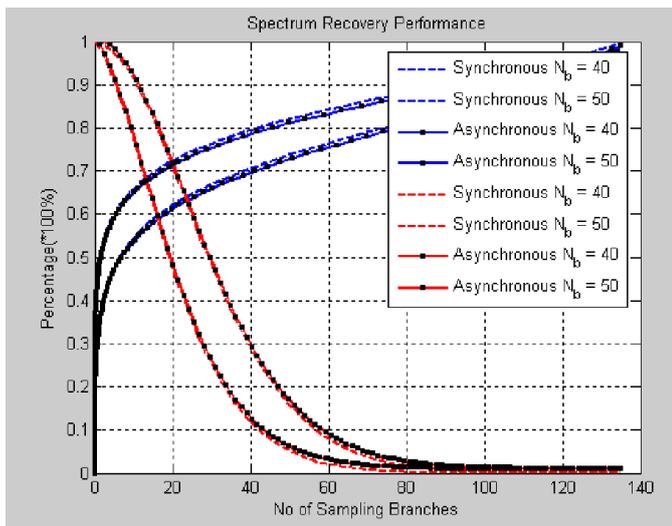


Fig.7. Comparison of spectrum recovery performance of synchronous samplers and asynchronous samplers with Nb=40 and Nb=50 over AWGN channels with average SNR=10dB

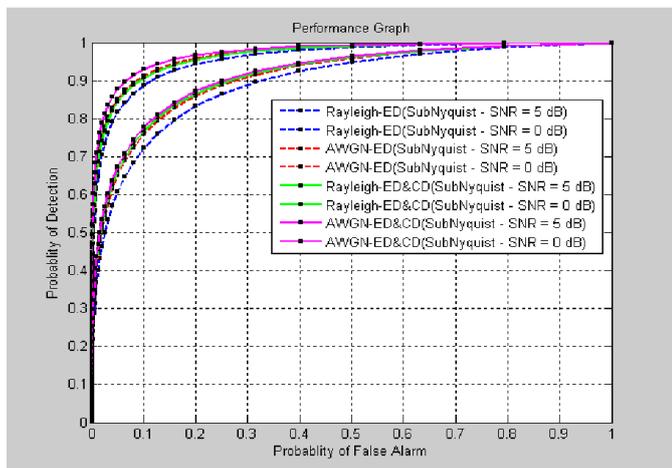


Fig.8. The detection performance of combined MASS over AWGN and Rayleigh fading channels with the number of subbands Nb=30

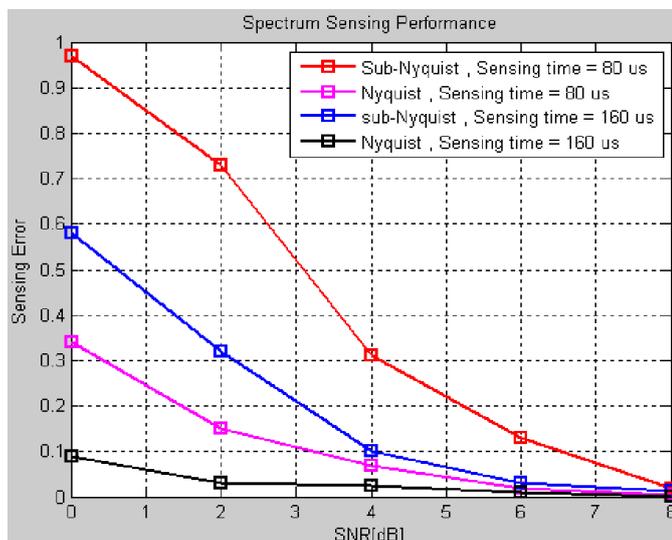


Fig.9. The detection performance of combined MASS as a function of SNR at different sensing times 80us and 160 us when the downsampling factor L=20

and the observation time is μ s. From the figure it is inferred that the both the asynchronous and synchronous samplers perform similarly when detection is considered. Fig illustrates that as the number of sampling branches increases better spectrum sensing performance can be achieved. Because with more sampling branches a higher accuracy in spectrum recovery can be obtained. Fig.8 shows the detection performance of combined MASS over AWGN and Rayleigh fading channels. When signal to noise ratio is zero decibels (db) detection performance of combined MASS over fading channels, is roughly as same as non-fading AWGN channels. This is because the strength of the signal is mostly masked by noise. Also, the detection performance of combined MASS over AWGN channel is better than that of fading channel when SNR=5db. Fig. also shows that the performance of combined MASS over Rayleigh fading channel is poorer in comparison to the case of AWGN channel.

We provide a simulation setup for calculating sensing time by varying the down sampling factor. We assume that there are $M=5$ sub bands in the frequency range $(0, f_{max}) = [0, 1.0]$ GHz. hence the nyquist rate is $f_{max} = 1/T_o = 1$ GHz. the bandwidth of each sub band are 5MHz. The down sampling factor is chosen as $L=20$, corresponding to the Sub-Nyquist rate of $1/LT_o = 75$ MHz ($> B_{max} = 5$ MHz). FFT size considered here is $N=12000$ and 24000 corresponding to the sensing time $Z = NL T_o = 80$ us and 160 us. Fig.9 depicts the detection performance of combined MASS as a function of SNR at different sensing times 80us and 160 us when the downsampling factor $L=20$. From the figure it is inferred that longer sensing time leads to improved sensing accuracy hence reduced sensing error. A similar system possessing the same performance is CS based system. But the disadvantage of CS based approach is that it requires pseudo random sequence generator as compression devices at the each CR node. Also lack of synchronization leads to false spectral recovery. But for two-stage MASS no separate device is required for generating measurement matrix and the synchronization requirements also relaxed. The spectral recovery and robustness against fading is improved in combined MASS even at very low SNR with slight increase in implementation complexity which is contributed by cyclostationary detector.

Conclusion

In this paper, we have proposed a two-stage wide band SS system i.e. two stage MASS. The basic procedure of sampling the wide band SS at different sub Nyquist rates by parallel low rate sampler is accomplished followed by spectrum recovery and detection by energy detection and cyclostationary detection. The prior knowledge needed is an upper bound on the number of sampling branches and the frequency range of interest CS based spectrum recovery is followed. Simulation results have shown that improved MASS has outstanding compressive capability compared with Nyquist sampling system. Compared with other available sub Nyquist sampling system of CS based system the two-stage MASS has been seen to be against lack of synchronization and have superior performance in AWGN and Rayleigh fading scenarios.

REFERENCES

[1] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun.*, vol. 6, 1999.

- [2] F.C.C., "Spectrum policy task force," *IEEE Trans. Inf. Forens. Security*, pp. 02-155, Nov 2002.
- [3] F.C.C., "In the matter of unlicensed operation in the TV broadcast bands," *Second Report and order and Memorandum opinion and Order*, no. FCC-08-260A1, Nov.2008.
- [4] C. Bazelon, "Licensed or unlicensed: The economic considerations in increment spectrum allocations," *New Frontiers in Dynamic Spectrum Access Networks, 2008 DySPAN 2008. 3rd IEEE Symposium on*, pp1-8, Oct. 2008.
- [5] Mitola, J., I. and J.Maguire, G.Q., "Cognitive Radio: making software radios more personal," *IEEE Pers., Commun.* Vol6, no 4, pp. 13-18, Aug. 1999.
- [6] I.F. Akyildiz, W.Y. Lee, M.C. Vuran, and S.Mohanty, "Nextgeneration/dynamic spectrum access/Cognitive radio Wireless Networks: a Survey," *Comput. Netw.*, vol 50, no.13, pp.2127-2159, 2006
- [7] C.R. Stevenson, G. Chouinard, Z. Lei, W.Hu, S.J. Shellhammer, and W. Caldwell, IEEE 802.22: The first cognitive radio wireless regional area network standard," *IEEE Commun. Mag.*, Jan 2009.
- [8] Y.-C. Liang, K.-C. Chen, G. Y. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.
- [9] S. Chaudhari, V. Koivunen, and H.V. Poor, "Autocorrelation-based decentralized sequential detection of OFDM signals in cognitive radios," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2690–2700, Jul. 2009.
- [10] Md. AliHussain, Md. Mastan and Syed Umar, "Quality of Service Issues in Wireless Ad-Hoc Network (IEEE 802.11B)", *International Journal of Computer Science & Information Security (IJCSIS)*, Vol.8 No.3 June 2010, ISSN: 1047 – 5500.
- [11] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," *IEEE Signal Process. Mag.*, vol. 29, no. 3, pp. 101–116, May 2012.
- [12] H. Sun, A. Nallanathan, J. Jiang, D. Laurenson, C.-X. Wang, and H. Poor, "A novel wideband spectrum sensing system for distributed cognitive radio networks," in *Proc. IEEE Global Telecommun. Conf.*, Houston, TX, Dec. 2011, pp. 1–6.
- [13] Z. Tian and G. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," in *Proc. IEEE Cognitive Radio Oriented Wireless Netw. Commun.*, Mykonos Island, Greece, Jun. 2006, pp. 1–5.
- [14] Z. Quan, S. Cui, A. H. Sayed, and H. V. Poor, "Optimal multiband joint detection for spectrum sensing in cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 57, no. 3, pp. 1128–1140, Mar. 2009.
- [15] Z. Quan, S. Cui, A. H. Sayed, and H. V. Poor, "Wideband spectrum sensing in cognitive radio networks," in *Proc. IEEE Int. Conf. Commun.*, Beijing, China, May 2008, pp. 901–906.
- [16] B. Farhang-Boroujeny, "Filter bank spectrum sensing for cognitive radios," *IEEE Trans. Signal Process.*, vol.56, no.5, pp.1801–1811, 2008.
- [17] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.
- [18] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.
- [19] H. Sun, A. Nallanathan, J. Jiang, and H. V. Poor, "Compressive autonomous sensing (CAsE) for wideband spectrum sensing," in *Proc. IEEE Int. Conf. Commun.*, Ottawa, Canada, Jun. 2012, pp. 5953–5957.
- [20] Z. Tian and G. Giannakis, "Compressed sensing for wideband cognitive radios," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, Honolulu, HI, Apr. 2007, pp. 1357–1360.
- [21] Y. Polo, Y. Wang, A. Pandharipande, and G. Leus, "Compressive wide-band spectrum sensing," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, Taipei, Taiwan, Apr. 2009, pp. 2337–2340.
- [22] F. Zeng, C. Li, and Z. Tian, "Distributed compressive spectrum sensing in cooperative multihop cognitive networks," *IEEE J.Sel. Topics Signal Process.*, vol. 5, no. 1, pp. 37–48, Feb. 2011.
- [23] F. Zeng, Z. Tian, and C. Li, "Distributed compressive wide band spectrum sensing in cooperative multi-hop cognitive networks," in *Proc. IEEE Int. Conf. Commun.*, Cape Town, South Africa, May 2010, pp. 1–5.
- [24] Z. Tian, E. Blasch, W. Li, G. Chen, and X. Li, "Performance evaluation of distributed Compressed wideband sensing for cognitive radio networks," in *Proc. Int. Conf. Inf. Fusion*, Cologne, Germany, Jul. 2008, pp. 1–8.
- [25] H. Sun, D. Laurenson, and C.-X. Wang, "Computationally tractable model of energy detection performance over slow fading channels," *IEEE Commun. Lett.*, vol. 14, no. 10, 9 pp.924–926, Oct. 2010.
- [26] W. A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory*. Upper Saddle River, NJ: Prentice-Hall, 1986.
- [27] D. L. Donoho and M. Elad, "Optimally sparse representation in general (nonorthogonal) dictionaries via ℓ_1 minimization," *Proc. Nat. Acad. Sci. USA*, vol. 100, no. 5, pp. 2197–2202, 2003.
- [28] Hongjian Sun, Wei-Yu Chiu, Jing jiang, Srugam Nallanathan and H.Poor, "Wideband Spectrum sensing with sub-Nyquist sampling in Cognitive Radios" *IEEE Transactions on Signal Processing*, vol.60, no.11, pp.6068-6073, Nov-2013.
